SOULÉ'S

INTERMEDIATE — ✳ — PHILOSOPHIC

✳RITHMETIC.

SOULÉ'S

Intermediate, Philosophic Arithmetic,

EMBRACING

ORAL AND WRITTEN PROBLEMS

INVOLVING THE GREAT PRINCIPLES OF THE SCIENCE OF NUMBERS,
AND THEIR APPLICATION TO PRACTICAL COMPUTATIONS.

THE PHILOSOPHIC SYSTEM

IS USED THROUGHOUT THIS WORK.

THIS SYSTEM GIVES *Strength, Acuteness, Expansion*, AND *Depth* TO
the Mind, AND THUS PREPARES IT FOR ACTIVE SERVICE IN
THE *field* OF *practical Mathematics* AND UPON THE
HIGHEST PLANES OF THOUGHT.

CONTRACTIONS IN NUMBERS

AND PRACTICAL WORK IN PERCENTAGE, INTEREST, MENSURATION
OF SURFACES AND SOLIDS, CONSTITUTE SPECIAL
FEATURES OF THE BOOK.

IT IS REPLETE WITH *new practical problems*, AND IT *sparkles* WITH
THE RAREST GEMS OF THE SCIENCE OF NUMBERS.

THE METRIC SYSTEM

IS ALSO FULLY PRESENTED AND CLEARLY ELUCIDATED.

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PREFACE.

PROGRESS AND EVOLUTION is a law of the Universe, and while it maintains throughout all the fields of Education in the domain of Practical Mathematics, it is working wondrous and beneficent changes.

The science of Arithmetic is one of the queenly products of the human mind. It is the front door to the grand Temple of Mathematics, in which are treasured some of the most beautiful principles of logic and the most profound truths contained in the vast kingdom of thought.

With man, Arithmetic has come down the evolving centuries compounding itself with his thought and occupation, and unfolding its beauties as the human race advanced on the planes of civilization. Receiving new inspirations and contributions from profound thinkers as the ages passed by, it has attained to the dignity of a queenly science before whose throne the logic of mind and the industries of the world worship. The erudite Hindoo and the learned Egyptian communed at its shrine. Before it, Thales and Pythagoras reasoned with uncovered heads; at its feet, Plato laid the genius of his great mind; and Aristotle and Archimedes employed their eloquence and philosophy in unfolding its mysteries. Newton and La Place adorned it with modern thought, and Pestalozzi leavened it with the great principle of analytic reason.

This Pestalozzian process of analytical reasoning has been, during the past 70 years, utilized by more than a hundred authors. Colburn, Emerson, Hobart, Stewart, McCormick, Ray, Greenleaf, Dodd, Perkins, Brooks, Davies, Robinson, Thompson, and all other authors since 1810, have used in various modified forms, more or less, the analytic methods of

Pestalozzi, and thus they have moved on with the evolving world and performed a measure of efficient missionary work in the pagan realms of practical mathematics.

The analytical method of Pestalozzi is the foundation of the golden Philosophic System presented in this and the author's other works on the Science of Numbers.

In the higher evolution of this system, the author employs and combines the three great processes of acquiring knowledge and eliciting truth, viz.: comparison, analysis, and synthesis. These three processes constitute the trinity of Mathematics; and by an ingenious use of them the author is believed to have advanced the science to loftier planes and to more rational methods than were ever before achieved. In no ancient or modern work on numbers have comparison, analysis, and synthesis been woven into such a chain of logical and philosophical reasoning as is herein presented.

By the Philosophic System all the difficult scientific statements of true proportion are avoided, all the objections raised against the purely analytic method are obviated, the somewhat entangling cause and effect method is surpassed, and that monstrosity of a process which "considers whether the answer is to be greater or less than the third term" is anathematized and consigned to the shades of the dark ages of arithmetic.

By the Philosophic System, all the arbitrary rules which overload the organ of memory and prevent the expansion of the higher faculties of causality and comparison, are abandoned, and the reasoning organs of the mind are brought into action, thereby capacitating the learner not only to produce the result of problems, but to observe fine distinctions, reason logically, and deduce correctly; thus qualifying for a high plane of usefulness, not only in the fields of mathematics, but in all the other vocations of life.

The Philosophic System is believed to be the most valuable improvement yet made to impart a thorough knowledge of the principles of numbers and to capacitate the student to

utilize the same in the practical affairs of business life. But notwithstanding its superiority and the fact that its advocates include many of the most profound mathematical minds, yet, like every other improvement or discovery in education, commerce, art, or science, it has some opponents and is regarded with indifference by those who are satisfied with the non-progressive and non-reasoning methods of past ages.

For nearly thirty years the author has labored with tongue and pen in the development of the Philosophic System of Arithmetic, and has tested its superior merits in the school room and lecture hall with over 7000 students; and from a full knowledge of its advantages, he conscientiously assures his co-laborers in the mathematical field of education, that a more thorough knowledge of the science of numbers can be imparted, and in far less time, by this system than by the usual methods and systems of work.

It stands without a peer in the annals of mathematics, and it is believed that the day is not far distant when its banner will wave in triumph from every spire, pinnacle, and dome of the Temple of Practical Mathematics.

In the above criticism, it will be observed that the charges are made against methods, and not against authors. The author of this work recognizes worth, scholarship, and genius in all co-authors. His attacks are made against old non-reasoning, non-progressive methods—methods which are as valueless to practical and progressive mathematics, as were the golden calves and bronze idols in the Oriental Temples to true and progressive religion. It is believed that such methods should be dethroned and demolished, and that in their place and upon their ruins should be erected methods founded upon reason and in consonance with the evolving thought of the age in which we live.

This work is designed as an introductory work to the Philosophic System and to the author's advanced treatise on Practical Mathematics, which is now undergoing revision.

It is especially prepared to meet the requirements of elementary and higher intermediate classes, and also contains much practical work of rare value to advanced students. It is believed to possess superior merit on the following points: 1. In the arrangement and character of the mental exercises and the logical methods of mental training. 2. In the extent, variety, practical and scientific character of the problems. 3. In the philosophical elucidation of subjects. Logical reasons are given for multiplying and dividing both abstract and denominate numbers, whole and fractional. This reasoning is contained in no other work, ancient or modern.

. The tables of Weights and Measures are far more extended than in any other work of corresponding grade.

The subjects of Percentage, Interest, and Mensuration of Surfaces and Solids are presented and philosophically elucidated in a manner never before published.

The Metric System is a special feature of the work, and is more fully treated in all particulars, addition, subtraction, multiplication, division, reduction, etc., than in any other work before the public.

Bills and Invoices of various forms for many departments of business constitute a special feature of the work.

The work contains an appendix embracing contractions in numbers. This part is replete with the most valuable methods known of handling whole and fractional numbers.

In the selection of the material and the elements for its problems, this work does not present the toys and playthings of the nursery, nor does it confine itself to the articles bought and sold on 'change. Instead of gyrating in the non-practical and non-progressive paths described by its hundreds of predecessors, it has diverged into new channels and derived the facts and elements of many of its problems from Geography, History, and Chronology; from Educational and Commercial Statistics; from Natural Philosophy, Astronomy, Geology, and Chemistry; from Anatomy, Physiology, and

Hygiene; from the Statistics of the Municipal, State, and National Governments; and from many other departments of scientific knowledge. Through this means, the work is rendered far more interesting, and as it brings into use organs of the mind different from those which consider the computation of numbers only, it thereby imbues the mind of the learner with much valuable information without the cost of additional study or the expenditure of additional time.

The entire work sparkles with the rarest gems of the science of numbers, and teaches that a new truth is better than an old error, and that facts and reasons are better than fallacious theories, however ancient or renowned.

To obviate a formal introduction to the work, the subject matter usually classed under that head has been presented in remarks and discussions throughout the work, in connection with the various topics treated.

By reason of this arrangement, the remarks and discussions have been made more pertinent, and will be read at the time when the student most needs them.

Review questions of the most searching kind follow each subject treated. In compliance with the wishes of a large number of teachers, brief general directions have been given for solving problems, independently of the reasoning processes.

These general directions follow immediately after the full elucidation of each subject, and it is believed they will be of service to the learner.

The work is a complete revision and an enlargement of the author's Introductory Philosophic Arithmetic, which has received the highest commendations from both teachers and pupils.

The author avails himself of this occasion to extend his thanks to his associate instructors for their kindly aid in proof-reading. To his short-hand amanuensis, Miss Carrie McGuigin, he acknowledges his indebtedness for valuable services. To his friend and former teacher, W. A. Beer, he

extends his thanks for assistance in preparing some of the problems in Denominate Numbers. To his faithful young friend and teacher, W. W. Weiss, he extends his earnest thanks for valued services in proof-reading and in re-working problems. These services were cheerfully rendered and will be gratefully remembered.

Soliciting for the work a thorough examination and a just measure of its merits, with the earnest hope that it may prove acceptable, and be of service in unfolding the principles of the beautiful science of numbers, and aid in advancing the interests of the rising generation, it is now submitted to the public.

THE AUTHOR.

NEW ORLEANS, MAY 14. 1886.

SOULÉ'S

INTERMEDIATE, PHILOSOPHIC ARITHMETIC.

DEFINITIONS.

1. **A Definition** is the meaning or import of a word or words expressed by other words.

2. **Science** is classified knowledge.

3. **Art** is the practical application of the principles of science, according to prescribed methods.

4. **Quantity** is anything that can be increased or diminished.

5. **A Unit** is a single thing of whatsoever denomination or nature, as one orange, one pound, etc.

6. **A Number** is a unit or a collection of units.

7. **Like Numbers** are those which express units of the same kind. Thus: five apples and six apples, seven bales and nine bales, are like numbers.

8. **Unlike Numbers** are those which express units of different kinds. Thus: four, seven hours, ten peaches, are unlike numbers.

9. **The Unit of a Number** is one of the collection of units forming that number. Thus the unit of *twelve* hats is *one* hat; of *five* is *one;* of *four* pounds, *one* pound.

10. **Numbers** are expressed by words, figures, or letters.

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11. An Abstract Number is one in which the kind of unit or quantity is not designated. Thus: three, four, five, etc.

12. A Denominate or Concrete Number is one in which the kind of unit is designated. Thus: two pounds, five yards, nine dollars, etc.

13. A Compound Number is a denominate number expressed in two or more denominations. Thus: five years, four months, and eight days; two miles, five furlongs, and ten rods; two yards, two feet, and five inches.

14. An Arithmetical Complement of a Number is the difference between the number and a unit of the next higher order. Thus: 3 is the arithmetical complement of 7; 26 is the arithmetical complement of 74; 19 is the arithmetical complement of 981.

15. An Arithmetical Supplement of a Number is the difference between the number and a unit of the next lower order. Thus: 7 is the arithmetical supplement of 17; 12 is the arithmetical supplement of 112.

16. A Problem is a question proposed or given for solution.

17. An Axiom is a self-evident truth.

18. The Premise is the proposition, declaration, truth, or fact which is asserted as the basis or predicate of a question or problem.

19. A Theorem is a truth to be proved.

20. A Philosophic Solution is, in this work, a full numerical statement showing, step by step, how the result of a problem is obtained, with a logical reason for each conclusion reached in the solution.

This process of solution and reasoning from the

premises and facts of all problems constitutes the
Philosophic System of Arithmetic.

It is the complete evolution of the Pestalozzi
system of reasoning given to the world nearly
ninety years ago, and upon which the author of
this book has labored for more than a quarter of a
century.

21. A Solution Statement, or an Operation, is
a statement of the figures employed in solving a
problem.

22. A Formula is the expression, by symbols, of
general principles applicable to the operations of
particular problems.

23. Philosophy is the knowledge of phenomena
as explained by, and resolved into, causes and
reasons, powers and laws.

24. Arithmetic is the science of numbers: or to
define it more extendedly, it is that branch of
Mathemathics which treats of the properties and
relations of numbers when expressed by the aid of
figures, either singly or combined. These princi-
ples and relations of numbers combined with the facts
relating to problems, are applied, by the reasoning
powers of man, to the solution of all numerical
problems of business affairs and of practical life.

25. Figures.—Figures in arithmetic, are charac-
ters used to represent numbers. The ten Arabic
figures which we use, are

Naught or Cipher	One	Two	Three	Four	Five	Six	Seven	Eight	Nine
0	1	2	3	4	5	6	7	8	9

By properly combining these ten figures, all pos-
sible numbers may be represented.

The 1, 2, 3, 4, 5, 6, 7, 8, and 9 are sometimes called
digits. They are also called the *significant figures,*
because each signifies a number when alone.

The naught (0) is so called, because by itself it

does not signify or express any number. It expresses number only when used in connection with other figures.

26. Value of Figures.—Figures have two values; a *simple* and a *local* value: thus when we write 1, independent of other figures, it has only a simple value, representing one unit; but when we write it to the left of another figure or figures, thus, 13 or 145, it has a *local* value as well as a simple value. This local value depends on the scale or system of numbers employed and its location in the scale.

27. A Scale or System of Numbers, in Arithmetic, is a succession of units, increasing and decreasing according to some established custom in the operations of numbers. Thus there is the binary, the trinary, etc., etc., and the decimal, the duodecimal, etc., etc.

28. In these four named different scales the value of three ones (111) would be as follows: In the *binary scale or system* the first 1 on the right is *one;* the second 1 is *two;* the third 1 is *four;* making *seven* altogether.

29. In the *trinary system or scale,* the first 1 on the right is *one;* the second 1 is *three;* and the third 1 is *nine;* making *thirteen* altogether.

30. In the *decimal scale* the first 1 is *one;* the second 1 is *ten;* and the third 1 is *one hundred;* making in all *one hundred eleven.*

31. In the *duodecimal scale* the first 1 is *one;* the second 1 is *twelve;* and the third 1 is *one hundred forty-four;* making in all *one hundred fifty-seven.*

From the foregoing we see that the *system* or *scale* derives its name from the ratio of value given to each succeeding figure from the right toward the left.

32. The **Decimal Scale or System** is one in which the rate or law of increase and decrease is always *ten.* This system is in general use and derives its name from the Latin word *decem,* which means *ten.*

33. Order of Figures.—The successive places occupied by figures are called orders. Thus in the Decimal System, a figure in the first place is called a figure of the *first* order, or of the order of *units;* a figure in the second place is a figure of the second order, or of the order of *tens;* in the third place, of the third order, or of the order of *hundreds;* and so on, each figure next to the left belonging to a distinct order, the unit of which is *tenfold* the size or value of a unit of the order of the figure on its right.

34. From the above we see 1st; that *ten* units of any order in a number, in the Decimal System, make *one* unit of the next higher order.

2nd. That moving a figure one place to the left, increases its representative value *tenfold.*

3rd. That moving a figure one place to the right, decreases its representative value *tenfold.*

35. Notation is a method of *writing* numbers. There are two systems, the *Arabic* and the *Roman.*

36. By the **Arabic** Notation, numbers are expressed or written by *figures.* This system is in general use and is so called because it was introduced into Europe by the Arabians, in the 10th century.

37. By the **Roman** Notation, numbers are expressed or written by letters. This system is now used chiefly to number chapters and divisions of books. It is so called because it was used by the ancient Romans.

38. Numeration is naming the places which figures occupy. **Reading Numbers** is expressing their value orally.

There are two systems of numerating, or reading numbers,—the French and the English.

The *French* system is the one in general use in the United States and on the Continent of Europe.

The *English* system is that generally used in England and in English Provinces.

FRENCH SYSTEM OF NUMERATION.

39. The French system separates figures into groups of three figures each, and gives a different name to each period, thus:

Digit	Label
2	Hundreds of Octillions.
8	Tens of Octillions.
4,	Octillions.
5	Hundreds of Septillions.
6	Tens of Septillions.
1,	Septillions.
3	Hundreds of Sextillions.
3	Tens of Sextillions.
8,	Sextillions.
1	Hundreds of Quintillions.
7	Tens of Quintillions.
9,	Quintillions.
6	Hundreds of Quadrillions.
0	Tens of Quadrillions.
4,	Quadrillions.
9	Hundreds of Trillions.
3	Tens of Trillions.
2,	Trillions.
4	Hundreds of Billions.
8	Tens of Billions.
7,	Billions.
2	Hundreds of Millions.
6	Tens of Millions.
3,	Millions.
1	Hundreds of Thousands.
9	Tens of Thousands.
6,	Thousands.
5	Hundreds.
4	Tens.
8	Units.

The number is read two hundred eighty-four octillions, five hundred sixty-one septillions, three hundred thirty-eight sextillions, one hundred seventy-nine quintillions, six hundred four quadrillions, nine hundred thirty-two trillions, four hundred eighty-seven billions, two hundred sixty-three millions, one hundred ninety-six thousand five hundred forty-eight.

The periods above Octillions, in regular order, are Nonillions, Decillions, Undecillions, Duodecillions, Tredecillions, Quatuordecillions, Quindecillions, Sexdecillions, Septendecillions, Octodecillions, Novemdecillions, Vigintillions, etc.

ENGLISH SYSTEM OF NUMERATION.

40. The English system of numeration separates the figures into groups or periods of six figures each, and designates each period by a distinct name, thus:

The number is read two hundred eighty-four thousand five hundred sixty-one quadrillions, three hundred thirty-eight thousand one hundred seventy-nine trillions, six hundred four thousand nine hundred thirty-two billions, four hundred eighty-seven thousand two hundred sixty-three millions, one hundred ninety-six thousand five hundred sixty-eight.

Period	Digit	Place
Period of Quadrillions.	2	Hundreds of Thousands of Quadrillions.
	8	Tens of Thousands of Quadrillions
	4	Thousands of Quadrillions.
	5	Hundreds of Quadrillions.
	6	Tens of Quadrillions.
	1,	Quadrillions.
Period of Trillions.	3	Hundreds of Thousands of Trillions.
	3	Tens of Thousands of Trillions.
	8	Thousands of Trillions.
	1	Hundreds of Trillions
	7	Tens of Trillions.
	9,	Trillions.
Period of Billions.	6	Hundreds of Thousands of Billions.
	0	Tens of thousands of Billions.
	4	Thousands of Billions.
	9	Hundreds of Billions.
	3	Tens of Billions.
	2,	Billions.
Period of Millions.	4	Hundreds of Thousands of Millions.
	8	Tens of Thousands of Millions.
	7	Thousands of Millions.
	2	Hundreds of Millions.
	6	Tens of Millions.
	3,	Millions.
Period of Units.	1	Hundreds of Thousands.
	9	Tens of Thousands.
	6	Thousands.
	5	Hundreds.
	6	Tens.
	8	Units.

By examining and comparing the two systems, it will be observed that they are the same to the ninth figure or the hundreds of millions, but at that figure a variation is made. Hence, if we wish to know the value of numbers higher than hundreds of millions, when we hear them spoken or see them in print, we must know whether they are expressed according to the French or the English system of numeration.

THE ROMAN SYSTEM OF NOTATION.

41. In the Roman system of notation the letter I represents *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred*, and M, *one thousand.*

The intermediate and succeeding numbers are expressed according to the following principles:

FIRST.—Every time a letter is repeated, its value is repeated; thus II represents *two;* XX represents *twenty.*

SECOND.—When a letter of *lesser* value is placed before one of *greater* value, the lesser is taken from the greater; if placed after the greater, it is to be added to it. Thus, IV represents *four*, while VI represents *six;* XL represents *forty*, LX represents *sixty.*

THIRD.—A line or bar —, placed over a letter, increases its value a thousand *times.* Thus $\overline{I}$ represents *one thousand;* $\overline{X}$ represents *ten thousand;* $\overline{L}$ represents *fifty thousand;* $\overline{C}$ represents *one hundred thousand*, and $\overline{M}$ represents *one million.*

TABLE OF ROMAN CHARACTERS.

I	one.	XXV	twenty-five.
II	two.	XXVI	twenty-six.
III	three.	XXVII	twenty-seven.
IV	four.	XXVIII	twenty-eight.
V	five.	XXIX	twenty-nine.
VI	six.	XXX	thirty.
VII	seven.	XL	forty.
VIII	eight.	L	fifty.
IX	nine.	LX	sixty.
X	ten.	LXX	seventy.
XI	eleven.	LXXX	eighty.
XII	twelve.	XC	ninety.
XIII	thirteen.	C	one hundred.
XIV	fourteen.	CC	two hundred.
XV	fifteen.	CCC	three hundred.
XVI	sixteen.	CCCC	four hundred.
XVII	seventeen.	D	five hundred.
XVIII	eighteen.	DC	six hundred.
XIX	nineteen.	DCC	seven hundred.
XX	twenty.	DCCC	eight hundred.
XXI	twenty-one.	DCCCC	nine hundred.
XXII	twenty-two.	M	one thousand.
XXIII	twenty-three.	MM	two thousand.
XXIV	twenty-four.	MDCCCLXXXVI	1886.

Notation and Numeration,

OR

WRITING AND READING NUMBERS.

42. **To Write, or Notate Numbers,** begin at the left and write the figures of each period in their proper place, filling the vacant orders, if any, with naughts.

43. **To Read, or Numerate Numbers,** begin on the right and point the number into periods of three figures each. Then commence at the left and read in succession each period with its name.

44. **To Verify the Notation,** or **Writing,** numerate the number and see if it agrees with the number given.

45. EXERCISES IN NOTATION AND NUMERATION.

1. Write six; eight; ten; fourteen; forty-two; ninety-nine; one hundred nine.

2. Write five hundred twenty-two; twenty; one hundred twenty-two; thirty-seven.

3. Write five; fifty-five; fifty-six; sixty-five; forty-seven; seventy-four; eighty-four; forty-eight.

4. Write all the numbers between one and one hundred one; between one hundred fifty and two hundred thirty-three.

5. Write two; twenty; twenty-two; two hundred; two hundred two; two hundred twenty-two.

6. Write thirty; forty; fifty; sixty; seventy; eighty; ninety; one hundred.

7. Write one; ten; one hundred ten; two hundred; three hundred; four hundred four; five hundred fifty.

8. Write six hundred; seven hundred; eight undred; nine hundred; ten hundred.

9. Write one thousand; two thousand; three thousand three; three thousand three hundred; three thousand thirty; three thousand three hundred three; three thousand three hundred thirty-three.

10. Read the following numbers:

307	1328	34546	142462	81562111
170	2813	10010	555055	27624555
200	3218	40101	606060	11111110
220	8123	19703	770077	40404040
202	7890	88888	809010	3333333333
322	9087	90909	100107	1050607082

11. Write 1 unit; 1 ten; 1 hundred; 1 unit and 1 ten; 1 hundred, 1 ten, and 1 unit.

12. Write 1 unit, 2 tens, and three hundreds; 4 units, 5 tens, and 6 hundreds; seven hundreds, 8 tens, and 9 units.

13. Write 0 units, 1 ten, and 0 hundreds; 4 units, 0 tens, 0 hundreds, and 4 thousands.

Write the following numbers in figures:

14. One thousand, six hundred ninety-four.

15. Eighteen hundred seventy-seven.

16. Twenty-four hundred six.

17. Three hundred forty-one thousand, twenty-two.

18. Sixty-five million, one hundred thirty-two thousand, three hundred eighty-seven.

19. Twelve billion, sixteen million, forty-three thousand, one hundred eleven.

20. Nine hundred thousand, three hundred fifty.

21. Six million, one hundred sixty-nine thousand, four hundred thirty-seven.

22. Seventy-six million, four hundred thousand, one hundred.

23. Twenty-two billion, one hundred three million, five hundred seventy-six thousand, one hundred two.

24. One hundred two trillion, one hundred twenty-five million, four hundred three.

25. Eight trillion, seven billion, seventy-six.

26. Write 208 million, 18 thousand, one unit.

27. Write 10 billion, 8 million, 103 thousand, eleven.

28. Write 100 sextillion, 1 quintillion, 100 quadrillion, 10 trillion, 111 billion, 1 million, 10 thousand, 10 units.

29. Write 87 million, 14; 5 thousand, 5.

30. Write eleven thousand, 11 hundred, 11; 16 thousand, 16 hundred, 16.

Write the following numbers in figures:

31. Four billion, fourteen million, nine.

32. One hundred one million, twenty thousand.

33. Sixty-seven trillion, seventy-six.

34. Nine thousand, nine hundred, ninety.

35. Five hundred twenty million, one.

36. Thirty thousand million, thirty.

37. One hundred eleven octillion.

38. Two hundred two vigintillion.

39. Fill all the orders of figures with 8's from the order of units to the order of octillions, both inclusive, point off, and read the same according to the French and the English systems of numeration.

Write in figures the following numbers, and numerate them according to the English system of numeration:

40. Four hundred twenty-three thousand, five hundred fourteen.

41. Six hundred nineteen thousand one hundred fifty-two million, twenty-one thousand forty-seven.

42. Fifty-three billion, two hundred twelve thousand twenty-six million, seventy-five thousand three hundred eighty-four.

43. 1342 trillion, 11122 billion, 14 million, 19 units.

44. 1 quadrillion, 1 trillion, 1 million, 1.

Read the following numbers:

XXXIV.	LI.	C.	CCC.	DC.	MMDCLI.
XXXV.	LXI.	CI.	CD.	DLCIX.	MMMXC.
XXXVI.	LXIX.	CX.	CDXIV.	M.	$\overline{\text{M}}$MCCX.
XLIX.	XC.	CL.	D.	MC.	$\overline{\text{MCXM}}$DX.

Write, in the Roman System of Notation, the following numbers:

9, 12, 14, 37, 49, 83, 108. 519, 1519, 14704, 88976, 13140363, 1001001001.

SYNOPSIS FOR REVIEW.

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Define the following words and phrases :

1. Definition. **2.** Science. **3.** An Art. **4.** Quantity. **5.** A Unit. **6.** A Number. **7.** Like Numbers. **8.** Unlike Numbers. **9.** The Unit of a Number. **10.** An Abstract Number. **11.** A Denominate Number. **12.** A Compound Number. **13.** An Arithmetical Complement of a Number. **14.** An Arithmetical Supplement of a Number. **15.** A Problem. **16.** An Axiom. **17.** The Premise. **18.** A Theorem. **19.** A Philosophic Solution. **20.** The Philosophic System. **21.** An Operation. **22.** A Formula. **23.** Philosophy. **24.** Arithmetic. **25.** Figures. **26.** Value of Figures. **27.** A Scale. **28.** The Binary Scale. **29.** The Trinary. **30.** The Decimal Scale. **31.** The Duodecimal. **32.** Decimal Scale or System. **33.** Order of Figures. **34.** Decimal System of Figures. **35.** Notation. **36.** Arabic Notation. **37.** Roman Notation Numbers. **38.** Numeration. **39.** French System of Numeration. **40.** English System of Numeration. **41.** Roman System of Notation. **42.** Write, or Notate Numbers. **43.** Read, or Numerate Numbers. **44.** Verify the Notation.

SIGNS AND SYMBOLS.

46. Signs and Symbols are used to abridge arithmetical operations. They also indicate some relationship existing among numbers, and what operation is to be performed.

The signs in general use in Arithmetic are as follows:

$$+, \; -, \; \times, \; \div, \; =, \; (\;), \; \text{or} \; \underline{\quad}, \; ., \; :, \; ::, \; \therefore, \; 16^2$$
$$\sqrt{}, \; ?, \; 1^\circ, \; 2^\circ, \; 3^\circ, \; ,.$$

47. The perpendicular or Greek Cross, $(+)$ is the sign of *Addition;* it is called *plus,* and is read *plus,* or *and.* It means more, and indicates that the numbers between which it is placed are to be added. Thus $7+9$ is read 7 plus 9, and means that 7 and 9 are to be added. When used after a number, thus, $5+$, which is read 5 plus, it means 5 and a small excess.

48. The horizontal line $(-)$ is the sign of *Subtraction,* and is called *minus.* It means *less,* and indicates, when placed between two numbers, that the one that follows it is to be taken from the one before it. Thus, $7-3$ equals 4.

49. The oblique or Saint Andrew's Cross $(\times)$ is the sign of *Multiplication.* It is read *multiply by,* or *times.* It indicates that the numbers between which it is placed are to be multiplied together. Thus, 7×5 is read, 7 multiplied by 5, or 5 times 7.

50. The horizontal line with a point above and below it, $(\div)$ is the sign of *Division.* It is read *divided by.* It indicates, when placed between two numbers, that the one before it is to be measured or divided by the one after it. Thus, $12\div4$ equals 3.

51. The parallel horizontal lines (=) are the sign of *Equality.* It is read *equals* or *is equal to.* It indicates that the quantities between which it is placed are equal. Thus, $5+3=12-4$. A statement of this kind is called an *equation*, because the quantity of $5+3$ is equal to $12-4$.

52. The ( ) or —— is the sign of *Aggregation.* The first is the *Parenthesis*, the second is the *Vinculum.* They are both used for the same purpose. They indicate that the numbers within the parenthesis or below the vinculum, are to be considered as *one quantity.* Thus $16-(5+4)=7$, or $16-\overline{5+4}=7$.

53. The single point or (.) is the *Decimal* sign. It indicates that the numbers which follow it are *tenths, hundredths*, etc. Thus, .5, .05 are read 5 *tenths, 5 hundredths.*

54. The (:) is the sign of *Ratio.* It is read, *is to* or *the ratio of.*

55. The (::) is the sign of *Proportion.* It is read, *as*, or *equal.* Thus, $3:6::5:10$, read 3 is to 6 as 5 is to 10.

56. The (16²) is the sign of *Involution.* The small figure to the right and top of the number indicates the power to which the number is to be raised. Thus, 8^2 indicates that 8 is to be raised to the second power or taken as a factor twice. Thus, $8\times8=64$. 8^3 indicates that the third power of 8 is required. Thus, $8\times8\times8=512$.

57. The Radical sign ($\sqrt{}$) is the sign of *Evolution.* It indicates that some root of a number is to be found. Thus $\sqrt{64}$ indicates that the *square root* of 64 is to be found. $\sqrt[3]{512}$ indicates that the *cube root* of 512 is required. $\sqrt[4]{4096}$ indicates that the *fourth root* of 4096 is to be extracted. The small

figure within the branches of the Radical sign is called the *index* and indicates what root is required.

Other signs of Evolution, now often used, are as follows:

thus, $256^{\frac{1}{2}}$ indicates that the square root is to be extracted.

$625^{\frac{1}{3}}$ indicates that the cube root is to be extracted.

$146^{\frac{3}{2}}$ indicates that the square root of the cube of the number is required.

58. The (.•.) is the sign of *Deduction*. It is read, *therefore, hence,* or *consequently.*

59. The *Interrogation* (?) is the sign meaning *what,* or *how many.* It signifies that the answer to the question asked is to be found.

60. The expressions 1°, 2°, 3°, denote *first, second, third,* etc.

61. The sign of the comma (,) is the sign of *Numeration.* It is used to separate large numbers into periods, to facilitate the reading of them.

SIGNS AND ABBREVIATIONS.

62. The following are the principal signs and abbreviations in general use among merchants and business men:

@ At.

%₀ Account.

1¹ One and one-quarter.

1² One and one-half.

1³ One and three-quarters

℣ Per.

℔ Pound (weight).

$ Dollar or dollars

¢ Cent or cents.

% Per cent. or per centum

Amt. Amount.

Bbl. Barrel.

Doz. Dozen.

B. L. Bill of Lading.

Blk. Black.

Shipt. Shipment.

Sunds. Sundries.

Dft. Draft.

Com. Commission

Do. The same

Co. Company.

Cr. Credit or creditor.

Dr. Debit or debtor.

Gal. Gallons.

Ps. Pieces.

Yds. Yards.

Fr't Freight.

Rec'd Received.

Pay't Payment.

Inst. This month.

Prox. The next month.

Ult. The last month.

£ Pound Sterling.

O. K. All Right.

Fr. Franc, French coin.

Fwd. Forward.

Bal. Balance.

Cons't Consignment.

Hhds. Hogsheads.

Mdse. Merchandise.

/ Shillings, thus ²/₆ two shillings and sixpence.

Mk. Marks, the German monetary unit.

√ Check mark, correct, approved.

⊕ Cifrao, used to separate the milreis from the reis in Brazil money.

17 doz. $\$_{10}^{4}$, $\$_{12}^{6}$, $\$_{15}^{7}$ =17 doz., 4 of which are at $10 per doz., 6 @ $12, and 7 @ $15.

8 doz. $\frac{2}{4}$ @ 5/, $\frac{5}{5}$ @ ⁴/₆ = 2 doz. No. 4 @ 5 shillings per doz., and 6 doz. No. 5 @ 4 shillings sixpence per dozen.

4

ADDITION.

63. Addition—*Increasing*—is the process of uniting two or more numbers of the same name or kind, into one equivalent number.

64. The number obtained by this process is called the **Sum** or **Amount.**

65. The **Sign of Addition** is a perpendicular cross, $+$, called plus; it means more; thus, $7 + 9$ is read, 7 plus 9, and indicates that 7 and nine are to be added. When used after a number, thus, $5 +$, which is read 5 plus, it means 5 and a small excess.

66. The **Sign of Equality** is $=$. It is read *equals*, or *equal to*, and denotes that the numbers between which it is placed are equal to each other; thus $7 + 9 = 16$ means that 7 and 9 added are equal to 16. The expression is read, 7 plus 9 equals 16.

67. A **Numerical Equation** is an equality between two numerical expressions, which though differing in form from each other, are equivalent. Each expression is called a term of the equation. Thus $5 + 8 = 13$ is a numerical equation in which the $5 + 8$ is called the first member of the equation and 13 the second member, and both are called the terms of the equation.

68. *Principle of Addition.* Numbers of the same kind, order, or character only, can be added. Thus we cannot add 2 apples and three oranges; or 5 pounds of sugar and 6 boxes of peaches; or 6 units and 5 hundreds; or $\frac{1}{2}$ and $\frac{3}{4}$, etc. We can only add apples to apples, oranges to oranges, sugar to sugar, peaches to peaches, units to units, hundreds to hundreds, halves to halves, fourths to

(2 6)

fourths, etc. We can *collect* together things of different kinds, apples, peaches, oranges, etc., but by collecting them together we do not increase the number or sum of either, and hence there is no addition.

69. ADDITION TABLES.

TABLE No. 1.

NOTE.—In learning these tables and in handling all numbers, all intermediate words and thoughts that occur between the numbers to be combined, should be omitted. Thus, instead of saying or thinking that 2 and 2 are 4, 3 and 5 are 8, etc., say or think 4; 8; etc.

EXPLANATION.—In this table we show 20 different combinations of the 9 significant figures, to produce results from 1 to 9. It may be said that three 1's make 3, three 2's make 6, etc., and that they are regular combinations; but we see by the table that two 1's are 2, and that two 2's are 4, etc. Hence, though the table does not contain all the possible combinations, it does contain all that are essential and of value in this connection.

1   1
―
2   1
    1
―
3   1
    2
―
4   2.1
    2.3
―
5   1.2.
    4.3
―
6   1.2.3
    5.4.3
―
7   1.2.3
    6.5.4
―
8   1.2.3.4
    7.6.5.4
―
9   1.2.3.4
    8.7.6.5

ADDITION TABLE No. 2.

10	1.2.3.4.5 9.8.7.6.5	EXPLANATION.—In this table we show the 25 different combina-
11	2.3.4.5 9.8.7.6	tions of the 9 significant figures,
12	3.4.5.6 9.8.7.6	the sum of which equals *ten* or more. To attain rapidity in add-
13	4.5.6 9.8.7	ing, it is absolutely necessary that the learner should be so familiar
14	5.6.7 9.8.7	with these combinations that he can instantly see the result with-
15	6.7 9.8	out adding, i. e. he must know the result by the combination, just as
16	7.8 9.8	he knows the value of 4 or 5, by the combination of lines forming
17	8 9	the figure, or as he knows the pronunciation of a word without
18	9 9	spelling it.

☞The rapid increasing and decreasing operations in the science of numbers, depend largely upon the capacity of the calculator to apprehend instantly and apply accurately, the result of two or more figures, no matter how they are to be combined. And the object of these tables is to aid the learner in acquiring the desired capacity.

70. ## ADDITION AND SUBTRACTION TABLE.

TABLE III.

	1 &'=9	1 &'=8	1 &'=7	1 &'=6	1 &'=5	1 &'=4	1 &'=3	1 &'=2
1 &"	1 " 9	1 " 8	1 " 7	1 " 6	1 " 5	1 " 4	1 " 3	1 " 2
2 &"	2 " 9	2 " 8	2 " 7	2 " 6	2 " 5	2 " 4	2 " 3	
3 &"	3 " 9	3 " 8	3 " 7	3 " 6	3 " 5	3 " 4		
4 &"	4 " 9	4 " 8	4 " 7	4 " 6	4 " 5			
5 &"	5 " 9	5 " 8	5 " 7	5 " 6				
6 &"	6 " 9	6 " 8	6 " 7					
7 &"	7 " 9	7 " 8						
8 &"	8 " 9							

EXPLANATION.

In this table, we present the 35 combinations of the significant figures, in which the difference between each is to be supplied by the learner. This is a very important table for rapid work in subtraction, by the addition method, and should receive careful attention.

71. ADDITION AND SUBTRACTION TABLE.
TABLE IV.

1 & ? = 100	26 & ? = 100	51 & ? = 100	76 & ? = 100
2 " " 100	27 " " 100	52 " " 100	77 " " 100
3 " " 100	28 " " 100	53 " " 100	78 " " 100
4 " " 100	29 " " 100	54 " " 100	79 " " 100
5 " " 100	30 " " 100	55 " " 100	80 " " 100
6 " " 100	31 " " 100	56 " " 100	81 " " 100
7 " " 100	32 " " 100	57 " " 100	82 " " 100
8 " " 100	33 " " 100	58 " " 100	83 " " 100
9 " " 100	34 " " 100	59 " " 100	84 " " 100
10 " " 100	35 " " 100	60 " " 100	85 " " 100
11 " " 100	36 " " 100	61 " " 100	86 " " 100
12 " " 100	37 " " 100	62 " " 100	87 " " 100
13 " " 100	38 " " 100	63 " " 100	88 " " 100
14 " " 100	39 " " 100	64 " " 100	89 " " 100
15 " " 100	40 " " 100	65 " " 100	90 " " 100
16 " " 100	41 " " 100	66 " " 100	91 " " 100
17 " " 100	42 " " 100	67 " " 100	92 " " 100
18 " " 100	43 " " 100	68 " " 100	93 " " 100
19 " " 100	44 " " 100	69 " " 100	94 " " 100
20 " " 100	45 " " 100	70 " " 100	95 " " 100
21 " " 100	46 " " 100	71 " " 100	96 " " 100
22 " " 100	47 " " 100	72 " " 100	97 " " 100
23 " " 100	48 " " 100	73 " " 100	98 " " 100
24 " " 100	49 " " 100	74 " " 100	99 " " 100
25 " " 100	50 " " 100	75 " " 100	

EXPLANATION.

We present this table to aid the learner in instantly seeing the difference between 100 and any number from 1 to 99. It is of special value in addition and subtraction, and all who expect to become rapid Calculators must be proficient in this character of work.

These tables constitute the alphabet of numbers, and render obsolete the disgusting and mind weakening practice of counting fingers, birds on limbs, ducks in ponds, apples on trees, or rabbits in the yard, etc., which is so often seen in primary arithmetics.

DRILL EXERCISES.

72. Name the *unit* result of the following numbers.

3	1	2	4	1	6	3	7	1	8	3
4	3	3	2	5	2	6	2	2	1	8

4	5	6	7	3	4	1	4	2	9	3
4	3	2	2	9	5	6	3	5	0	2

9	8	9	8	9	8	9	8	8	7	9	5	6
9	8	6	6	4	4	2	2	9	8	7	9	7

3	8	7	6	8	5	5	6	9	4	7	5
9	3	3	4	4	7	5	6	1	7	7	8

3	4	5	1	6	9	8	2	3	6	5
5	2	2	3	3	2	1	5	7	4	7
7	9	8	7	8	2	4	6	8	9	3

7	1	8	4	6	5	4	9	7	9	7
4	8	2	7	7	8	5	6	8	8	5
5	8	9	9	8	9	8	8	5	6	9

2. Write all the combinations of two figures, that make 10, 11, 12, 13, 14, 15, 16, 17 and 18.

3. Commence with 1 and orally add thereto 2, and continue to add 2 to the successively occurring sums, until you produce 31. Thus 3, 5, 7, 9, 11, 13, etc.

4. Commence with 1 and in like manner add 3 until you produce 31. Thus 4, 7, 10, 13, etc.

5. Commence with 1 and in like manner add 4 until you produce 41.

6. Commence with 1 and in like manner add 5 until you produce 51.

7. Commence with 1 and in like manner add 6 until you produce 61.

8. Commence with 1 and in like manner add 7 until you produce 71.

9. Commence with 1 and in like manner add 8 until you produce 81.

10. Commence with 1 and in like manner add 9 until you produce 91.

11. . Orally add by 2's until you produce 20.

12.	"	"	3's	"	"	30.
13.	"	"	4's	"	"	40.
14.	"	"	5's	"	"	50.
15.	"	"	6's	"	"	60.
16.	"	"	7's	"	"	70.
17.	"	"	8's	"	"	80.
18.	"	"	9's	"	"	90.
19.	"	"	10's	"	".	100.

20. Commence at 1 and orally add 3 and 5, alternately, until you produce 100.

21. Commence at 1 and orally add 4 and 7, alternately, until you produce 100.

Add by 2's, 3's, and 4's alternately from 0 to 99.

73. Add the following problems:

5+6+8=?	14+ 7+ 3=?	22+ 8+ 1=?
7+4+9=?	21+ 0+ 2=?	12+ 9+ 7=?
6+7+5=?	7+12+ 0=?	4+ 9+ 8=?
9+8+8=?	26+ 9+12=?	20+40+ 3=?
4+7+9=?	8+23+ 5=?	22+14+ 4=?
8+9+6=?	10+15+16=?	17+19+18=?

What is the sum of 5 apples and 10 oranges?

What is the amount of four 0's plus three 0's?

WRITTEN PROBLEMS IN ADDITION.

74. Add the following numbers; 6376, 564, 309, 485, and 5092.

OPERATION.

Thousands. Hundreds. Tens. Units.
6376
564
309
485
5092
———

Sum 12,826
132

EXPLANATION.—In all addition problems, we first write the numbers so that units of the same order stand in the same column, i. e. *units* in the units, or first column; *tens* in the tens, or second column, *hundreds* in the hundreds, or third column, and so on through the numbers. We then begin at the units, or first column and add the columns separately. In adding the first column, we commence with the 2 and 5, and name only the successive results; thus, 7, 16, 20, 26, which is 2 *tens* and 6 *units;* the 6 we write in the first place, or column of units, and place the 2 *tens* which is to be carried to the column of tens, directly below the 6 in a small figure. Then adding the 2 *tens* to the tens column, we say, 11, 19, 25, 32—which is 3 *hundreds* and 2 *tens;* the 2 *tens* we write in the column of tens, and place the 3 *hundreds*, which is to be carried to the hundreds column, directly under it. Then adding the 3 *hundreds* to the hundreds column, we say, 7, 10, 15, 18, which is 1 *thousand* and 8 *hundreds;* the 8 *hundreds* we write in the hundreds column, and the carrying figure, 1 thousand, directly under. Then adding the 1 *thousand* to the fourth, or thousand column, we say, 6, 12, which is 1 *ten thousand* and 2 *thousands,* and this being the last column to add, we write the figures in their respective columns and produce 12826 as the *sum* of all the numbers.

When adding, set the result in pencil figures, being careful to place the carrying figure or figures directly beneath the unit figure of each column added, as shown in the preceding problem.

PROOF OF ADDITION.

75. The best proof of the correctness of addition is to be proficient in your work, and then re-add the columns in the reverse direction.

5

GENERAL DIRECTIONS FOR ADDITION.

76.   From the foregoing elucidations, we derive the following general directions for addition:

1.   *Write the numbers so that units of the same order stand in the same column.*   See explanation, page 33.

2.   *Begin at the units, or first column on the right, and add each column separately, writing the unit result under the column added, and carrying the tens, if any, to the tens column.   At the last column write the full sum of the column.*   See explanation, page 33.

3.   *To add horizontally, the numbers are not written in columns of like orders; and the result, or sum, is written to the right of the numbers.*

77.   What is the sum of each of the following groups of numbers ?

(1)	(2)	(3)	(4)	(5)
4304	780	890	777	9040
291	1261	706	888	1288
643	537	73	999	9907
98	309	4009	666	6543
1400	6987	8888	645	2018

78.   Add the following numbers horizontally:

1.   248, 3936, 409, 1278, 97, 563, 9210.

2.   325, 1468, 87, 911, 1809, 19068, 54.

3.   72, 13615, 41848, 1905, 8, 9763.

79.   Add the following groups of numbers:

(1)	(2)	(3)	(4)	(5)	(6)
818	412	582	328	809	981
390	297	578	346	523	350
970	318	757	386	605	269
276	824	420	672	848	789
752	932	731	793	945	696
843	373	542	864	397	136
865	576	853	965	684	169
129	876	684	448	976	295
768	444	743	404	666	468
904	102	915	151	217	687
972	814	686	148	879	825
114	331	637	263	516	951
346	554	917	295	259	784
545	161	650	161	896	122
622	197	411	461	864	440
749	490	237	874	565	450
717	876	349	898	150	414
222	902	489	769	514	654
234	396	698	243	446	789
166	484	228	174	576	458
365	235	433	952	489	747
272	386	949	683	394	636
729	624	687	574	407	241
955	897	762	956	812	477
177	477	849	658	798	681
——	——	——	——	——	——

(7)	(8)	(9)	(10)	(11)	(12)
864	677	595	849	539	257
363	305	249	283	377	476
629	420	463	327	762	426
145	982	830	651	235	684
174	217	221	543	856	492
144	326	232	502	950	343
176	111	151	113	446	602
767	871	387	438	834	182
644	512	516	455	540	955
747	814	247	328	919	858
156	376	331	633	358	989
106	468	281	624	149	855
872	189	828	581	268	954
694	177	986	491	662	126
788	885	817	888	693	136
866	264	918	992	682	564
944	294	289	202	355	163
922	896	259	548	223	764
116	597	381	365	521	921
911	814	329	208	530	515
866	277	678	662	874	735
179	476	640	764	528	393
129	716	821	287	584	556
659	802	457	848	625	888
778	584	587	255	262	932
—	—	—	—	—	—

(13)	(14)	(15)	(16)
778999	979644	156563	1333182
115224	130466	994544	9979667
964892	898567	836869	7391573
578678	787543	234246	3517569
577594	964432	765183	8598674
668678	699678	345927	2513756
669657	978321	654678	3454210
539886	678789	456432	7656754
664756	564673	345718	5467856
795568	895437	765391	5645781
699689	569128	673123	7893344
689786	678982	437987	3216675
688968	869771	566789	4569911
935789	668339	544321	6543344
778896	956234	891389	9576677
659669	195842	219720	1539902
363769	957454	625221	6662234
351994	573367	431348	4235564

17. Add 6, 8, 9, 7, 6, 8, 5, 4, 9, 4, 8, 7, 6, 9, 14, 19, 18, 27, 38, 47, 59, 65, 74, 83, 92.   Ans. 632.

18. Add 528, 791, 14389, 888, 91361, 587, 301, 7004, 52800, 7106, 42881.   Ans. 218,636.

19. Add 476010, 51873, 98, 48932, 3581427, 67843, 21050, 3672.   Ans. 4,250,905.

20. Add 63, 94, 85, 74, 63, 52, 41, 39, 48, 57, 66, 75, 84, 93, 27, 18, 60, 80, 19, 88, 99, 77, 66, 55, 44, 33, 22, 11, 98, 97, 96, 86, 76, 65, 54, 43.   Ans. 2248.

21. Add seven million four thousand ninety-six, and three hundred eighty-seven thousand five hundred sixty-two.   Ans. 7391658.

22. Find the sum of 4888765, 92238, 1600084, 8888888, 99999999999, 4100000808707 and 222222333-333444444.   Ans. 222226533349723125.

23. Find the sum of 999999999, 88888888, 7777777, 666666, 55555, 4444, 333, 22, 1, and sixty-three million.   Ans. 1160393685.

24. Add 789, 679, 987, 140018, 191070, 871230432, 49706, 40000, 80000000, and eleven hundred eleven.
Ans. 951654792.

25. Add five hundred thousand nine hundred thirty nine, and eleven thousand eleven hundred eleven.   Ans. 513050.

ADDITION OF DOLLARS AND CENTS.

80. Dollar and Cent Signs.—The dollar sign is $, and the cent sign is ℒ. When the dollar sign is placed before numbers, they are read as dollars. Thus $45 is read 45 dollars. When the cent sign is placed after numbers, they are read as cents. Thus 14ℒ is read 14 cents. When dollars and cents are written together, the cents are separated from the dollars by a point (.), and the sign of cents is omitted. Thus $16.45 is read 16 dollars and 45 cents.

Since there are 100 cents in 1 dollar, *cents* always occupy *two* places and only two, in connection with dollars. When the number of cents is less than 10, a *naught* must be used to fill the *tens* column, or the first place at the right of the point. Thus 8 dollars and 5 cents are written $8.05.

When cents only are written, they are expressed, as follows: 25 cents, or 25ℒ, or $.25.

When writing numbers representing dollars and cents for the purpose of addition, they must be set so that dollars will be under dollars, and cents

under cents, in the regular order of units, tens, hundreds, etc., and the points (.) that separate dollars and cents must be in a vertical line.

The dollar sign ($) and the point (.) should never be omitted when writing dollars and cents, except when writing in books, or on paper which contains dollar and cent columns.

81. The United States Monetary units are as follows:

10 mills (m.)	= 1 cent, ⨍.
10 cents	= 1 dime, d.
10 dimes	= 1 dollar, $.
10 dollars	= 1 eagle, E.

PROBLEMS.

	(1)	(2)	(3)	(4)	(5)
82. Add	$14.50	$34.16	$75.	$ .88	$180.40
	8.	9.08	4.45	11.	48.08
	4.25	14.83	67.06	5 13	91.16
	12.15	8.	.35	7.02	7.05
	$38.90	$66.07	$146.86	$24.03	$326.69

	(6)	(7)	(8)	(9)	(10)
Add	$321.	$521.16	$ 9.45	$ 431.	$194.15
	640.80	83.25	80.	124.	8.05
	9.13	19.30	17.	381.	73.75
	75.20	8.	.65	569.	6.13
	100.05	4.07	6.10	827.	.95
	$1146.18	$635.78	$113.20	$2332.	$283.03

11.   Add $8.12, $9, $.50, $3.40, $37.05, $.75, and $12.12.   Ans. $70.94.

12.   Add $43.10, $17, $5, 48¢, 75¢, $11, $24.14, $3.   Ans. $104.47.

13.   Add $108, $97.16, $84.12, $.75, $8, $6.40, 25¢, $18.   Ans. $322.68.

14.   Add $550.10, $671.23, $794.98, $88, $45¢, 5¢, $3.10.   Ans. $2137.91.

15.   Add $999.99, $888.88, $777.77, $666.66, $555.55, $444.44, $333.33, $222.22, $111.11, and 1¢.
                                                                 Ans. $4999.96.

16.   Add $987.65, $876.54, $765.43, $654.32, $543.21, $123.45, $234.56, $345.67, $456.78, $567.89, $678.90, and $789.   Ans. $7.23.40.

17.   Pritchett bought a hat for $2, a coat for $9.50, a pair of shoes for $2.75, a pair of pants for $4, a vest for $1.75, and had $41.05 left.   How much money had he at first?   Ans. $61.05.

18.   Miss Smith paid for a broom 35¢, for soap $1.60, for starch 75¢, for matches 5¢, for salt 15¢, for sugar $1.50, for rice $2, for butter 80¢, Graham flour $1.25, and for a Hygienic cook book $1.   What was the sum paid for all?   Ans. $9.45.

19.   Baltar paid for a reader $1.35, for an arithmetic $1.50, for a history $2, for a set of drawing instruments $3.70, for paper $.60, for pens $.15, for ink $.05, for a pair of Indian clubs $3.50, and for the Boy's Own Book $1.   What did all cost?
                                                                 Ans. $13.85.

20.   Hornor paid $1.75 for Chesterfield's letters; $1.80 for Cutter's Anatomy, Physiology, and Hygiene; $1.75 for Comb's Constitution of Man; $1.25 for How to Read Character by Wells; $1.50 for Nordhoff's Politics for Young Americans; $1.75 for Phy-

sical Perfection by Jacques; $4 for Plutarch's Lives; $8 for Shakspeare's Works; $2 for the Literary Reader; $6 for Carey's Social Science; $5 for Parson's Laws of Business; $5 for Soulé's Philosophic Work on Commercial and Exchange Calculations; and $1 for Cushing's Manual. How much did he pay for all? Ans. $40.80.

21. If you should travel by rail 160 miles, by steamer 214, and walk 8, how far would you travel? Ans. 382.

22. A planter raises 9842 pounds of sugar, 2351 pounds of cotton, 1827 pounds of rice, 3840 bushels of corn, 325 bushels of sweet potatoes, and 194 bushels of beans. How many pounds and how many bushels does he raise in all? Ans. 14020 pounds, 4359 bushels.

23. Conrad loaned to Purcell, $9; to Gresham, $3.50; to Hanna, 75¢; to Mitchell, 85¢; to Sweeney, 5¢; to Bothic, $1; to Keen, 25¢; to Abbott, 75¢; to Prophet, 50¢. What sum did he loan to all? Ans. $16.65.

24. Keen has $143.05; Couret, $91; McCoard, $18.30; Bush, 90¢; Nevers, 25¢; Fischer, $5.05; Beck, $9; Meyers, $6; Levy, $7; Brown, $7; Rice, $45; Shotwell, $27; Wise, $6.80; Moffett, $5.50; Lindsey, $88.70. How much have all? Ans. $460.55.

25. A merchant bought four adjacent lots of ground for $6850. He built, thereon, a house which cost $11875. Paid for fences, $912; for flagging, $1819.55; for furniture, $3481.12. How much did the whole cost? Ans. $24937.67.

26. If you pay $175 for a horse, $450 for a carriage, $75 for a set of harness, $38 for a saddle and bridle, and $6.50 for a whip, what will the whole cost? Ans. $744.50.

6

27.   A planter has 54 cows, 321 sheep, 174 mules, 23 horses, 42 oxen, 43 calves, 7 colts.   How much live stock has he altogether?   Ans. 664.

28.   A merchant bought at one time 250 barrels flour for $1500; at another, 345 barrels for $2415; and at another, 200 barrels for $1625.   How many barrels did he buy and what was the total cost?
                                             Ans. 795 Bbls.,
                                                      $5540 Cost.

29.   The weight of ten bales of cotton is as follows: 481, 503, 398, 462, 470, 479, 401, 397, 463, and 511 pounds.   What is the total weight?   Ans. 4565.

30.   Bought at one time 43 yards of calico and 32 yards of silk; at another, 104 yards of calico and 24 yards of silk; at another, 96 yards of calico and 48 yards of silk.   How many yards of each kind did I buy?   Ans. Calico, 243; Silk, 104.

31.   Paid $425 for a lot of sugar, $120 for rice, and $75 for potatoes.   Sold the sugar at a profit of $41, and the rice and potatoes at cost.   What did I get for the whole?   Ans. $661.

32.   From New Orleans to the Rigolets is 31 miles; hence to Montgomery, 18; hence to Bay St. Louis, 3; hence to Pass Christian, 6; hence to Mississippi City, 13; hence to Biloxi, 9; hence to Ocean Springs, 4; hence to East Pascagoula, 16; hence to St. Elmo, 21; hence to Mobile, 20.   How many miles to Mobile?   Ans. 141.

33.   From New Orleans to Kenner is 10 miles; hence to Manchac, 27; hence to Ponchatoula, 11; hence to Hammond, 4; hence to Amite, 16; hence to Tangipahoa, 10; hence to Osyka, 10; hence to Magnolia, 10; hence to McComb City, 7; hence to Summit, 3; hence to Bogue Chitto, 10; hence to Brookhaven, 10; hence to Beauregard, 11; hence to

Crystal Springs, 19; hence to Terry, 9; hence to **Jackson,** 15; hence to Madison, 13; hence to Canton, 11. How many miles is it to Canton?
<div align="right">Ans. 206.</div>

34. A young man paid $125 for a year's tuition **at college,** $22.50 for books, lost $40, and has $378.35 on hand. How much had he at first?
<div align="right">Ans. $565.85.</div>

35. A boy gave Jane 6 oranges, Kate 4, John 3, he ate 2, and had 5 remaining. How many had he at first?
<div align="right">Ans. 20.</div>

36. Louisiana contains 41255 square miles; Mississippi, 47156; Texas, 237504; Arkansas, 52198; Tennessee, 45600; Kentucky, 37680; Alabama, 50722; Georgia, 52009; South Carolina, 29385; North Carolina, 50704; Missouri, 67380; Virginia, 61352; Maryland, 11124; Florida, 59268; California, 188982. How many square miles in the fifteen states?
<div align="right">Ans. 1032319.</div>

37. The population of London is 3832441; Paris, 1988806; St. Petersburg, 667963; Rio Janeiro, 274972; Constantinople, 600000; Vienna, 726105; Berlin, 1122385; Lisbon, 203681; Tokio, or Jeddo, 594283; Bombay, 644405; Madrid, 397690; Glasgow, 555289; Dublin, 249486; Amsterdam, 308948; Brussels, 391393; Stockholm, 169429; Copenhagen, 273727; Cairo, (Egypt), 327462; Tunis, 125000; Naples, 450804; Liverpool, 552425; Rome, 303383; City of Mexico, 236500; Barcelona, 249106. What is the population of all?
<div align="right">Ans. 15245683.</div>

38. The length of the Mississippi River is 4200 miles; of the Nile, 4000; Amazon, 3750; Yenisei, 3400; Obi, 3000; Yang-tse-Kiang, 3320; Niger, 3000; Lena, 2700; Amoor, 2650; Volga, 2000; Ganges, 1600; Brahmapootra, 2300; La Plata, 2300; Mackenzie, 2300; St. Lawrence, 2000; Saskatchewan,

1900; Orinoco, 1550; Columbia, 1020; Colorado, 600; Yukon, 1600; Red River, 1500. What is the combined length of all? Ans. 50690.

39. Lake Superior is 400 miles in length; Lake Michigan, 320; Lake Huron, 240; Lake Erie, 240; Lake Ontario, 180; Lake Baikal, 375; Lake Pontchartrain, 40. What is the combined length of all?
Ans. 1795.

40. Mount Everest of the Himalaya chain in Asia, and the highest point on the globe, is 29062 feet high; Mt. St. Elias, the highest mountain in North America, is 17900 feet; Mt. Illampu, the highest mountain in South America, is 24812 feet; Mt. Blanc, the highest mountain in Europe, is 15780 feet; Mt. Kilima Njaro, the highest mountain in Africa, is 20065 feet; Mt. Kosciusko, the highest mountain in Australia, is 7176 feet. What is the combined height of all? Ans. 114795 feet.

41. By the census of 1880, the population of New York was 1206577; Philadelphia, 847170; Brooklyn, 566689; St. Louis, 350518; Chicago, 503185; Baltimore, 332313; Boston, 369832; Cincinnati, 255809; New Orleans, 216090; San Francisco, 233959; Buffalo, 149500; Washington, 147293; Newark, 136508; Louisville, 123758; Mobile, 48602; Galveston, 24126; Memphis, 78433. What is the population of all combined? Ans. 5590362.

42. The standing army of the United States is 32000; of Great Britain and Ireland, 192000; of France, 454000; of the German Empire, 402000; of Russia, 766000; of Spain, 284000; of Switzerland, 201000; of Italy, 205000; of Brazil, 25000; of Mexico, 21000; of Turkish Empire, 93000; of Sweden, 150000; of Holland, 62000; of Portugal, 33000; of Belgium, 40000. How many men in all?
Ans. 2960000.

43. Homer was born 733 years before the Christian Era. How many years from the birth of Homer to the year 1886. Ans. 2619.

44. The Mayor of the City of New Orleans receives a yearly salary of $3500; the Treasurer, $3500; the Commissioner of Public Works, $3500; the Comptroller, $3500; the Commissioner of Police and of Public Buildings, $3000; the City Attorney, $3500; the City Surveyor, $2500; the City Superintendent of Public Schools, $3000; the City Superintendent of Fire Alarm Telegraph, $1800. What is the salary of all these officers? - Ans. $27800.

45. The Governor of Louisiana receives a salary of $4000 per annum; the Lieutenant Governor, $8 per day during the 60 days' session of the Legislature; the Secretary of State receives $1800 per annum; the Auditor of Accounts, $2500; the State Treasurer, $2000; the Attorney General, $3000; the five Justices of the Supreme Court, $5000 each; the two Judges of Criminal Court in N. O., $4000 each; the two Judges of the Court of Appeals in N. O., $4000 each; the five Judges of the Civil District Court, Parish of Orleans, $4000 each; the State Superintendent of Education, $2000. What is the salary of all these officers, including the per diem of the Lieutenant Governor? Ans. $76780.

46. From August 31st, 1875, to Sept. 1st, 1876, the production of Sugar in Louisiana was as follows:

Parish of Livingston, 4 hogsheads; St. Tammany, 16; East Feliciana, 37; Lafayette, 187; West Feliciana, 339; Vermillion, 609; Avoyelles, 1582; St. Landry, 1768; St. Martin, 1884; Orleans, 1041; St. Bernard, 2097; East Baton Rouge, 2544; Rapides, 2453; Pointe Coupee, 2762; Iberia, 3632; Jefferson, 3671; West Baton Rouge, 4155; St. Charles, 5808; St. John, 8335; Plaquemines, 9068; Iberville, 9814;

Lafourche, 11302; Terrebonne, 10888; St. James, 13437; Ascension, 14267; St. Mary, 14318; Assumption, 14712. How many hogsheads were produced during the year?   Ans. 140730.

47.   From New Orleans to Carrollton is 7 miles; hence to Donaldsonville, 71; hence to Plaquemines, 32; hence to Baton Rouge, 20; hence to Port Hudson, 23; hence to Bayou Sara, 12; hence to mouth of Red River, 40; hence to Natchez, 72; hence to Rodney, 45; hence to Grand Gulf, 18; hence to Vicksburg, 61; hence to the Louisiana Line, 97; hence to Helena, 230; hence to Columbus, 329; hence to Cairo, 20; hence to Cape Girardeau, 50; hence to St. Louis, 151. How many miles to St. Louis by river?   Ans. 1278 miles.

48.   From New Orleans to the mouth of Red River is 210 miles; hence to Black River, 40; hence to Alexandria, 110; hence to Grand Ecore, 120; hence to Grand Bayou, 95; hence to New Hope, 60; hence to Waterloo, 30; hence to Shreveport, 35. How many miles to Shreveport by river?
                       Ans. 700 miles.

49.   The Cotton Crop of the Southern States from 1880, to Sept. 1, 1885, was as follows:
1880-'81, 6605750 bales, of which
          New Orleans received....1606184 bales.
1881-'82, 5456048 bales, of which
          New Orleans received....1190711 bales.
1882-'83, 6949756 bales, of which
          New Orleans received....1690709 bales.
1883-'84, 5713200 bales, of which
          New Orleans received....1529188 bales.
1884-'85, 5655900 bales, of which
          New Orleans received....1521755 bales.
How many bales were produced in the five years, and how many of them did New Orleans receive?
   Ans. 30380654 bales.—N. O. received 7538547 bales.

50. From New Orleans to MacDonoughville is 1 mile; hence to Algiers, 1; hence to Old Spanish Fort St. Leon, 16; hence to Poverty Point, 18; hence to Point Celeste, 7; hence to Pointe-à-la-Hache, 3; hence to Sixty Mile Point, 15; hence to Quarantine, 9; hence to Bolivar Point, 3; hence to Forts St. Philip and Jackson, 2; hence to The Jump, 10; hence to Head of Passes, 11; hence to Pilot Town, 10; hence to Port Eads, 1. How many miles from New Orleans to Port Eads?   Ans. 107 miles.

51. From New Orleans to Algiers Depot is 1 mile; hence to Gretna, 3; hence to Jefferson, 9; hence to St. Charles, 6; hence to Boutte, 6; hence to Bayou des Alemedes, 8; hence to Raceland, 8; hence to Ewing's, 6; hence to Lafourche, 6; hence to Terrebonne, 3; hence to Chucahoula, 6; hence to Tigerville, 5; hence to L'Ourse, 4; hence to Bayou Bœuf, 3; hence to Ramos, 3; hence to Morgan City, 4; hence to Galveston, 240. How many miles to Galveston?   Ans. 321 miles.

52. Twenty-four peaches were eaten; 5 being spoiled were thrown away; and 32 remained in the basket. How many were there at first?   Ans. 61.

53. A man was 26 years of age when he was married. How old will he be when he has been married 14 years?   Ans. 40 years.

54. A young man graduated from college when he was 22 years of age. He married 6 years afterwards, and 2 years afterwards he was presented with a son. What will be his age when the son is 21 years old?   Ans. 51 years.

55. A lady paid $6.50 for a dress, $8 for a shawl, $4 for a bonnet, and $3.75 for a pair of shoes. What was the total cost?   Ans. 22.25.

56.   A boy sold his pony for $45, and lost $15 by the sale.   What did the pony cost him?   Ans. $60.

57.   A merchant paid for a lot of goods $580; he sold them and gained $190.   How much did he receive for them?   Ans. $770.

58.   Henry is 16 years old, James is 3 years older, and William is 2 years older than James.   How old are James and William?

Ans. James 19, William 21.

59.   The internal framework of the human body consists of bones, which united by strong ligaments, constitute the *skeleton.*   In the skull are 8 bones; in the face, 14; in each ear, 3; in the tongue, 1; in the trunk and spinal column and pelvis, 55; in each shoulder, 2; in each arm, 3; in each wrist, 8; in the palm of each hand, 5; in each thumb, 2; in each finger, 3; in each leg, 4; in each ankle, 7; in each foot 5; in each great toe, 2; in each of the other toes, 3; and there are 32 teeth.   How many bones in the whole body?   Ans. 240.

60.   How many pupils in a school in which there are 6 grades, the first containing 63; the second, 58; the third, 27; the fourth, 49; the fifth, 35; the sixth, 24?   Ans. 256.

61.   Bothick has $420; Conrad has $130 more than Bothick; and Prophet has as much as Bothick and Conrad together.   What sum have all three?

Ans. $1940.

OPERATION INDICATED.

Bothick has	$420	$420 Bothick.
	$130	$550 Conrad.
	———	$970 Prophet.
Conrad has	$550	———
	$420	$1940 Ans.
	———	
Prophet has	$970	

62. Keen, Soulé, and Abbott form a copartnership. Keen invests $3400; Soulé, $4000; and Abbott $500 more than both Keen and Soulé. What is the capital of the firm?   Ans. $15300.

OPERATION INDICATED.

Keen invests $3400	$3400 Keen.
Soulé " $4000	$4000 Soulé.
	$7900 Abbott.
Keen & Soulé invest $7400	
500	$15300 Ans.
Abbott invests $7900	

63. A father gave his son seven thousand eight hundred dollars; his daughter, nineteen hundred and fifty dollars; and his wife, three thousand five hundred more than he gave to both the son and the daughter. What sum did he give away?

Ans. $23000.

SYNOPSIS FOR REVIEW.

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Define the following words and phrases:

46. Signs and Symbols. What are they used for? 47. Sign of Addition. 48. Of Subtraction. 49. Of Multiplication. 50. Of Division. 52. The Parenthesis and Vinculum. 53. The Period. 54. The Ratio Sign. 55. Sign of Proportion. 56. The Sign of Involution. 57. The Radical Sign. 58. Sign of Deduction. 59. The Interrogation. 60. The Sign for *First*, *Second*, etc. 61. Sign of the Comma. 62. @, ⅜, 1³, $, ⊄, %, Amt., lb., bbl., doz., do., yd., ps., Co., Dr., Cr., gal., O. K. 63. Addition. 64. Sum, or Amount. 65. Sign of Addition. 66. Sign of Equality. 67. Numerical Equation. 68. Principle of Addition. 69. The Alphabet of Numbers. 75. Proof of Addition. 80. To add Dollars and Cents.

SUBTRACTION,

83. **Subtraction** is the process of finding the difference between two numbers of the same kind.

84. The result obtained by subtraction is called the **Difference,** or **Remainder.**

85. The greater number is called the **Minuend,** which means a number to be decreased.

86. The lesser number is called the **Subtrahend,** which means the number to be subtracted.

87. The **sign of subtraction** is a horizontal line, —. It is read *minus* and means less.

When this sign is placed between two numbers, it indicates that the number *after* it is to be subtracted from the number *before* it. Thus 8 — 3 is read 8 minus 3=5, or 8 less 3=5.

88. The **Principle** governing all problems in subtraction is, that like numbers and units of the same order only, can be subtracted, the one from the other.

89. To **Prove** the operation of subtraction, add the remainder to the subtrahend: if the sum is equal to the minuend, the work is correct. ·

90. Subtraction is the reverse of addition, and by it we find what number added to the lesser of two numbers will produce the greater,

(51)

91. SUBTRACTION TABLE.

2—2=0	3—3=0	4—4=0	5—5=0
3—2=1	4—3=1	5—4=1	6—5=1
4—2=2	5—3=2	6—4=2	7—5=2
5—2=3	6—3=3	7—4=3	8—5=3
6—2=4	7—3=4	8—4=4	9—5=4
7—2=5	8—3=5	9—4=5	10—5=5
8—2=6	9—3=6	10—4=6	11—5=6
9—2=7	10—3=7	11—4=7	12—5=7
10—2=8	11—3=8	12—4=8	13—5=8
11—2=9	12—3=9	13—4=9	14—5=9
6—6=0	7—7=0	8—8=0	9—9=0
7—6=1	8—7=1	9—8=1	10—9=1
8—6=2	9—7=2	10—8=2	11—9=2
9—6=3	10—7=3	11—8=3	12—9=3
10—6=4	11—7=4	12—8=4	13—9=4
11—6=5	12—7=5	13—8=5	14—9=5
12—6=6	13—7=6	14—8=6	15—9=6
13—6=7	14—7=7	15—8=7	16—9=7
14—6=8	15—7=8	16—8=8	17—9=8
15—6=9	16—7=9	17—8=9	18—9=9

92. ORAL EXERCISES.

1. Commence at 50 and orally count to 0 by continually subtracting 1, thus: 49, 48, 47, 46, 45, etc.

2. Commence at 50 and orally count to 0 by continually subtracting 2, thus: 48, 46, 44, 42, etc.

3. Commence at 51 and orally count to 0 by successively subtracting 3, thus: 47, 44, 41, 38, etc.

4. In like manner, commence at 50 and subtract respectively 4, 5, 6, 7, 8, 9, 10 until you produce 1, thus: 46, 41, 35, 28, etc.

5. Commence at 50 and subtract alternately 2 and 5 until you produce 1, thus: 48, 43, 41, 36, etc.

6. Commence at 50 and subtract alternately 8 and 3 until you produce 6, thus: 42, 39, 31, etc.

7.   60—2+12+6—2—2—3+5—8+11—6+9—
3 equal ?

8.   75+5—20—20—5+8+7—9+6—5+4—3+
2—45=?

93.   *To subtract one number from another, when any figure of the subtrahend is less than the corresponding figure of the minuend.*

1.   From 897 subtract 641.

OPERATION.

897 or 641
641   897

256   256

Explanation.—First write the numbers with the lesser *under* or *over* the greater, so that units of the same order stand in the same column.   Then commence with the units figure and subtract each order separately; thus 1 from 7 leaves 6; 4 from 9 leaves 5; 6 from 8 leaves 2.   By this work we obtain the difference, or remainder, 256.

Subtract the following:

278	843	384	978	425	9876
499	521	762	655	679	3456

94.   Demonstration—to prove that the difference between two numbers is the same as the difference between the two numbers when equally increased.

OPERATIONS.

6	6+3=9	5	5+2=7	23	23+10=33
4	4+3=7	2	2+2=4	11	11+10=21
—	Difference —	—	Difference —	—	Difference —
2 is the same. 2		3 is the same. 3		12 is the same. 12	

EXPLANATION.—Here we see that the difference between 6 and 4 is 2, and that the difference between 6 and 4 equally increased by 3, is also 2.   The operations with 5 and 2, and 23 and 11 show similar results.   Hence the law *that the difference between two numbers is the same as the difference between the two numbers when equally increased.*

The application of this numerical law is shown in the following problem, and it governs all operations in subtraction, where the subtrahend figure of any order exceeds the minuend figure of the same order.

95.   *To subtract one number from another, when any figure of the subtrahend is greater than the corresponding figure of the minuend.*

1.   From 4173 subtract 2346.

FIRST OPERATION.

	Thousands. Hundreds. Tens. Units.			Thousands. Hundreds. Tens. Units.
Minuend	4173		Subtrahend	2346
Subtrahend	2346	or	Minuend	4173
Difference	1827			1827

Explanation.—Having written the numbers with the lesser under or over the greater, so that units of the same order stand in the same column, we commence at the right hand to perform the operation.

We first observe that 6 *units* cannot be taken from 3 *units*. We therefore, according to the foregoing numerical law, mentally increase the 3 *units* by 10 *units* making 13 *units;* from this we subtract the 6 units and set the remainder, 7 units, in the line of difference.   Then, as we added 10 *units* to the minuend, we now, to compensate therefor, mentally add 1 *ten*, the equivalent of 10 *units*, to the *tens* figure of the subtrahend, and say 5 from 7 leaves 2, which we write in the line of difference.

We next observe that 3 *hundreds* cannot be taken from 1 *hundred;* and, therefore, for reasons above given, we mentally add 10 *hundreds* to the 1 *hundred* making 11 *hundreds,* and then say 3 from 11 leaves 8.   Then having added 10 *hundreds* to the *hundreds* figure of the minuend, we now mentally add 1 *thousand,* the equivalent of the 10 *hundreds,* to the *thousands* figure of the subtrahend and say 3 from 4 leaves 1.   This completes the operation and gives 1827 as the difference between the two numbers.

The foregoing is the only rational and true method of subtraction, and it should be universally sub-

stituted for the absurd and unmathematical "*bor-rowing*" method, which is given by nearly all the authors of arithmetics now before the public.

SECOND OPERATION.

4173		2346
2346	or	4173
----		----
1827		1827

Explanation.—We will here perform the operation by addition, which is a simpler and better method than the preceding, and consists simply in adding to the subtrahend such a number as will make it equal to the minuend. Thus commencing with the unit figure of the subtrahend, or smaller number, we say, 6 and 7 make 13; and write the 7 in the *units* place of the difference; then carrying 1, we say 5 and 2 make 7, and write the 2 in the *tens* column of the difference; then we say 3 and 8 make 11, and write the 8 in the *third* column, or *hundreds* place of the difference; then carrying 1, we say 3 and one make 4, and write the one in the *fourth* place of the difference. This completes the operation.

2. From 73245 subtract 1228.

FIRST OPERATION.

```
73245
 1228
------
72017
```

Explanation.—Here we say 8 from 15 leaves 7; 3 from 4 leaves 1; 2 from 2 leaves 0; 1 from 3 leaves 2; 0 from 7 leaves 7.

SECOND OPERATION.

```
73245
 1228
------
72017
```

Explanation.—Here we say 8 and 7 make 15; 3 and 1 make 4; 2 and 0 make 2; 1 and 2 make 3; 0 and 7 make 7.

3. From 56802 subtract 50531.

FIRST OPERATION.

```
56802
50531
-----
 6271
```

Explanation.—Here we say 1 from 2, 1; 3 from 10, 7; 6 from 8, 2; 0 from 6, 6; 5 from 5, 0; which being the last figure on the left has no value, and hence is not written.

SECOND OPERATION.

```
56802
50531
-----
 6271
```

Explanation.—Here we say 1 and 1 =2; 3 and 7=10; 6 and 2=8; 0 and 6=6; 5 and 0=5 The naught is not written for the reason given in the first solution.

GENERAL DIRECTIONS FOR SUBTRACTION.

96. From the foregoing elucidations, we derive the following general directions for subtraction:

1. *Write the numbers so that units of the same order stand in the same column.*

2. *Begin at the unit figure and take successively each figure of the subtrahend from the figure of the corresponding order of the minuend.*

3. *When a figure of the subtrahend is greater than the figure of the same order in the minuend, add 10 to the minuend figure, perform the subtraction, and then add 1—the equivalent of the 10—to the next subtrahend figure.*

PROBLEMS.

97. Write the following groups of numbers as they are here written, and subtract the lesser from the greater of each group:

(1)	(2)	(3)	(4)	(5)	(6)
467	1807	3842	607	3001	6879
342	4251	1291	8013	1009	9640

Subtract the following numbers:

7. From 5307 take 309. Ans. 4998.
8. From 1090 take 1009. Ans. 81.
9. From 7608 take 3705. Ans. 3903.
10. From 184240 take 39460. Ans. 144780.
11. From 41074089 take 1875429.

Ans. 39198660.

12. From 9876543210 take 1234567890.

Ans. 8641975320.

98. TO SUBTRACT DOLLARS AND CENTS.

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1. What is the difference between $483 and $51.65. Ans. $431.35.

OPERATION.

$483.00
51.65
———
$431.35

Explanation.—In all problems of this kind we first write the numbers in the same manner as when adding dollars and cents, with dollars under dollars, and cents under cents, so that units of the same order stand in the same column, and the points in a vertical line.

When there are no cents in the minuend, we fill the place of cents with naughts.

The operation of subtraction is performed with dollars and cents, the same as with other numbers.

99. What is the difference between the numbers in each of the following groups?

(1)	(2)	(3)	(4)	(5)	(6)
$16.25	$8.00	$.75	$41.04	$10.50	$1.93
9.38	3.75	.59	6.61	4.78	.47
$6.87	$4.25				

(7)	(8)	(9)	(10)	(11)
$681.85	$127.05	$248.00	$49.11	$8529.09
99.33	105.50	181.15	9.89	2798.17

(12)	(13)	(14)	(15)	(16)
$576.00	$87.45	$482.68	$14.00	$279107.16
132.85	19.38	2246.10	68.25	91020.48

TO SUBTRACT HORIZONTALLY.

---

100. Subtract the following numbers horizontally:

(1)
81—52=32.

(2)
143—62=81.

(3)
721—95=626

(4)
62—18=?

(5)
87—39=?

(6)
643—214=?

(7)
$486.43—$91.35=?

(8)
$327.15—$168.08=?

(9)
$386.14—$169.87=?

---

101.   PROBLEMS IN SUBTRACTION.

---

1. Paid for rice $5500, and for sugar $6875.40. How much more was paid for sugar than for rice?
                          Ans. $1375.40

2. Bought a lot of flour for $2225, and sold the same for $2800. What was the gain?
                          Ans. $575.

3. It is 700 miles to Shreveport and 320 to Galveston. How much farther is it to Shreveport than to Galveston?   Ans. 380 miles.

4. The ant has fifty eyes, and the dragon fly 12000. How many more has the dragon fly than the ant?   Ans. 11950 eyes.

5. The total coinage of gold and silver at the different mints of the U. S. during the fiscal year ending June 30th, 1875, was $43854708. Of this amount $33553965 was gold. What was the amount of silver coined?   Ans. $10300743.

6 A student had 40 problems to work, and **worked** 17. How many has he yet to work?

Ans. 23.

7. Man has 26 bones in each foot, and 27 in each **hand.** How many more has he in the hand than in the foot? Ans. 1.

8. Sound travels through the air at the rate of 1118 feet per second, and a bullet fired from a rifle travels 1750 feet per second. How much faster does the ball travel than sound?

Ans. 632 feet per second.

9. Physiologists have determined, with the aid of the microscope, that the lungs of a man contain not less than 600,000,000 air cells; they have also determined that a single drop of human blood contains more than 4,000,000,000 of corpuscles. How many more corpuscles in one drop of blood than air cells in the lungs? Ans. 3,400,000,000.

10. Geologists have demonstrated that the formation of the stalactites and stalagmites in the **Mammoth Cave** of Kentucky, required not less than **75000** years of time; and that the wearing away of the rock of Niagara Falls, by friction, from Queenstown where they first were after the glacial epoch, to their present location, 7 miles above, required at least 40000 years. How much longer did it require to form the stalactites and stalagmites, than for the Falls of Niagara to recede to their present location?

Ans. 35000 years.

11. The average age of the deceased graduates of Harvard College is 58 years. The average age of all the people of Massachusetts, who die after they reach 20, is only 50. How many years of increased life does education give the Harvard **graduate?** Ans. 8 years.

12. The average age of the deceased Presidents and Professors of Yale College is 65 years. The average age of all adults in Connecticut is but 51 years. How much longer do the Presidents and Professors live, than do the other classes of citizens, and what are some of the reasons therefor?

Ans. 1st, 14 years.

Ans. 2d, Length of life and good health are measurably in proportion to the extent of education, because the highly educated man knows better than the man of little education, how to take care of the bones, muscles, and organs of his body. He knows what, when, and how much to eat, drink, and breathe, to work, rest, and sleep. By reason of his superior knowledge, he can multiply comforts, and guard against and conquer many of the enemies of his nature.

13. The Equatorial diameter of the earth is 7925.65 miles, and the Polar diameter is 7899.17 miles. How much greater is the Equatorial diameter than the Polar? Ans. 26.48 miles.

How many years from the date of each of the following events to the present year?

14. Quills first used for writing, 636.

15. Figures used by the Arabs, borrowed from the Indians, 813.

16. High towers first erected on churches, 1000.

17. Glass windows first used in England, 1180.

18. Chimneys built in England, 1236.

19. Spectacles invented by Spina, 1299.

20. Woolen cloths first made in England, 1331.

21. Muskets used in England, 1421.

22. Printing invented, 1436.

23. Almanacs first published in Buda, 1460.

24. America was discovered in 1492.

25. Tobacco discovered in St. Domingo, 1496.

26.   Spinning-wheel invented at Brunswick, 1530.
27.   Steel needles first made in England by an East Indian, 1545.
28.   Telescopes invented by Jansen, 1590.
29.   Laws of Falling Bodies, discovered by Galileo, 1591.
30.   Decimal Arithmetic invented at Bruges, 1602,
31.   Circulation of blood discovered by Harvey, 1619.
32.   Newspapers first published, 1630.
33.   Coffee brought to England, 1641.
34.   Steam engines invented by the Marquis of Worcester, 1649.
35.   Gravitation discovered by Newton, 1687.
36.   Lightning Rods invented by Franklin, 1752.
37.   Spinning-jenny invented by Hargreaves, 1767.
38.   Cotton first planted in the United States, 1769.
39.   Power loom invented by Cartwright, 1784.
40.   Cotton first spun in America, 1787.
41.   Cotton gin invented by Whitney, 1793.
42.   Steel pen invented by Wise, 1803.
43.   Steam first used to propel boats, by Fulton, in America, 1807.
44.   First locomotive was made at Liverpool, 1829.
45.   Electro-Magnetic Telegraph invented by Morse, of America, 1832.
46.   The electric telegraph was first used in the United States in 1844.
47.   Sewing machine invented by Howe, 1846.
48.   Type-writer invented by Sholes, Soulé and Glidden, 1867.
49.   Telephone invented, 1876.
50.   The Stenograph invented by Bartholomew, 1882.

51. In 1830, there were 23 miles of railroad in operation in the United States; in 1880, there were 93671 miles of railroad. What was the number of miles increase during the half century?

<div align="right">Ans. 93648 miles.</div>

52. Physicists have determined that to produce the color, *dark red*, 395,000,000,000,000 of ethereal waves strike the eye per second; and to produce *violet*, 760,000,000,000,000 of ethereal waves strike the eye per second. How many more waves per second are required to produce violet than dark red?   Ans. 365,000,000,000,000.

53. The army of the Duke of Wellington, at the battle of Waterloo, consisted of 26661 Infantry, 8735 Cavalry, 6877 Artillery, and 33413 Allies. Napoleon's army at the same battle was composed of 48950 Infantry, 15765 Cavalry, and 7732 Artillery. What was the whole number in each army; which Commander had the larger army, and what was the excess?   Ans. Wellington, 75686.

<div align="right">Napoleon, 72447.</div>

<div align="center">Wellington had 3239 more than Napoleon.</div>

54. The following named officers of the United States receive salaries as follows:

President, . . .	$50000	
Vice-President, . .	8000	
Cabinet Ministers, .	8000	
Chief Justice of Supreme Court, . .	10500	
Justices of Supreme Court, ...	10000	
Senators and Representatives,	5000	
(with mileage extra).		
Admiral, ...	13000	
Vice-Admiral, . .	9000	
Rear Admiral, . .	6000	
Commodores, . .	5000	
Captains, . . .	4500	
Commanders, . .	3500	
Lt. Commanders, 2800 to 3000		
Lieutenants in the Navy, . .	2400 to 2600	
Generals, . . .	13000	
Lt. Generals, . .	11000	
Maj. Generals, . .	7500	
Brig. Generals, . .	5500	
Colonels, . .	3500	
Lt. Colonels, . .	3000	
Majors, ...	2500	
Captains in Cavalry and Artillery, . .	2000	
Captains in Infantry,	1800	
Adjutants, . . .	1800	
Quartermasters, . .	1800	
1st Lieutenants in Cavalry and Artillery,	1600	
1st Lieut. in Infantry,	1500	

How much more does the President receive than **each of** the other officers named?

    Ans. To the first two: $42000 more than the
         Vice-President, and the Cabinet Minister

55. General George Washington was born in 1732 and died in 1799; General R. E. Lee was born in 1807 and died in 1870. How much older was General Washington than General Lee, when he died?   Ans. 4 years.

56. What is the difference between 23222 and 11 thousand 11 hundred and 11?   Ans. 11111.

<div align="center">

OPERATION INDICATED.

$23222 - \overline{11000 + 1100 + 11} = 11111$ Ans.

</div>

57. What number must be added to 68741 to make a million?   Ans. 931259.

58. Philadelphia has 153151 buildings; New Orleans 35600. How many more has Philadelphia than New Orleans?   Ans. 117551.

59. James, who is 23, is 7 years older than Henry. How old is Henry?   Ans. 16 years.

60. Fishel has $500 which is $150 more than I, and I have $75 more than Keiffer. How much has Keiffer, and how much have I?   Ans. Keiffer has $275.
                                       I have $350.

61. There are two parties who owe me $8000, and one of them owes $4250. The other wishes to pay me $1700 on account. How much will he then owe?   Ans. $2050.

62. A speculator bought a lot of apples for $215, and sold them at such a price, that if he had gotten $22.50 more, he would have gained as much as they cost him. How much did he sell them for?   Ans. $407.50.

SOLUTION STATEMENT.

$215 Cost of apples.
  2 times the cost=

$430; which would have been the selling price
      to gain as much as they cost.
$22.50 deducted, leaves

$407.50 the real selling price.

63. From New Orleans to Vicksburg is **401** miles, and to Natchez, 277 miles. How far is it from Natchez to Vicksburg?   Ans. 124 miles.

64. What is the difference between one million, seventeen thousand seven, and one thousand, sixteen hundred sixteen?   Ans. 1014391.

65. The sum of two numbers is 1463, and one of the numbers is 628. What is the other?

Ans. 835.

66. The velocity of our earth on its yearly voyage through space, around the sun, is 99733 feet per second; the velocity of a 12 pound cannon ball fired from a gun with an average charge of powder is 1734 feet per second. How many feet farther does the earth travel, in each second, than a cannon ball?   Ans. 97999 feet, or 18 miles and 2959 feet.

67. What number is that to which, if 17821 be added, the sum will be 37907?   Ans. 20086.

68. At an election, the defeated candidate received 23742 votes; but had he received 5112 votes more, he would have been elected by 1000 majority. How many votes did the elected candidate receive?   Ans. 27854.

OPERATION INDICATED.

$23742 + 5112 - 1000 = 27854$ Ans.

69. How many years have elapsed since the birth of the following named persons:

Zoroaster, according to Aristotle, was born B. C. 5429; Abraham, B. C. 2000; Menes, B. C. 2000; Moses, B. C. 1570; Solomon, B. C. 1033; Homer, B. C. 1000; Lycurgus, B. C. 850; Thales, B. C. 640; Solon, B. C. 638; Pythagoras, B. C. 600; Confucius, B. C. 551; Buddha, or Guatama, B. C. 500; Sophocles, B. C. 495; Socrates, B. C. 470; Plato, B. C. 429; Aristotle, B. C. 384; Demosthenes, B. C. 382; Alexander, B. C. 356; Euclid, B. C. 323; Cicero, B. C. 106; Seneca, B. C. 5; Plutarch, A. D. 50; Justinian, A. D. 483.

70. A father divided his plantation consisting of 4500 acres, between his five sons—Albert, Edward, William, Frank, and Robert. To Albert he gave 800 acres; to Edward he gave 150 acres more than he gave Albert; to William he gave 100 acres less than he gave Edward; to Frank he gave as much as he gave Edward; and the remainder he gave to Robert. How many acres did Robert receive?

Ans. 950 acres.

SYNOPSIS FOR REVIEW.

Define the following words and phrases:

83. Subtraction. **84.** Difference, or Remainder. **85.** Minuend. **86.** Subtrahend. **87.** Sign of Subtraction. **88.** Principle of Subtraction. **89.** To Prove Subtraction. **94.** What is the Numerical Law regarding the Difference between Two Numbers? **95.** To Subtract by Addition. **96.** General Directions for Subtraction. **98.** To Subtract Dollars and Cents.

9

MULTIPLICATION.

102. **Multiplication** is the process of increasing one of two numbers as many times as there are units in the other. Or, differently explained, it is a short method of performing addition.

103. The number to be multiplied is called the **multiplicand.**

104. The number which shows how many times the multiplicand is to be increased, or repeated, is called the **Multiplier.**

105. The result obtained by multiplying is called the **Product.**

106. The multiplicand and multiplier are called **Factors.** The meaning of the word factor is *maker*, or *producer*.

107. The **Sign of Multiplication** is an oblique cross, ×. It shows that the numbers, between which it is placed, are to be multiplied together, and is read *multiplied by*, or *times*. Thus 8×3, is read 8 multiplied by 3, or 3 times 8.

(66)

Changing the order of the factors does not change the product or result. Thus 8×3 may be read 3 times 8, or 8 times 3.

108. Principles of Multiplication. 1. In all cases the multiplier must be regarded as an abstract number. Two denominate numbers cannot be multiplied together as denominate numbers. Thus, we cannot multiply 5 apples by 5 apples, 10 cents by 10 cents, 12 yards by 8 pounds, or 6 boxes by $2. All such questions are absurd and insolvable.

2. In all multiplication operations, the *product* is the *same,* in *name* or *kind,* as the multiplicand.

109. To **Prove** the operations of multiplication, repeat the work or multiply the multiplier by the multiplicand. If the result is the same as the first, the work is probably correct.

110.   MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32.	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100
11	22	33	44	55	66	77	88	99	110
12	24	36	48	60	72	84	96	108	120
13	26	39	52	65	78	91	104	117	130
14	28	42	56	70	84	98	112	126	140
15	30	45	60	75	90	105	120	135	150
16	32	48	64	80	96	112	128	144	160
17	34	51	68	85	102	119	136	153	170
18	36	54	72	90	108	126	144	162	180
19	38	57	76	95	114	133	152	171	190
20	40	60	80	100	120	140	160	180	200

Explanation.—We recommend this table as being far superior to the one presented in other School and College Text Books of the country, and urge all who aspire to proficiency in computing numbers to learn it. In learning this table, or in the use of it, we caution the calculator against the use of all intermediate words, whether he speaks or thinks them; thus, instead of saying or thinking, 9 times 3 are 27; 17 times 6 are 102, &c., say or think, 9, 3, 27; 17, 6, 102, &c.

In reading we do not stop to spell orally or mentally the words that compose the sentences; from the combination of the letters we see what the words are, without looking specially at each individual letter; and to read or operate with rapidity in the combination of numbers, we must omit all superfluous talk or thought.

111.   ORAL AND WRITTEN EXERCISES.

1.   1 Orange cost 5 cents.   What will 4 oranges cost?   Ans. 20 cents.

SOLUTION STATEMENT BY ADDITION.

$5¢ + 5¢ + 5¢ + 5¢ = 20¢$, or thus:

$$5¢$$
$$5¢$$
$$5¢$$
$$5¢$$

———

$20¢$ Ans.

2.   If 1 hat cost $2, what will 3 hats cost?

Ans. $6.

3.   At 8 cents per yard, what will 5 yards cost?

Ans. $40¢$.

This and all similar problems may be solved by addition, in the same manner as the preceding one.

The addition method, though quite easily performed in these problems, would be too difficult and lengthy for practical work with larger numbers; and hence another and briefer method is adopted, as shown below.

112.   THE PHILOSOPHIC METHOD.

At this advance of our work, we introduce, and throughout the book shall continue to use the logic of numbers—the Philosophic System of solving problems.   (See page 10).

By this system, the reasoning faculties of the mind are brought into action, invigorated, strengthened, and capacitated to see fine distinctions, to consider conditions, to investigate facts, to reason logically, and to deduce correct conclusions from not only the premises and conditions of problems, but upon all matters and questions that the changing affairs of this world's life may present for consideration.

1. 1 orange cost 5 cents. What will 4 oranges cost?
                                          Ans. 20 cents.

SOLUTION STATEMENT.

5¢
4
—
20¢ Ans.

Reason.—One orange cost 5 cents. Since 1 orange cost 5 cents, 4 oranges will cost 4 times as much, which is 20 cents.

2. If 1 hat cost $2, what will 3 hats cost?
                                          Ans. $6.

SOLUTION STATEMENT.

$2
3
—
$6 Ans.

Reason.—1 hat cost $2. Since 1 hat cost $2, 3 hats will cost 3 times as much, which is $6.

3. At 8 cents per yard, what will 5 yards cost?
                                          Ans. 40¢.

SOLUTION STATEMENT.

8¢
5
—
40¢ Ans.

Reason.—1 yard cost 8 cents. Since 1 yard cost 8 cents, 5 yards will cost 5 times as much, which is 40¢.

The Reason, Why, and Wherefore, continued.

Question.—How do you know, that if 1 yard cost 8 cents 5 yards will cost 5 times as much?

Answer.—By the exercise of my judgment—by the use of the reasoning faculties of the mind.

Question.—What do you mean, in this connection, by judgment?

Answer.—The conclusion arrived at by the operations of the mind, after duly considering the premise, the facts, and the conditions of the problem.

Question.—What do you mean by premise or premises?

Answer.—The proposition, declaration, truth, or fact which is asserted as the basis, or predicate, of

a question. In this problem, the premise is, *one yard cost* 8 *cents.*

Question.—Why will 5 yards cost 5 times as much as 1 yard?

Answer.—Because 5 is five times as much as 1.

Question.—What kind of reasoning is the foregoing?

Answer.—Analogical and axiomatical. Analogical, because there is analogy, relationship, or likeness existing between the cost of 1 yard and the cost of 5 yards. Axiomatical, because, the premise and question considered, the conclusion is self-evident.

Question.—What is reason?

Answer.—The faculty or power of the human mind, by which truth is distinguished from falsehood, right from wrong, and by which correct conclusions are reached by considering the logical relationship which exists between the premises, the facts, and the conditions of particular statements and questions. *

4.   What will 4 books cost at 20 cents each?

SOLUTION STATEMENT.

20¢
4
—
80¢ Ans.

Reason.—1 book cost 20¢. Since one book cost 20¢, 4 books will cost 4 times as much.

Questions.—1.   How do you know that 4 books will cost 4 times as much as 1 book? 2.   What do

---

*Note.—The last two of the preceding questions and answers may be too profound for young learners to fully comprehend; but though they may not have the strength of mind to thoroughly understand their full meaning, they will nevertheless derive great benefit by repeated reflect on and exercise thereon.

None of the foregoing questions are intended for minds under 8 to 10 years of age, according to native brain capacity. Science teaches and experience proves, that the mental or brain capacity of both young and old is as different as their physical strength. And predicating our belief upon long experience and much study of the human mind, we claim that the brain should perform but little labor before the age of 8 or 10 years has been attained.

you mean by judgment? 3. Why will 4 books
cost 4 times as much as 1 book? *

5.   At 15 cents per dozen, what will 3 dozen cost?

SOLUTION STATEMENT.

15¢	*Reason.*—1 dozen cost 15¢.
3	Since 1 dozen cost 15 cents,
—	3 dozen will cost 3 times as
45¢ Ans.	much.

Questions.—1. How do you know that 3 dozen
will cost 3 times a much as 1 dozen? 2. Why will
3 dozen cost 3 times as much as 1 dozen? 4. What
do you mean by judgment?

6.   If 1 hat cost $3, what will 9 hats cost?

SOLUTION STATEMENT.

$ 3	*Reason.*—1 hat cost $3.
9	Since 1 hat cost $3, 9 hats
—	will cost 9 times as much.
$27 Ans.	

Questions.—1. How do you know they will? 2.
Why will they? 3. What is judgment in this
connection?

7.   Flour is $8 per barrel, what will 12 barrels
cost ¿

SOLUTION STATEMENT.

$ 8	*Reason.*—1 barrel cost $8.
12	Since 1 barrel cost $8, 12
—	barrels will cost 12 times as
$96 Ans.	much.

Questions.—How do you know this? 2. Why
will they? 3. What do you understand by judg-
ment in this connection?

---

* NOTE.—The answers given to like questions in the preceding problems, are
the proper answers to these and all similar questions.

8. Bought 9 pounds of sugar at 8¢ per pound. What was the cost of the whole?

SOLUTION STATEMENT.

<div>

8¢

9

——

72¢ Ans.

</div>

Reason —1 pound cost 8¢. Since 1 pound cost 8 cents, 9 pounds will cost 9 times as much.

Questions.—1. How do you know this? 2. Why will it?

9. At $7 per cord, what will 123 cords cost?

SOLUTION STATEMENTS.

1st.	2d.
$ 7	123
123	7
——	——
$861 Ans.	$861 Ans.

Explanation.—In the second statement, for convenience, the multiplicand is used as the multiplier.

Reason.—1 cord cost $7. Since 1 cord of wood cost $7, 123 cords will cost 123 times as much.

Questions.—1. How do you know this? 2. Why will it?

10. If an employé receives $3 per day for services and he works 22 days, how much money has he earned?

SOLUTION STATEMENT.

<div>

22

3

——

$66 Ans.

</div>

Reason.—1 day's service is worth $3. Since 1 day's service is worth $3, 22 days' services are worth 22 times as much; or, since he receives $3 for 1 day's work, for 22 days' work he will receive 22 times as much.

Questions.—1. How do you know this? 2. Why will he? 3. What do you mean by judgment?

11. There are 60 minutes in an hour. How many minutes are there in a day of 24 hours?

10

SOLUTION STATEMENT.

$$\begin{array}{r} 60 \\ 24 \\ \hline 1440 \text{ Ans.} \end{array}$$

Reason.—In 1 hour are 60 minutes. Since there are 60 minutes in 1 hour, in 24 hours there are 24 times as many.

Questions.—1. How do you know this? 2. What do you mean by judgment?

The reasoning of these problems and the answers to the questions following the reasoning, should be repeated until the mind is fully capacitated to solve, in like manner, all similar problems.

Solve the following problems in like manner as the foregoing, and write the reason for each:

12. At $6 per cord, what will 34 cords of wood cost? Ans. $204.

13. Paid $4 per barrel for potatoes and bought 47 barrels. What did they cost? Ans. $188.

14. What will 7 yards cost, at 12¢ per yard? Ans. 84¢.

15. What will 4 books cost, at 20¢ each? Ans. 80¢.

16. At 13¢ per dozen, what will 6 dozen cost? Ans. 78¢.

17. If 1 box cost $3, what will 23 boxes cost? Ans. $69.

18. Flour is worth $7 per barrel. What are 25 barrels worth? Ans. $175.

19. 12 inches make a foot. How many inches in 16 feet? Ans. 192 inches.

20. 4 quarts make a gallon. How many quarts in a barrel that holds 42 gallons? Ans. 168 quarts.

21. If you buy 15 boxes of peaches @ $2 per box, what will they cost? Ans. $30.

22. If you buy 7 pencils at 5 cents each and hand to the seller 50¢, how much change ought you to receive?   Ans. 15¢.

23. A merchant bought 23 barrels of apples at $4 per barrel, and paid $65 on account. How much does he still owe?   Ans. $27.

113. *To Multiply Abstract Numbers and Give Reasons Therefor.*

1. Multiply 7 by 6.

OPERATION.

7
6
——
42 Ans.

Explanation and Reason.—Plato tells us that one is the basis of all things; and hence it is the basis of all numbers. Multiplication is the process of repeating one number as many times as there are units—*ones*—in another. Considering these facts, we first multiply the 7 by 1, and in the product obtain a premise for our argument. Thus, axiomatically 1 time 7 is 7. Since 1 time 7 is 7, 6 times 7 is 6 times as many, which is 42. This is the long looked for reason for the multiplication of abstract numbers.

Multiply the following numbers, and write the reason:

8×5;   17×12;   23×16;   234×157.

114. *To Multiply, When the Multiplier Consists of Only One Figure.*

1. What is the product of 947 multiplied by 6?

OPERATION.

Multiplicand 947
Multiplier   6
              ——
Product   5682

Explanation.—In all problems of this kind, we write the multiplier under the units figure of the multiplicand, and then commencing with the units figure we say, 6 times 7 are 42, which is 4 *tens* and 2 *units;* the 2 *units* we write in the *units* place of the product and retain in the mind the 4 *tens* to add to the column of *tens;* we next say 6 times 4 are 24 plus the 4 *tens* retained in the mind, are 28, which is 2 *hundreds* and 8 *tens;* the 8 *tens* we write in the *tens* column of the pro-

duct, and retain in the mind the 2 *hundreds* to add to the column of *hundreds*. We then say 6 times 9 are 54, plus 2 *hundreds* are 56, which is 5 *thousand* and 6 *hundreds*, which we write respectively in the *thousands* and *hundreds* columns of the product. This completes the operation and gives a product of 5682.

In practice, instead of saying 6 times 7 are 42, 6 times 4 are 24, etc., we should only name the result of the combination. Thus, 42, 24, etc. *In handling figures, we should always pronounce the result of the combinations without naming the figures that make the result, just as we pronounce words without spelling or naming the letters that make the words.*

Perform the following multiplications:

	(2)	(3)	(4)	(5)
Multiplicand	543	983	2769	76895
Multiplier	7	8	5	9
Product	3801	7864	13845	692055

	(6)	(7)	(8)	(9)
	8764	2987	9876	85421
	5	8	7	9

	(10)	(11)	(12)	(13)
	46532	58674	9861	81453
	14	15	17	19

14.   What will 4 pianos cost at $425 each?
<div align="right">Ans. $1700.</div>

15.   At $65 each, what will 9 wagons cost?
<div align="right">Ans. $585.</div>

16.   What will 7 lots of ground cost, at $1875 each?
<div align="right">Ans. $13125.</div>

17. At $6 per barrel, what will be the cost of 245 barrels of flour?

SOLUTION STATEMENT.

Multiplier   245
Multiplicand   6
———
$1470 Ans.

Reason.—1 barrel of flour cost $6. Since 1 barrel of flour cost $6, 245 barrels will cost 245 times as much. The $6 is the real multiplicand, but in the operation we use it as the multiplier. This we do for convenience in performing the operation, in all problems where the multiplicand is less than the multiplier. The result is the same whichever factor we use as a multiplier.

18. What will 42 dozen boxes cost, at $9 per dozen?   Ans. $378.

19. At $7 a piece, what will 48 chairs cost?   Ans. $336.

20. A clerk receives $75 per month. If he spends $40 per month, how much can he save in one year, or twelve months?   Ans. $420.

21. Multiply 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 together.   Ans. 0.

22. Multiply $2 \times 8 \times 9 \times 10 \times 1 \times 0 \times 5 \times 7$.   Ans. 0.

23. Bought 44000 pounds of cotton at 8¢ per pound and sold it at 9¢ per pound. What was the gain?   Ans. $440.

24. What is the difference in the cost of 150 sheep at $4 a head, and 80 head of cattle at $12 a head.   Ans. $360.

115.   *To Multiply, when the Multiplier Consists of More Than One Figure.*

1.   What is the product of 397 multiplied by **653?**

Multiplicand	397
Multiplier	653

1st Partial product by 3 units, $1191 = 3$ times the multiplicand
2d Partial product by 5 tens, $1985 = 50$ times   "
3d   "   "   6 h'ds., $2382 = 600$ "

Total product $259241 = 653$ "   "

Explanation.—In all problems of this kind, we first write the multiplier under the multiplicand, so that units of the same order stand in the same column, and then multiply by one figure at a time. We first multiply by the *units* figure, then the *tens, hundreds*, and so on in regular order through the multiplier; then add the several partial products together and thus obtain the required product.

In this problem, we first multiply by 3, the *units* figure, in the same manner as explained in the first problem where there was but one figure in the multiplier, and obtain 1191 as the first partial product. This we write below the multiplier so that units of the same order stand in the same column.

Next we multiply by the 5 *tens;* we say 5 times 7 are 35, which is 3 *hundreds* and 5 *tens;* we write the 5 *tens* in the *tens* column directly below the multiplying figure, and reserve in the mind the 3 *hundreds* to add to the *hundreds* column. We then say 5 times 9 are 45 + 3 *hundreds*, which were reserved, are 48 *hundreds* which is 4 *thousands* and 8 *hundreds;* we write the 8 *hundreds* in the column of *hundreds*, and reserve the 4 *thousands* to add to the *thousands* column. We then say 5 times 3 are 15 plus 4 *thousands*, reserved, are 19 *thousands*, which is 1 *ten thousand* and 9 *thousands*, and which we write in their respective columns.

We then, in like manner, multiply by the 6 *hundreds* in the multiplier, being careful to write the first figure obtained (2) in the *hundreds* column, directly under the 6 of the multiplier, and the other figures in their respective columns, *thousands,*

ten thousands, and *hundred thousands.* We then add the *partial products* together and obtain 259241, as the whole product of 397 multiplied by 653.

In practice, remember to name or think only the results of the numerical combinations, when adding or multiplying.

EXAMPLES.

2. **Multiply 3426 by 457.**

OPERATION.

	Millions. Hund. of thous. Ten Thousands. Thousands. Hundreds. Tens. Units.
Multiplicand	3426
Multiplier	457

1. Partial prod. by 7 *units,* 23982 =7 times the multiplicand
2. Partial prod. by 5 *tens,* 17130 =50 times the multiplicand
3. Partial prod. by 4 *h'ds,* 13704 =400 times the multiplicand

Whole product   1565682 =457 times the multiplicand

3. **Multiply 647 by 58.**

OPERATION.
```
    647
     58
  ─────
   5176
   3235
  ─────
  37526  Ans.
```

5. **Multiply 28433 by 4172**

4. **Multiply 21794 by 2365**

OPERATION.
```
  21794
   2365
 ──────
 108970
 130764
  65382
  43588
 ──────
 51542810 Ans.
```

6. **Multiply 989769 by 248193**

Multiply the following numbers:

7. 483 by 569	10. 581 by 76	13. 671 by 508
8. 924 by 237	11. 1847 by 84	14. 8765 by 2046
9. 1683 by 328	12. 2346 by 127	

Operation of the 13th
problem.

```
       671
       508
      ─────
      5368
      3355
      ─────
    340868  Ans.
```

Explanation.—In all problems where there are naughts in the multiplier, we multiply by the significant figures only, for the reason that the product of any number by 0 is 0.

116.   *To Multiply, when either the Multiplicand or Multiplier, or both, have naughts on the right.*

1.   Multiply 463 by 200.

OPERATION.

```
     463
     200
    ─────
   92600
```

Explanation.—In all problems of this kind, we write the significant figures so that units of the same order stand in the same column, and write the naughts on the right of the significant figures. We then multiply the significant figures and annex to the product as many naughts as there are in the multiplier or multiplicand, or in both. The basis or reason of this is, that the removal of a figure or number one place to the left increases its value *ten* fold. The annexing of a naught removes the significant figures one place to the left, thereby increasing them *ten* fold; and hence, annexing a naught is in effect multiplying by 10. For the same reason, annexing two naughts is multiplying by 100, the annexing of 3 naughts is multiplying by 1000, etc., for other powers of 10. In this problem we first use the multiplier 2 *hundred* as 2 *units;* hence the first partial product, 926, was 100 times too small. We then, by annexing the two naughts, multiplied it by 100, and obtained 92600 as the correct product.

2.   Multiply 3400 by 26.

OPERATION.

```
     3400
       26
    ──────
      204
       68
    ──────
    88400  Ans.
```

3.   Multiply 940 by 4700.

OPERATION.

```
      940
     4700
    ──────
      658
      376
    ───────
   4418000  Ans.
```

4. Multiply 5020 by 420

| OPERATION. |
| 5020 |
| 420 |
| —— |
| 1004 |
| 2008 |
| ———— |
| 2108400 Ans. |

5. Multiply 82000 by 483.

82000	or	483
483		82000
——		——
246		966
656		3864
328		————
————		39606000
39606000 Ans.

6. Multiply 842 by 600.
7. Multiply 1208 by 1020.
8. Multiply 9900 by 707.
9   Multiply 23500 by 12030.
10. Multiply 1000 by 6208.
11. Multiply 81000 by 90200.
12. Multiply 45670 by 5780.
13. Multiply 987000 by 49.

117. *To Multiply by the Factors of a Number.*

Factors of a number are such numbers as will, when multiplied together, produce the number. Thus 6 and 6 are the factors of 36; 7 and 8 are the the factors of 56.

PRINCIPLE.

The product of any number of Factors will be the same, in whatever order they may be multiplied.

1. Multiply 2435 by 42.

| OPERATION. |
| 2435 |
| 7 |
| ———— |
| 17045 |
| 6 |
| ———— |
| 102270 Ans. |

Explanation.—In all problems of this kind, we separate the multiplier into two or more factors; then multiply the multiplicand by one of the factors, the resulting product by another factor, and so on, until we have used all the factors. The last product will be the correct product.

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2. Multiply 781 by 63.
3. Multiply 3140 by 36.
4. Multiply 588 by 81

5. Multiply 480 by 361.
6. Multiply 1756 by 125.
7. Multiply 3281 by 128.

118.   *To Multiply, when the Multiplicand or Multiplier Contains Dollars and Cents.*

1.   Multiply $342.15 by 6.

OPERATION.
$342.15
6
‾‾‾‾‾‾
Product $2052.90

Explanation.—In all problems of this kind, we multiply in the regular manner, then prefix the dollar sign ($) and place the point (.) two places from the right. Our answer is then in dollars and cents.

2.   What will 1682 pounds of sugar cost, at 9¢ per pound?   Ans. $151.38.

3.   A merchant's monthly expenses are $1342.75. What are they for 12 months?   Ans. $16113.00.

4.   It costs a family $2.30 a day for marketing. What will be the expense for 30 days?
                                      Ans. $69.00.

5.   What will 37 boxes of oranges cost, at $3.75 per box?   Ans. $138.75.

6.   At 16 cents per pound, what is the value of 23780 pounds of cotton?   Ans. $3804.80.

7.   If it costs $17500 to construct one mile of railroad, what will be the cost to build 364 miles?
                                      Ans. $6370000.

8.   What will 875 tons of railroad iron cost, at $55 per ton?   Ans. $48125.

GENERAL DIRECTIONS FOR MUTIPLICATION.

119. From the foregoing elucidations, we derive the following general directions for multiplication:

1. *Write the multiplier under the multiplicand, so that units of the same order stand in the same column, and draw a line beneath.*

2. *When the multiplier consists of one figure, begin at the units and multiply each figure of the multiplicand by the multiplier. Write in the product line the units of each result, and add the tens, if any, to the next result.*

3. *When the multiplier consists of two or more figures, begin at the units figure and multiply successively, each figure of the multiplicand by each figure of the multiplier, placing the right hand figure of each partial product under that figure of the multiplier which produced it.*

4. *Draw a line beneath the several partial products and add them together; the sum will be the required product. If there are any decimals in the factors, point off as many figures from the right of the product as there are places of decimals in the multiplicand and multiplier.*

PROOF.—1st. Carefully review the work. 2nd. Multiply the multiplier by the multiplicand; if the results are the same, the work is probably correct.

MISCELLANEOUS PROBLEMS IN MULTIPLICATION.

120. 1. What is the value of the following numerical expression?

$$(43-6)+(8\times2). \qquad \text{Ans. 53.}$$

2. What is the product of $8+7$, multiplied by $204-101$? Ans. 1545.

3. What is the product of $16+\overline{18-10}$ by $\overline{12\times2}$? Ans. 576.

4. What is the difference between $50-(5\times4)$ and $25+4-8$? Ans. 9.

5. Multiply $240-\overline{50+22}$ by $\overline{14\times16}-\overline{112-65}$. Ans. 29736.

6. Multiply 16 thousand 16 hundred 16, by 11 thousand 11 hundred forty and 11. Ans. 214052016.

7. Multiply the following:

$$9\times8\times7\times6\times5\times4\times3\times0\times2\times1.$$

Ans. 0.

8. What will 6 dozen dozen boxes cost, at one half a dozen dozen cents per box? Ans. $622.08.

OPERATION INDICATED.

$12=1$ doz.; 12 times $12=144=1$ doz. doz.; then 6 times $144=864=6$ doz. doz.

$6=$ one-half doz.; $12=1$ doz.; then 6 times $12=72$ $=$ one-half doz. doz.

$864\times72\rlap{/}{c}=\$622.08$ Ans.

or, thus:

$12\times12\times6=864=6$ doz. doz.
$12\times6 \qquad = 72=$ one-half doz. doz.

$\$622.08=$Ans.

9. Multiply one million twenty-six, by nineteen thousand seven hundred ten. Ans. 19710512460.

10. One cubic foot contains 1728 cubic inches. How many cubic inches in 324 cubic feet?
Ans. 559872.

11. One square foot contains 144 square inches. How many square inches in 95 square feet?
Ans. 13680.

12. One gallon contains 231 cubic inches. How many cubic inches in a cistern that holds 3500 gallons? Ans. 808500.

13. One bushel contains 2150.42 cubic inches. How many cubic inches in 20 bushels?
Ans. 43008.40.

14. One mile contains 5280 feet. How many feet in 25 miles? Ans. 132000.

15. Allowing the year to contain 365 days, how many days in 21 years? Ans. 7665.

16. The human heart beats 4200 times an hour. How many times does it beat in 10 years, there being 24 hours in one day, and allowing 365 days in each year? Ans. 367920000.

OPERATION INDICATED.

365, days, × 10, years, × 24, hours, × 4200, beats, or pulsations, = the answer.

17. Sound travels 1118 feet per second. How far will it travel in ten minutes, there being 60 seconds in a minute? Ans. 670800 feet.

18. A railroad train runs 25 miles an hour. How far will it go in 3 days, allowing 3 hours for lost time in stoppages? Ans. 1725.

19. Light travels 192500 miles per second. **How** many miles will it travel in 1 day, there being 24 hours in a day, 60 minutes in an hour, and 60 seconds in a minute? Ans. 16632000000.

OPERATION INDICATED.

24, hours, × 60, minutes, × 60, seconds, × 192500, miles per second,＝the answer.

Or 60, minutes, ×24, hours,＝minutes. 60, seconds, × the? ?minutes,＝seconds. 192500, miles, × the? ? seconds＝the answer.

20. If a person respires 17 times in a minute, how many times will he breathe in a day?
Ans. 24480.

21. If a person inhales 1 gallon of air at each respiration, and respires 17 times per minute, how many gallons will he inhale in 24 hours?
Ans. 24480.

22. At $17 per ounce, what is the value of 9 pounds of gold, there being 12 ounces in a pound Troy, or Mint, weight? Ans. $1836.

23. How many pounds of coffee in 180 bags, if each bag contains 162 pounds? Ans. 29160.

24. How many pounds of cotton in 87 bales, if each bale weighs 475 pounds? Ans. 41325.

25. What will 27893 pounds of tobacco cost, at 56 cents per pound? Ans. $15620.08.

26. What will 1870 acres of land cost, at $18 per acre? Ans. $33660.

27. The Senate and House of Representatives of the State of Louisiana consist of 137 members who receive $4 per day. The regular session continues 60 days. What is the yearly expense for the salaries of the State's law makers? Ans. $32880.

28. A contractor has 865 men employed at $1.50 per day. What are the weekly wages of all for 6 days' labor?   Ans. $7785.

29. What will it cost to build 37428 cubic yards of levee, at 45 cents per cubic yard?
                                        Ans. $16842.60.

30. A steamboat arrives with 3840 bales of cotton, 1320 sacks cotton seed, and 580 barrels molasses. Her freight charges are $2 per bale for cotton, 25¢ per sack for cotton seed, and 50¢ per barrel for molasses. What is the amount of her freight bills?   Ans. $8300.

Note.—Make the Solution Statement and write the Reason for the first four following problems:

31. A drayman charges 75 cents a load, and he has hauled 63 loads. How much is due him?
                                        Ans. $47.25.

32. What will it cost to slate the roof of a house containing 52 squares, at $13.25 per square?
                                        Ans. $689.

33. The walks around a dwelling contain 129 square yards. What will it cost to flag them with German flags, at $3.10 per square yard?
                                        Ans. $399.90.

34. What will it cost to pave a street, containing 20000 square yards, with stone, at $4.75 per square yard?   Ans. $95000.

35. Bought 2180 barrels of coal at 48¢ per barrel. What was the cost?   Ans. $1046.40.

36. Multiply 5 billion 16, by 5 million 1 thousand.
                        Ans. 25005000080016000.

37. A hogshead of sugar contains 1085 pounds. How many pounds in 107 hogsheads of equal weight? Ans. 116095.

38. A planter produced 68 bales of cotton. If the average weight of the bales was 460 pounds, and the cotton sold for 13 cents per pound, how much money would it bring? Ans. $4066.40.

39. What will 3 cases, containing 2 dozen pairs each, of shoes cost @ $2.90 per pair? Ans. $208.80.

40. If it costs $1.50 a day to support one person, what will it cost to support a family of 13 for one year, or 365 days? Ans. $7117.50.

41. There are 35600 dwellings in New Orleans. Allowing 7 persons to each dwelling, what would be the population of the city? Ans. 249200.

42. A merchant sold *three dozen dozen* ladies' hose at *one-quarter of a dozen dozen* cents a pair. How much did he receive for them? Ans. $155.52.

See problem 8 page 84, for aid to work the above problem.

43. The pressure of the atmosphere is 15 pounds on every square inch of surface. The exterior surface of a man of average size is about 2500 square inches. How many pounds weight does he sustain? Ans. 37500 pounds.

44. How many dollars are 375 $10 gold pieces worth? Ans. $3750.

45. What is the value of 2146 dimes? Ans. $214.60.

46. What is the value of 1010 quarter dollars? Ans. $252.50.

47. What is the value of 728 nickels?

<div align="right">Ans. $36.40.</div>

48. What is the value of 1612 half dollars?

<div align="right">Ans. $806.00.</div>

49. Light travels 192500 miles a second, and it requires 100000 years to travel to us from some of the fixed stars that are seen with the telescope. Allowing 365 days, 5 hours, 48 minutes, and 49 seconds to a year, and remembering that there are 24 hours in a day, 60 minutes in an hour, and 60 seconds in a minute, how far distant are such stars?

<div align="right">Ans. 607470883250000000 miles.</div>

OPERATION INDICATED.

24, hours, × 365, days, + 5 hours = 8765 hours.

60, minutes × 8765, hours, + 48 minutes = 525948 min.

60, seconds, × 525948, min., + 49, seconds, = 31556929 seconds in 1 year.

31556929 sec. × 100000, years = 3155692900000 seconds in 100000 years.

3155692900000 sec. × 192500, miles, = the answer.

<div align="center">or, thus:</div>

365 ×
24 + 5 hours.
_____

8765 hours, ×
60 + 48 minutes.
_____

525948 minutes, ×
60 + 49 seconds.
_____

31556929 seconds in 1 year, ×
100000
_____

3155692900000 seconds in 100000 years, ×
192500
_____

607,470,883,250,000,000 miles, Ans.

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50. During the fiscal year ending Sept. 1, 1876, there were received 30181 hogsheads of tobacco. If each hhd. contained 12 pounds of poison, how many pounds of poison were there in the whole? Ans. 362172.

51. The circumference of the earth is nearly 25000 miles; the distance to the sun is 3800 times as many miles. How far is it to the sun? Ans. 95000000.

52. 4875 is the thirteenth part of a number. What is the number? Ans. 63375.

53. The sun is 1384500 times as large as the earth; the earth is 45 times as large as the moon. How many times is the sun as large as the moon? Ans. 62302500.

54. A man's receipts are $1800 a year and his disbursements are $1125 a year. How much are his net receipts in 3 years? Ans. $2025.

55. It is estimated by Astronomers that 7500000 visible meteors fall upon the earth daily; it is also estimated that the average weight of each is 100 grains. From these figures, and allowing 365 days to the year, what is the annual growth of the earth in weight by the accession of the visible meteoric matter? Ans. 273750000000 grains.

SYNOPSIS FOR REVIEW.

Define the following words and phrases:

102. Multiplication. 103. Multiplicand. 104. Multiplier. 105. Product. 106. The meaning of Factors. 107. The Sign of Multiplication. 108. Principles of Multiplication. 109. Proof of Multiplication. 112. The Philosophic Method. 112. Reason. 112. Judgment. 112. Premises. 112. Analogical. 112. Axiomatical. 113. Reason for Multiplying Abstract Numbers. 114. To Multiply, when the Multiplier consists of only one figure. 115. To Multiply, when the Multiplier consists of more than one figure. 116. To Multiply, when either the Multiplicand or Multiplier, or both, have naughts on the right. 117. To Multiply by the Factors of a Number. 118. To Multiply, when the Multiplicand or Multiplier contains Dollars and Cents. 119. General Directions for Multiplication.

DIVISION.

121. Division is the process of finding how
many times one number is equal to another, which
is used as a unit of measure. Or, differently defined,
it is the process of finding one of the factors of a
given product when the other factor is known.

122. From the first and the better definition,
we see that Division is a process of *measuring* some
numbers by other numbers. And that it is not the
process of finding how many times one number is
contained in another, as is taught by nearly all the
authors of Arithmetics. One number cannot go
into another, however small the one or large the
other; and the questions of division of numbers
do not warrant a definition so unmathematical,
indefinite, and illogical.

The following questions and operations will eluci-
date the point:

1. You have $6. and I have $2. How many times
is your money equal to mine?

Is it not clear, by the terms of the question, that
your sum of money is to be compared with, and
measured by, my $2.? And the contracted thought
to do this is, $6 is equal to $2, 3 times.

The full thought, recognizing 1, or *unity*, as the
basis of all numerical computations would be as
follows: $6. is equal to $1, 6 times. Since $6. is
equal to $1., 6 times, it is equal to $2, one-half the
number of times, which is 3.

2. Again.—You have 8 yards of cloth and I have 4 yards. How many times is your quantity of cloth equal to mine?

Here it is plain that my 4 yards is made, by the terms of the question, the unit of measure; and your 8 yards is the thing to be measured. And the contracted thought to do the work would be, 8 yards is equal to 4 yards, 2 times.

The full logical reason is as follows: 8 yards are equal to 1 yard 8 times. Since 8 yards are equal to 1 yard 8 times, they are equal to 4 yards instead of 1, the *fourth* part of the number of times, which is 2.

3. Divide 10 by 5.

In this problem the real question is, how many times is 10 equal to 5: *not how many times 5 can go into* 10.

5 is the unit of measure and 10 is the number to be divided or measured.

The contracted reasoning is, 10 is equal to 5, 2 times; *not 5 is contained or goes into* 10, *2 times.* The full logical reason is as follows: 10 is equal to 1, 10 times. Since 10 is equal to 1, 10 times, it is equal to 5, instead of 1, the fifth part of the number of times, which is 2.

This process of reasoning is applicable to every possible question of division, and is the only logical reasoning that can be given for abstract numbers. Yet strange to say it has escaped the attention or the approbation of all other authors of Arithmetics. With due modesty, we claim some merit for its first introduction, and earnestly commend it to the thoughtful consideration of authors, of students, and of the public.

123. The **Dividend** is the number to be measured, or the number to be divided.

124. The **Divisor** is the number used as the *unit of measure,* or the number by which we *divide.*

125. The **Quotient** is the result of the division, and shows how many times the dividend is *equal to the* divisor.

126. The **Remainder** is the number left after dividing dividends, which are not multiples of the divisor, or which are not an exact number of times equal to the divisor. It must always be less than the divisor.

127. The **Sign of Division** is a horizontal line with a point above and below, thus ÷. It is read *divided by,* or *is equal to ;* and it indicates that the number before it, is to be divided by the number after it; thus 25÷5, is read 25 *divided by* 5, or 25 *is equal to* 5?

128. Division is also indicated by a horizontal line, a vertical line, or a curved line, when placed between the dividend and the divisor. Thus,

$$\frac{36}{9} \qquad 9\,|\,36, \qquad 9)36, \qquad 36(\frac{9}{}-$$

are all read 36 divided by 9, or how many times is 36 equal to 9?

PRINCIPLES OF DIVISION.

129. 1. When the *divisor* and *dividend* are both denominate or both abstract numbers, the quotient will be an abstract number.

2. When the divisor is an abstract number and the dividend a denominate number, the quotient will be a denominate number.

3. When there is a remainder it is a part of the dividend, and is therefore the same in name or kind.

4. Multiplying the dividend or dividing the divisor *multiplies* the quotient.

5. Dividing the dividend or multiplying the divisor divides the quotient.

6. Multiplying or dividing both the divisor and dividend by the same number does not change the quotient.

130. Proof.—Multiply the quotient by the divisor and to the product add the remainder, if any. If the result is equal to the dividend the work is correct.

131. The operation of Division may be performed, either by Addition or Subtraction. Thus, in the following problem:

How many times is 25 equal to 8?
         Ans. 3 times and 1 Remainder.

OPERATION BY ADDITION.	OPERATION BY SUBTRACTION.
25 is equal to 8, 1 time.	25 is equal to
25 " " 8, 2 times.	8, 1 time,
⎯⎯	⎯⎯
16	17
25 is equal to 8, 3 times,	8, 2 times,
⎯⎯	⎯⎯
24	9
1 Remain'r.	8, 3 times, and
⎯⎯	⎯⎯
25.	1 Remainder.

132.   ORAL EXERCISES.

1. How many times is 0 equal to 1? or $0 \div 1 = ?$
2.   "   "   1   "   0? or $1 \div 0 = ?$

Ans. An infinite number of times.

3. How many times is 1 equal to 1? or $1 \div 1 = ?$
4.   "   "   2   "   1? or $2 \div 1 = ?$
5.   "   "   3   "   1? or $3 \div 1 = ?$
6.   "   "   4   "   2? or $4 \div 2 = ?$
7.   "   "   8   "   2? or $8 \div 2 = ?$
8.   "   "   9   "   3? or $9 \div 3 = ?$
9.   "   "   12   "   4? or $12 \div 4 = ?$
10.   "   "   20   "   5? or $20 \div 5 = ?$
11.   "   "   24   "   6? or $24 \div 6 = ?$
12.   "   "   35   "   7? or $35 \div 7 = ?$
13.   "   "   56   "   8? or $56 \div 8 = ?$
14.   "   "   63   "   9? or $63 \div 9 = ?$
15.   "   "   72   "   9? or $72 \div 9 = ?$
16.   "   "   80   "   10? or $80 \div 10 = ?$
17.   "   "   88   "   11? or $88 \div 11 = ?$
18.   "   "   96   "   12? or $96 \div 12 = ?$
19. $\frac{36}{6} = ?$ 4)42 = ?   9)45 = ? $77 \div 7?$
      $\frac{94}{?} = ?$ 6)48 = ?   5)55 = ? $84 \div 12?$

20. How many times is 24 equal to 3, to 4, to 6, to 8, to 12, to 24?

21. How many times is 36 equal to 3, to 4, to 6, to 9, to 12, to 36?

22. How many times is 42 equal to 2, to 6, to 7, to 42?
23.   "   "   64   "   2, to 4, to 8, to 64?
24.   "   "   72   "   2, to 8, to 9, to 72?

(25)	(29)	(33)	(37)
18 = 6?	32 = 8?	54 = 9?	66 = 6?
(26)	(30)	(34)	(38)
24 = 8?	35 = 7?	48 = 6?	77 = 11?
(27)	(31)	(35)	(39)
27 = 9?	36 = 6?	63 = 7?	84 = 12?
(28)	(32)	(36)	(40)
30 = 6?	42 = 7?	72 = 8?	95 = 5?

41. $\dfrac{14}{2}=?$ $\quad\dfrac{21}{7}=?$ $\quad\dfrac{28}{4}=?$ $\quad\dfrac{64}{8}=?$ $\quad\dfrac{90}{9}=?$ $\quad\dfrac{88}{11}=?$

47. $4\,|\,16=?$ $\quad 5\,|\,45=?$ $\quad 7)56=?$ $\quad 9)72=?$

$$24\left(\dfrac{6}{=?}\right. \qquad 32\left(\dfrac{8}{=?}\right.$$

133. FRACTIONAL NUMBERS.

When we divide a unit or a number of units of any kind into equal parts, these parts are sometimes called fractions. The name of the equal parts varies according to the number of parts into which the thing or number was divided.

When the unit or number is divided into 2 equal parts, 1 of the parts is called *one-half*, and is written thus, $\frac{1}{2}$. If divided into 4 equal parts, 1 of the parts is called *one-fourth*, and is written thus, $\frac{1}{4}$; 3 of the parts are called *three-fourths*, and are written thus, $\frac{3}{4}$.

In like manner we obtain *fifths, sixths, sevenths, eighths, twelfths, sixteenths, twenty-firsts*, etc.

In writing fractional numbers in figures, we place the number which shows the *name* of the parts below a horizontal line as a *divisor*, and the number which shows *how many* parts are taken, or used, above the line as a *dividend*.

The following examples will fully elucidate this work:

Two-thirds, $\frac{2}{3}$. |Five-eights, $\frac{5}{8}$. |Seven-twelfths, $\frac{7}{12}$
Three-fourths, $\frac{3}{4}$.|Seven-ninths, $\frac{7}{9}$.|Nine-tenths, $\frac{9}{10}$.
Fifteenth-sixteenths, $\frac{15}{16}$. | Eleven-Eightieths, $\frac{11}{80}$.

How do you find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc. of any number?
How do you find $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, etc. of any number?

What is $\frac{1}{2}$ of 4?	What is $\frac{1}{5}$ of 15?	What is $\frac{2}{3}$ of 9?
" " $\frac{1}{2}$ of 6?	" " $\frac{1}{6}$ of 18?	" " $\frac{3}{4}$ of 12?
" " $\frac{1}{4}$ of 8?	" " $\frac{1}{7}$ of 28?	" " $\frac{7}{8}$ of 40?

13

134.   ORAL AND WRITTEN PROBLEMS.

1.   5 pounds cost 40 cents.   What was the cost of 1 pound?

SOLUTION STATEMENT.

¢
| 40
5 |
— | ——
| 8¢ Ans.

Reason.—5 pounds cost 40¢. Since 5 pounds cost 40 cents, 1 pound will cost the 5th part, which is 8 cents.

Reason, Why, and Wherefore, continued.

Question 1.—How do you know that if 5 pounds cost 40 cents, 1 pound will cost the 5th part?

Answer.—By the exercise of my judgment—by the use of the reasoning faculties of my mind.

Question 2.—What do you mean in this connection by judgment?

Answer.—The conclusion arrived at by the operations of the mind, after having duly considered the premise, the facts, and the conditions of the problem.

Question 3.—What do you mean by the premise or premises?

Answer.—The proposition, declaration, truth, or fact which is stated, or asserted, as the basis, or predicate, of a question.   In this problem the premise is, *5 pounds cost 40 cents.*

Question 4.—Why will 1 pound cost *one fifth* part as much as 5 pounds?

Answer.—Because 1 is the fifth part of five.

Question 5.—What kind of reasoning is the foregoing.

Answer.—(See answer to the same question on page 71).

Question 6.—What do you mean by reason?

Answer.—(See answer to the same question on page 71).

2.   If 12 yards cost 60 cents, what will 1 yard cost?

SOLUTION STATEMENT.

¢		*Reason.*—12 yards cost 60¢.
12	60	Since 12 yards cost 60 cents,
—	—	1 yard will cost the 12th
•	5¢ Ans.	part, which is 5 cents.

Questions.—1. How do you know this?   2. Why will it?   3. What do you mean by judgment?

3.   9 barrels of flour cost $72.   What was the cost of 1 barrel?

SOLUTION STATEMENT.

$		*Reason.*—9 barrels cost $72.
9	72	Since 9 barrels cost $72, 1
—	—	barrel will cost the 9th
	$8 Ans.	part.

Questions.—1. How do you know this?   2. Why will it?   3. What do you mean by judgment?

4.   Paid $18 for 6 days' labor.   What was the rate per day?

SOLUTION STATEMENT.

$		*Reason.*—6 days' labor cost
6	18	$18. Since 6 days' labor cost
—	—	$18. 1 day's labor will cost
	$3 Ans.	the 6th part.

Questions.—1. How do you know this?   2. Why will it?

5. The freight on 17 bales of cotton was $51. What was the freight per bale?

SOLUTION STATEMENT.

8
17 | 51
— | —
  | $3 Ans.

Reason.—The freight on 17 bales was $51. Since the freight on 17 bales was $51, on 1 bale it would be the 17th part, which is $3.

Questions.—1. How do you know this? 2. Why will it? 3. What do you mean by judgment?

6. If you divide 2 dozen oranges equally between 8 persons, how many oranges will you give each?

SOLUTION STATEMENT.

O.
8 | 24
— | —
  | 3 Oranges Ans.

Reason.—8 persons are to receive 24 oranges. Since 8 persons are to receive 24 oranges, 1 person will receive the 8th part, which is 3 oranges.

Questions.—1. How do you know this? 2. Why?

7. A railroad train runs 360 miles in 12 hours. What is the speed per hour?

SOLUTION STATEMENT.

M.
12 | 360
— | —
  | 30 miles Ans.

Reason.— In 12 hours 360 miles are run. Since 360 miles are run in 12 hours, in 1 hour the 12th part of the distance would be run, which is 30 miles. Or, since 12 hours' running give 360 miles, 1 hour's running will give the 12th part.

Questions.—1. How do you know this? 2. Why?

8.   **Rice** is 7 cents per pound. How many pounds **can you** buy for 35 cents?

1st SOLUTION STATEMENT.

lb

7 | 35

—

5 lbs. Ans.

Reason.—7 cents buy 1 pound of rice. Since 7 cents will buy 1 lb., 1 cent will buy the 7th part, and 35 cents will buy 35 times as much. Or, since 7 cents will buy 1 pound, for 35 cents you can buy as many pounds as 35 cents are equal to 7 cents.

2d SOLUTION STATEMENT.

7 | 35

—

5 lbs. Ans.

Questions.—1. How do you know this?   2. Why? **3.** What do you mean by judgment?

9.   At 9 cents per pound, how many pounds can **be** bought for 45 cents?   Ans. 5 pounds.

10.   Flour is worth $8 per barrel. How many **barrels** can be purchased for $56?

Ans. 7 barrels.

11.   For $.95, how many papers can you buy at 5 cents a paper?   Ans. 19 papers.

12.   At $3 a piece, how many chairs can be bought **for $36?**   Ans. 12 chairs.

13.   If the printer charges $1.50 to set 1 page of this book, how many pages can be set for $75?

Ans. 50 pages.

14.   The dividend is 42 and the divisor, 7. What **is the** quotient?   Ans. 6.

15.   The divisor is 8, the quotient 3, and the **remainder** 2. What was the dividend?   Ans. 26.

16.   8 | 64 = ? Write the full reasoning.

WRITTEN EXERCISES.

135.   *To Divide When the Divisor Does Not Exceed Twelve.*

1.   Divide 3648 by 5.

OPERATION.

Divisor 5) 3648 dividend.

Quotient 729, and 3 rem.

Explanation.—In all problems of this kind, we write the numbers as shown in the operation, and then begin on the left of the dividend to divide.   We begin on the left in order to carry the remainder, if any, of the higher order of units to the next lower order. In this problem, we first take the 3 *thousands*, and by inspection we see it is not equal to 5; we therefore unite it with the 6 *hundreds*, making 36 *hundreds*, which by trial multiplication and subtraction mentally performed, we find is equal to 5, 7 *hundreds* times and 1 remainder; the 7 we write in the *hundreds* column of the quotient line, directly under the 6, the last figure of the dividend used.   Then to the 1 remainder we mentally annex the 4 *tens*, making 14 *tens*, as the second partial dividend, and which, by mental multiplication and subtraction, we find it equal to 5, 2 *tens* times and 4 remainder; the 2 we write in the *tens* column of the quotient line, and to the 4 we mentally annex the *units* figure of the dividend, making 48 *units* as the third and last partial dividend; this we find, by mental multiplication and subtraction, to be equal to 5, 9 times and 3 remainder.

The remainder is usually expressed fractionally by writing it over the divisor; thus ⅗, which expresses the part of a unit of times that the remainder is equal to the divisor.

SHORT DIVISION.

136.   Operations in division according to the foregoing method, are called *short division*, because the multiplication and subtraction work, in finding the remainder of the partial dividends, were mentally performed.

2. **How many times is 846 equal to 6?**

OPERATION.

Divisor 6)846 **Dividend.**   *Explanation.*—In the pre-
ceding problem, we gave a
full and explicit explana-
Quotient 141   tion for each step of the
operation. In practice, much of the explanation therein
given is omitted, and the work performed thus: Commencing
with the left hand figure we say 8 is equal to 6, 1 time and 2
remainder; 24 is equal to 6, 4 times; 6 is equal to 6, 1 time.

Work the following indicated divisions:

(3)	(4)	(5)	(6)
7)847	8)12327	9)1085	11)2386
121	1540$\frac{7}{8}$	120$\frac{5}{9}$	216$\frac{10}{11}$

(7)	(8)	(9)	(10)
8)1471	4)11899	9)81018	12)10824

(11)	(12)	(13)	(14)
2344	7 | 93020	21345÷8	9)76451
3			

Divide the following numbers:

15.	9872 by 4	19.	10286 by 6
16.	1483 " 7	20.	48710 " 7
17.	1691 " 9	21.	10008 " 9
18.	41070 " 8	22.	199999 " 8

23. What is $\frac{1}{3}$ of $528? | 25. What are $\frac{3}{4}$ of $448?
24   " are $\frac{3}{5}$ of $1005? | 26.   "   " $\frac{7}{8}$ of $6448?

Operation for the 24th problem.	Operation for the 26th problem.
5)$1005	8)$6448
$ 201=$\frac{1}{5}$	$ 806=$\frac{1}{8}$
3	7
$603 Ans.	$5642 Ans.

27. How many apples can be bought for **$2.25,** at 5 cents a piece?   Ans. 45 apples.

28. At 15 cents a pound, how many pounds **can** you buy for **$3.15?**   Ans. 21 pounds.

29. Paid **$90** for 10 volumes of Chambers' Cyclopedia. What was the price of one volume?
Ans. **$9.**

30. If 8 men are to receive **$5791** in equal **parts,** what will be each man's share?   Ans. **$723⅞.**

31. The dividend is 63, and the quotient is **9.** What is the divisor?   Ans. 7.

32. The quotient is 15, the divisor 3, and **the** remainder 2. What is the dividend?   Ans. 47.

33. The quotient is 36, and the divisor 6. What is the dividend?   Ans. 216.

34. The dividend is 72, and the divisor is **4.** What is the quotient?   Ans. 18.

35. How many pounds of cotton, at 11 cents a pound, will be required to pay for 33 pounds of sugar @ 8 cents a pound?   Ans. 24.

137. *To Divide When the Divisor Exceeds Twelve.*

1. Divide 7387 by 36.

OPERATION.
Divisor, Dividend, Quotient.

$$36)\quad 7387 \quad (205\tfrac{7}{36}$$
$$72$$
———
$$187$$
$$180$$
———
7 remainder.

Explanation.—We first write the numbers, as shown in the operation, and commence to divide as explained in the first written example. But as the divisor is too large to be conveniently used mentally, we therefore write the operation of multiplying the divisor by the quotient figures, and subtracting the successive products from the several partial dividends. In performing the division we first

see by comparison, that 7 *thousands* are not equal to 36, and hence there will be no thousands in the quotient. We then annex to the 7 *thousands* the 3 *hundreds*, making 73 *hundreds* as the first partial dividend; this is equal to 36, 2 times, and a remainder; we write the 2 in the hundreds column of the quotient, multiply the divisor by it, write the product under, and subtract the same from, the 73 *hundreds* of the dividend. This work gives us 1 *hundred* remainder, to which we annex the 8 *tens*, making 18 *tens* as the second partial dividend; this partial dividend not being equal to 36, we write 0 (no tens) in the *tens* column of the quotient, and annex to the 18 *tens* the 7 *units*, making 187 *units* as the third and last partial dividend. This is equal to 36, 5 times and a remainder; we write the 5 in the quotient, multiply and subtract as we did with the first obtained figure of the quotient, and thus produce 7 remainder, which we write over the divisor as explained in short division.

LONG DIVISION.

Operations in division, according to the above method, are called *long division*, for the reason that the multiplication and subtraction work in finding the remainders of the partial dividends is written.

GENERAL DIRECTIONS FOR DIVISION.

138. From the foregoing elucidations, we derive the following general directions for the *operations* of division. For the process of reasoning given in connection with the *operations* or *solution statements*. we refer to pages, 98, 99, and 100.

1. *Draw a vertical or curved line, and write the dividend on the right and the divisor on the left.*

2. *Take the least number of the left hand figures of the dividend that are equal to or greater than the divisor, find how many times the same is equal to the divisor, and write the result in the quotient line.*

14

3. *Multiply the divisor by this quotient figure, subtract the product from the partial dividend used, and to the remainder annex the succeeding figure of the dividend and divide as before.*

4. *Proceed in like manner until all the figures of the dividend have been used.*

5. *When the partial dividend is not equal to the divisor, write a naught in the quotient, annex the succeeding figure of the dividend to the partial dividend, and proceed as before.*

6. *If there is a remainder after the last division, write it in the quotient, draw a line below it, and write the divisor underneath, as a part of the quotient.*

PROOF. Multiply the quotient by the divisor and to the product add the remainder, if any. If the result is equal to the dividend, the work is correct.

NOTE—1. The product referred to in No. 3, must never be *greater* than the partial dividend from which it is to be subtracted; if it is larger, the quotient figure is too large, and must be diminished.

2. The remainder, after each subtraction referred to in No. 3, must always be *less* than the divisor; if it is not, the last quotient figure is too small and must be increased.

3. The *order* of each quotient figure is the same as the lowest *order* in the partial dividends from which it was obtained.

2. How many times is 66804 equal to 53?

Ans. 1260$\frac{24}{53}$.

OPERATION.

Divisor, Dividend, Quotient,

5.3) 66804 (1260$\frac{24}{53}$

53

138

106

320

318

24

Proof.

1260 Quotient.

53 Divisor.

3780

6300

24 Remainder.

66804 Dividend.

3. What is the quotient of **107941÷396?**

OPERATION.

396)107941(272 Quotient.

792

2874

2772

1021

792

229 Remainder.

4. Divide 7167901 by 11267.

OPERATION.

11267)7167901(636 Quot't.

67602

40770

33801

69691

67602

2089 Remainder.

5. Divide 784 by 82.

STATEMENT.

82)784(9$\frac{46}{82}$ Ans.

6. Divide 91070 by 8761.

STATEMENT.

8761)91070(10$\frac{3460}{8761}$ Ans.

7. Divide 2461 by 74.
8. Divide 4809 by 91.
9. Divide 13872 by 263.
10. Divide 54123 by 1423.
11. Divide 628100 by 156.
12. Divide 10000 by 304.

13.   Divide 37021 by 2002.

14.   Divide 8888888 by 332311.

15.   Divide $6805 equally between 5 men. What will be the share of each?   Ans. $1361.

16.   What is the *sixty-fourth* part of $44800?
    Ans. $700.

17.   145 men picked 1305000 pounds of cotton. Supposing each picked an equal quantity, how much did each man pick?   Ans. 9000 pounds.

18.   A father gave his 7 sons a Christmas present of $353.50 to be shared equally among them. What was each one's share?   Ans. $50.50.

139.   *To Divide when there are Naughts on the Right of the Divisor.*

1.   Divide 2843 by 200.   Ans. $14\frac{43}{200}$.

OPERATION.

2|00)28|43
————
14 and 43 Rem.

Explanation.—Since by our scale of numbers they *increase* from *right* to *left* in a tenfold ratio, and *decrease* from *left* to *right* in a corresponding manner, it is clear that the removal of any order of figures from *left* to *right* diminishes its value *ten* times for each place of removal. And as previously shown page 80, that the *annexing* of naughts *multiplies* numbers, by removing them to places of higher value, so in like manner, *cutting figures off* from the *right* of a number removes the remaining orders to the *right*, and hence *decreases* them *tenfold* for every figure cut off. Hence to cut off *one* figure is dividing by 10; to cut off *two* figures divides by 100; to cut off *three* figures divides by 1000; and so on.

Considering these principles, in all cases of this kind we cut off the naughts from the right of the divisor and the same number of figures from the right of the dividend; and then divide the remaining figures of the dividend by the remaining figures of the divisor. When there is a remainder, annex the figures cut off, and we obtain the true remainder.

2. **Divide** 87931 by 1000. Ans. $87\frac{931}{1000}$.

OPERATION.

1|000) 87|931

Quotient 87 and 931 Remainder.

3. **Divide** 178 by 10.

OPERATION.

1|0) 17|8

Quotient 17—8 Remain'r.
Ans. $17\frac{8}{10}$.

4. **Divide** 6581 by 300.

OPERATION.

3|00) 65|81

Quotient 21—281 Rem.
Ans. $21\frac{281}{300}$.

5. **Divide** 71468071 by 341000.

OPERATION.

341|000)71468|071(209$\frac{199071}{341000}$ Ans.

682

3268
3069

199

6. Divide 8897600 by 8100. Ans. $1098\frac{38}{81}$.
7. Divide 1000000 by 10000. Ans. 100.
8. Divide 99999 by 9000. Ans. $11\frac{999}{9000}$.
9. Divide 33440 by 270. Ans. $123\frac{230}{270}$.
10. Divide 140817 by 6800. Ans. $20\frac{4817}{6800}$.

140. *To Divide by the Factors of a Number.*

1. Divide 936 by 24.

OPERATION.

4)936

---

6)234

39

Explanation.—In all problems where the divisor is a composite number, we may divide by the factors and thus shorten the operation. In this example the factors are 4 and 6, and we first divide by 4 which gives a quotient 6 times too large, for the reason that 4 is but $\frac{1}{6}$ of 24 the true divisor. We therefore divide this quotient by 6 and obtain the true quotient.

2. Divide 588 by 28.   The factors are 4 and 7.
Ans. 21.

3. Divide 6976 by 32.   The factors are 4 and 8.
Ans. 218.

4. Divide 2583 by 63.   The factors are 7 and 9.
Ans. 41.

5. Divide 10206 by 81.   The factors are 9 and 9.
Ans. 126.

6. Divide 11984 by 56.   The factors are 8 and 7.
Ans. 214.

141.   *To Find the True Remainders when Dividing
by the Factors of a Number.*

7.   Divide 1607 by 72, using the factors 3, 4, and
6, and find the true remainder.
Ans. 22 quotient, and 23 remainder.

FIRST OPERATION.

3) 1607
——
4) 535   2, 1st remainder.
——
6) 133   3, 2d remainder.
——
22   1, 3d remainder.

Explanation.—In this ex-
ample, using as divisors 3,
4, and 6, the factors of 72,
we obtain for remainders 2,
3, and 1.
The first remainder 2, is
clearly units of the given
dividend, and hence a part
of the true remainder.

The second remainder 3, being *fourths* of the second dividend
535, which are reciprocal *thirds* of the given dividend, it is
hence ⅓ of the reciprocal of ¼ of ⅓ = 9, of the given dividend
and true remainder.

The third remainder 1, being *sixths* of the third dividend
133, which are reciprocal *twelfths* of the given dividend, it is
hence ⅙ of the reciprocal of ⅙ of ¼ of ⅓ = 12, of the given
dividend and true remainder.   Therefore 2, the first remain-
der, plus 9, the unit value of the second remainder, plus 12,
the unit value of the third remainder = 23, the true remain-

der. Or we may obtain the true remainder without consider-
ing the reciprocal relationship of the quotients and divisors,
thus:

First remainder 2
Plus 2d remainder 3, ✕ the preceding divisor 3, = . 9
Plus 3d remainder 1, ✕ all the preceding divisors, 4 and 3=12

which added gives the true remainder 23

From the foregoing, we see that the true remainder may be
obtained by adding to the first remainder, the product of the
other remainders by all the divisors preceding the one which
produced it.

8. Divide 7851 by 64, using the factors 8 and 8.
Ans. 122 quotient, 43 remainder.

OPERATION.

8)7851

8)981—3, 1st remainder.

122—5, 2d remainder.

Explanation.—Here the 1st re-
mainder is 3, to which we add
the product of the 2d remain-
der 5, multiplied by the preced-
ing divisor 8, equals 40, making
43, the true remainder.

9. Divide 17803 by 96, using the factors 2, 3, 4,
and 4. Ans. $185\frac{43}{96}$.

OPERATION.

2)17803

3)8901—1

4)2967—0

4)741—3

185—1

Explanation.

1st remainder, . . . 1

2d remainder, 0✕2= . . 0

3d remainder, 3✕3✕2= . 18

4th remainder, 1✕4✕3✕2= 24

True remainder, . 43

10. Divide 27865 by the factors of 81.
Ans. $344\frac{1}{81}$.

11. Divide 101041 by the factors of 84.
Ans. $1202\frac{73}{84}$.

12. Divide 899 by the factors of 108.

Ans. $8\frac{35}{108}$.

13. If $4691 are divided equally between 35 men, what will each one receive? Ans. $134\frac{1}{35}$.

14. There are 32 quarts in one bushel. How many bushels are there in 1536 quarts?

Ans. 48 bushels.

15. A hogshead of wine contains 63 gallons. How many hogsheads in 2898 gallons?

Ans. 46 hogsheads.

MISCELLANEOUS PROBLEMS IN DIVISION.

142. 1. One of the factors of 10800 is 225. What is the other? Ans. 48.

2. What number multiplied by 137, will give 959137 for the product? Ans. 7001.

3. Multiplying 372 by an unknown number gives 44640. What is the number? Ans. 120.

4. What is the quotient of 9126 divided by 9?

Ans. 1014.

5. Divide four million eight thousand sixteen by MMDCXLIV. Ans. $1515\frac{2256}{2644}$.

6. The Great Church of St. Peter, in Rome, will accommodate 57000 people. The largest Church in New Orleans will accommodate 2500. How many times is the capacity of the church of St. Peter equal to that of the largest church in New Orleans?

Ans. $22\frac{20}{25}$.

7. Physiologists inform us that the man of average weight, 140 pounds, absorbs and discharges through the various organs of his body, 7 pounds

per day, of the different materials composing his body. Now, supposing that this waste continued, and no new material was absorbed or assimulated, how many days would it take for a man weighing 140 pounds, to waste away?   Ans. 20 days.

8.   What number is that to which, if sixteen be added, the sum multiplied by 8, and 13 subtracted from the product, the remainder will be 339?

Ans. 28.

OPERATION.

339+13=352.÷8=44—16=28 Ans.

The student will write the full reasoning.

9.   There is a number from which if you subtract 55, and divide the remainder by 12, your quotient will be 36.   What is that number?   Ans. 487.

OPERATION.

36×12=432+55=487 Ans.

The student will write the full reasoning.

10.   A merchant owes a debt of $1875, which he agreed to pay by weekly installments of $25.   He has made 55 payments.   How many more payments has he to make?   Ans. 20.

11.   The "Father of the Forest," as he lies in his tomb in the cemetery of the Calaveras Grove of Big Trees in California, measures 450 feet in length, and 112 feet in circumference at the larger end.   If you are 5 feet high, and 30 inches around the chest, how many times is the length of this great tree equal to your height, and how many times is its circumference equal to your girth of chest?

Ans. 90 times my length, and $44\frac{24}{30}$ times my circumference of chest.

12.   About *one-sixth* of man's weight is blood. How many pounds of blood in a man whose weight is 168 pounds?   Ans. 28 pounds.

13. A merchant bought 350 barrels of flour at
$6 a barrel, and sold it at $7.50 per barrel. The gain
he gave in equal parts to 4 worthy boys, to aid them
in obtaining an education. What was the cost and
the selling price of the flour, and how much money
did each boy receive?   Ans. $2100 cost, $2625
                selling price, $131.25 each boy received.

PARTIAL OPERATION.

350 × $6 = $2100 cost.

350 × $7.50 =   2625 sales.

4 | $525 gain. .

 | $131¼ each boy's share.

14. An acre contains 160 square rods. How
many acres in a plantation containing 123200 square
rods?   Ans. 770 acres.

15. A boy sold 50 oranges at 5¢ each, and there-
by gained $1.50. At what rate did he buy the
oranges?   Ans. 2¢ a piece.

16. How many times 136 will produce 1768?
                                Ans. 13.

17. Divide the product of 750 and 875, by their
difference.   Ans. 5250.

18. The diameter of the earth at the equator is
7925 miles. How long would it take a locomotive
to travel that distance, at the rate of 25 miles an
hour?   Ans. 317 hours = 13 days, 5 hours.

19. It is estimated that, by reason of intemper-
ance, the United States loses annually $98400000.
How many school houses costing $5000 each, and
how many libraries costing $3000 could be estab-
lished with this amount of money?
                        Ans. 12300 of each.

20. The first Atlantic Telegraph Cable, as originally made, cost $1258250. 10 miles of deep sea cable were made at a cost of $1450 per mile, and 25 miles of shore ends were made at a cost of $1250 per mile. The remainder cost $485 per mile. How many miles of Cable were made?

Ans. 2535 miles.

PARTIAL OPERATION.

$1450 × 10 = $14500.

$1250 × 25 = 31250.

———

$45750.

$1258250 — $45750 = $1212500 ÷ 485 =

2500 miles + 10 miles + 25 miles = 2535 miles Ans.

21. A grocer wishes to put 3335 pounds of sugar in 3 kinds of boxes, containing respectively 20, 50, and 75 pounds, and use the same number of boxes of each kind or size. How many boxes will he require?   Ans. 23 of each size.

22. The Northern Pacific Railroad from Lake Superior to Puget Sound, as located, is 2000 miles long. The estimated cost and equipment of the road, including interest, is $85277000. What will be the average cost per mile?   Ans. $42638.50.

23. The capacity of steam engines is measured by *horse power;* and 1 horse power is a force that will raise 33000 pounds, 1 foot in 1 minute. How much horse power has a steam engine that possesses a capacity of 1188000 pounds?   Ans. 36.

24. The average weight of man is 150 pounds, and about $\frac{1}{8}$ of this weight is blood. Allowing that the heart throws out 2 ounces of blood at each pulsation, that it beats 72 times a minute, and that 16 ounces make a pound, how long will it take the heart to circulate all the blood in the body?

Ans. $2\frac{112}{144}$ minutes,

25.   Ten freedmen agreed to pick 20000 pounds of cotton and receive for their labor ⅕ of the cotton picked.   After they had picked 7000 pounds, 4 freedmen quit, leaving the other 6 to finish the work. How much cotton is each entitled to when the work is finished?   Ans. 140 pounds each for those who left, and 573⅔ each for those who remained.

<center>OPERATION.</center>

5 | 7000 pounds picked by 10 freedmen.

10 | 1400 pounds due the 10 freedmen.

140 pounds due each of the 10 freedmen.
  20000 pounds to pick.
    7000   "   picked by 10.

  5)13000   "   to be picked by 6.

6 | 2600   "   due to the 6.

    433⅔   "   due each of the 6.
    140   "   "   "   "   10.

    573⅔   "   "   "   "   6.

26.   Prof. Wilson, a physician and physiologist, has counted in the skin of the palm of the hand, 3528 perspiratory pores to the square inch; but as there are less to the square inch on some other parts of the body, he estimates that 2800 is a fair average to allow to the square inch, for the whole surface of the body.   The average size man has 2500 square inches of body surface, which would give 7000000 perspiratory pores.   Through these pores fully 2 pounds of perspiration, water, refuse matter, and worn out tissue pass every 24 hours.   If a man weighs 150 pounds, how long will it take for matter

equal to the weight of the body to pass through the perspiratory pores, if they are kept open as they should be by daily bathing? Ans. 75 days.

27. A merchant bought 800 gallons of molasses at 65¢, and sold ½ of it at 72¢ a gallon. From the profit he bought his children a set of Cutter's Anatomical and Physiological Charts, and had $8.20 left. What did the charts cost? Ans. $19.80

OPERATION.

$$2)\overline{\quad 800 \text{ gallons @ } 65\text{¢} =}\qquad 2)\overline{\quad \$520 \text{ cost.}}$$

$$\$260 \text{ one-half cost.}$$

400 gallons @ 72¢ =   288 sales.

$$\overline{\quad\$28 \text{ gain.}}$$

$28 — $8.20 = $19.80 Ans.

The student should write a full explanation.

28. The circumference of our earth at the equator is 24899 miles, and the mean diameter of the earth is 7912 miles. How many times is the circumference as great as the mean diameter?

Ans. $3\frac{1119}{1912}$ times.

29. Our Earth is about 95000000 miles from the Sun, and Neptune, the most distant member of our Solar System, is about 2850000000. How many times as far as our earth is Neptune from the Sun?

Ans. 30.

30. The velocity of the Earth on its yearly voyage around the Sun is 99733 feet per second. The velocity of a cannon ball fired from a gun with an average charge of powder is 1750 feet a second. How many times as fast as the velocity of a cannon ball, is the velocity of our Earth?

Ans. $56\frac{1733}{1750}$.

31. Geo. Peabody of Mass. gave, while living, to 27 schools and colleges, library associations, benevolent societies, state public schools, etc., not including many private presents, $7875000.   Of this amount, $3300000 were given to the public schools of the South. What part of the whole specified donation did he give to the South?

Ans. $\frac{3300000}{7875000}$.

32. Stephen Girard, of Philadelphia, gave $6000000 for the founding and support of Girard College. Soulé's College in New Orleans is worth $40000. How many such colleges could be built with the amount of money given by Mr. Girard to establish one college?   Ans. 150.

33. The air which surrounds our earth, and of which we each inhale 600 gallons every hour, is composed of 4 parts of Nitrogen, and 1 part of Oxygen. How many gallons of each are there in a room 22 feet long, 21 feet wide, and 10 feet high, and which contains 34560 gallons of air?

Ans. 6912 Oxygen, 27648 Nitrogen.

Reasoning Solution.—Since by the terms of the problem the air is 4 parts Nitrogen and 1 part Oxygen, there are 5 parts of compound gas in each gallon of air; and since there are 5 parts in each of the 34560 gallons of air, there are as many *one parts* as 34560 gallons are equal to 5, which is 6912. Again, according to the facts of the problem, this number of *one parts* is Oxygen, and 4 times this *one* part, which is 27648, is Nitrogen. ∴ 6912 gallons are Oxygen, and 27648 gallons are Nitrogen.

34. A room contains 34560 gallons of air, and a man inhales 600 gallons per hour. How long will it take for 10 men to inhale the air in the room. -

Ans. $5\frac{4560}{6000}$ hours.

35. A room 16 feet long, 10 feet wide, and 8 feet high, contains 1280 cubic feet of air.   Every time a person breathes he throws out from his lungs a sufficient quantity of carbonic acid, or carbon di-oxide,

(a most deadly gas,) to pollute, or render poisonous and unfit for breathing, 3 cubic feet of air, and he breathes 17 times a minute. How long will it take for the air of a room of the above dimensions to become poisonous, if occupied by 5 persons, and no change of air is made by ventilation.

Ans. $5\frac{5}{255}$ minutes.

OPERATION INDICATED.

1280 cubic feet $\div (3 \times 17 \times 5) =$Ans.

36. A man produces by breathing at least 6 gallons of carbonic acid gas every minute; a single burning gas jet, 10 gallons; an ordinary stove, 60 gallons. How many gallons of carbonic acid gas will an audience of 1000 people, 2 heated stoves, and 50 burning gas jets produce in 3 hours, and how many times would the quantity fill a room 100 feet long, 50 feet wide, and 30 feet high?

Ans. 1191600 gallons. $1\frac{16059600}{259200000}$ time.

PARTIAL OPERATION—TO AID THE STUDENT.

6 gallons $\times$ 1000 (people) = 6000 gallons.
60 gallons $\times$ 2 (stoves) = 120 "
10 gallons $\times$ 50 (gas jets) = 500 "

$$100 \times 50 \times 30 \times 1728 =$$
$$259200000 \text{ cubic inches.}$$

$\left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\{$ 6620 gallons in 1 minute.
60
397200 galls. in 60 minutes
3   or 1 hour.

1191600 galls. in 3 hours.
231   cubic in. in 1 gal.

275259600 cubic inches.

$275259600 \div 259200000 =$ Ans.

<hr>

NOTE.—There are 60 minutes in an hour, 231 cubic inches in a gallon, and 1728 cubic inches in a cubic foot.

37. Astronomers estimate that 7500000 visible meteors fall upon the earth daily, the average weight of which is estimated to be 100 grains each. Allowing for an equal quantity of matter to be brought down by the invisible meteors and the ærolites, how many pounds a year does our earth increase in weight, there being 7000 grains in a pound, and 365 days in a year?

Ans. 7821428 5/7 pounds.

143. *Problems Involving the English Money Account.*

1. What will 13840 pounds of cotton cost, at 8 pence a pound. Ans. £461. 6s. 8d.

OPERATION.

```
  13840
     8d.
 ───────
12) 110720d.
 ───────
20) 9226   8d.
 ───────
£ 461   6s.
```

Explanation.—In this problem, the price is given in one of the subdivisions of the English monetary unit, and hence we must know what that unit and its subdivisions are, before we can solve the problem. The English monetary unit is the *Pound Sterling,* which is divided into 20 *Shillings;* each shilling is divided into 12 *Pennies,* and each penny into 4 *Farthings.* With this knowledge of English money, we can work all problems of the above character. In this example we first multiply the price of one pound by the number of pounds, and thus produce the value of the whole in pence. Then to reduce the *pence* to *shillings,* we divide them by 12, and obtain 9226 *shillings* and a remainder of 8, which being a part of the dividend is therefore *8d.* Then to reduce the *shillings* to *pounds,* we divide them by 20, and obtain 461 *pounds* and a remainder of 6, which being a part of the second dividend is therefore *6s.* In the English monetary system the following abbreviations are used: £. represents pounds, s. represents shillings, d. represents pence, and f. represents farthings.

2.   What is the value of 483 yards of cloth, at 16 shillings per yard?
Ans. £386. 8s.

OPERATION.

$$483$$
$$16$$

2ø) 772ø shillings.

£ 386 8s.

3.   What will 241 boxes cheese cost, at £3 per box?   Ans. £723.

OPERATION.

$$241$$
$$3$$

£ 723

4.   Sold 486 yards of calico at 5 pence a yard. What did it amount to ?   Ans. £10 2s. 6d.

5.   Bought 38495 pounds of good middling cotton at 7 pence a pound.   How much did it cost?
Ans. £1122 15s. 5d.

6.   What is the value of 850 barrels of flour at 34 shillings a barrel?   Ans. £1445.

7.   How much will 1812 tons of iron cost, at £52 4s. per ton?   Ans. £94586 8s.

8.   Bought 38421 pounds of cotton at 9 pence per pound.   What did it cost?   Ans. £1440 15s. 9d.

SYNOPSIS FOR REVIEW.

Define the following words and phrases:

121. Division. 122. If 2 yards cost $4, what will 1 yard cost? 122. Divide 6 by 3 and give the reasoning. 123. Dividend. 124. Divisor, or Unit of Measure. 125. Quotient. 126. Remainder. 127. Sign of Division. 128. Other Signs of Division. 129. The 6 Principles of Division. 130. Proof of Division. 131. How many ways may Division be performed? 131. What are they? 133. Fractional Numbers. 134. The Philosophy of Division. 135. How to Divide when the Divisor does not exceed twelve. 136. Short Division. 137. To Divide when the Divisor exceeds twelve. 137. Long Division. 138. Give the General Directions for Long Division, and all the Details for the full operation. 139. To Divide when there are naughts on the right of the Divisor. 140. To Divide by the Factors of the Divisor. 141. To find the True Remainder. 143. Operation in English Money.

144. MISCELLANEOUS PROBLEMS

Involving the Principles of Addition, Subtraction,

Multiplication, and Division.

1. The subtrahend is 216, and the remainder 184. What is the minuend? Ans. 400.

2. A grocer paid $350 for some tea and some coffee, and for the tea he paid $50 more than for the coffee. What did he pay for each?
Ans. Tea $200, coffee $150.

PARTIAL OPERATION—TO AID THE STUDENT.

$350—$50=$300÷2=$150 paid for coffee.
$150+$50=$200, amount paid for tea.

3. H. M. Hornor has 25 cents, and F. L. Richardson has four times as many lacking 10 cents. How many cents has Richardson? Ans. 90 cents.

4. A slate costs 15 cents; an arithmetic four times as much as the slate; and a philosophy twice as much, lacking 25 cents, as the slate and arithmetic. What did they all cost? Ans. $2.00.

15¢ cost of slate.
15¢×4=60¢ cost of arithmetic.
15¢+60¢=75¢=cost of slate and arithmetic.
75¢×2=$1.50=twice cost of slate and arithmetic.
$1.50—25¢=$1.25=cost of philosophy.
15¢+60¢+$1.25=$2.00 Ans.

5. If 2 men start from the same point and travel in opposite directions, one at the rate of 20 miles per day and the other at the rate of 25 miles per day, how many days will they travel before they are 495 miles apart? Ans. 11.

6.   What number multiplied by 4 will give the same product as 16 multiplied by 12?   Ans. 48.

7.   If the speed of the Steamer J. M. White is 15 miles per hour in still water, and the velocity of the river is 3 miles per hour, how far will she run up the river in 4 hours? How many miles down the river in 4 hours? How far if she runs in still water 4 hours?   Ans. 48 miles up; 72 miles down; 60 miles in still water.

8.   The sum of two numbers is 480, and their difference is 80.   What are the numbers?
Ans. 200, 280.

PARTIAL OPERATION—TO AID THE STUDENT.

480—80=400=the sum of two numbers less their difference.
400÷2=200=the lesser of two numbers.
200+80=280=the greater of two numbers.

9.   A man purchased a horse and a cow.   For the horse he paid $175, and for the cow $110 less than for the horse.   What did the cow cost?
Ans. $65.

10.   The lesser of two numbers is 224, and their difference 100.   What is the greater?   Ans. 324.

11.   The product of two numbers is 6450, and one of the numbers is 150.   What is the other.
Ans. 43.

12.   A merchant bought 415 yards calico at 10 cents per yard and sold it for 13 cents per yard. How much did he gain?   Ans. $12.45.

13.   The dividend is 37500 and the quotient 75. What is the divisor.   Ans. 500.

14.   A news boy sold 20 papers at 5¢ each, and with the money bought oranges at 4¢ each. How many oranges did he get?   Ans. 25.

15. A boy sold 5 chickens at 25¢ a piece, and 8 ducks at 50¢ each. He received in payment 3 pigeons at 30¢ each, and the balance in money. How much money did he receive? Ans. $4.35.

<div align="center">

OPERATION.

5 chickens @ 25¢ =	-	$1.25
8 ducks @ 50¢ =	-	4.00
Total amount due	-	$5.25
Cr. by 3 pigeons @ 30¢		.90
Balance due in cash		$4.35

</div>

16. The divisor is 37, the quotient 21, and the remainder 23. What is the dividend? Ans. 800.

17. The first battle of the Revolution was fought April 19, 1775. How many years, months, and days, have passed since then?

18. Chas. J. Sinnott has an orange orchard consisting of 480 trees, and each tree produces 5 barrels of oranges which are worth in the market $4 a barrel. What is the value of his orange crop?
Ans. $9600.

19. W. Couder bought a barrel of sirop de batterie containing 43 gallons at 95¢ per gallon; 4 gallons having leaked out he sold the remainder at $1.05 a gallon. How much did he gain by the transaction? Ans. $.10.

<div align="center">

PARTIAL OPERATION.

</div>

43 gallons @ 95¢ = - - - - $40.85 cost.
43 gals.—4 gals.=39 gals. @ $1.05 $40.95 sales.

<div align="right">.10 gain.</div>

20. R. S. Soulé bought 354 barrels of flour for $2478, and sold the same at $7.50 per barrel. How much did he gain? Ans. $177.

21. The Capital Stock of a Manufactory is $100000, which is divided into 200 shares. What are 5 shares worth?   Ans. $2500.

22. J. T. Richter sold to H. Roos 25 barrels of apples at $4 per barrel, and 124 barrels of potatoes at $3.25 per barrel. He received in payment 1 hogshead of sugar containing 1143 pounds at 8¢, and the remainder in money. How much money did he receive?   ·   Ans. $411.56.

23. A speculator bought 528 cords of wood at $6.50 per cord. He re-corded the wood so that it measured 579 cords, which he sold at $6.75 a cord. How much did he gain?   Ans. $476.25.

CANCELLATION.

145. **Cancellation** is the process of shortening the operations of division, or of the indicated result of multiplication and division operations combined, by rejecting equal factors from both dividend and divisor, or from both increasing and decreasing numbers.

The operation is performed by drawing a line across each factor *cancelled*, or cut out; thus, $\cancel{8}, \cancel{7}, \cancel{2\frac{1}{3}}$.

146. The **Principles of Cancellation**, are,

1. Rejecting, or cancelling a factor from any number is in effect dividing the number by that factor.

2. Rejecting, or cancelling equal factors from both dividend and divisor, or from. both increasing and decreasing numbers in an indicated result, does not change the quotient or result.

<div align="center">EXAMPLES.</div>

Divide $7 \times 3 \times 4$ by 7×4.

Operation by Cancellation.

$$\begin{array}{c} \cancel{7}|\cancel{7} \\ \cancel{4}|3 \\ |\cancel{4} \\ \hline 3, \text{ Ans.} \end{array}$$

Explanation.—In all problems where we have both multiplication and division operations to perform, we use a vertical or perpendicular line which we call the statement line.

This line is used to facilitate the work by separating the dividends and the divisors, or the increasing and the decreasing numbers. The dividends, or the increasing numbers, are always placed upon the right hand side of the line and the divisors, or decreasing, numbers are always placed upon the left hand side.

In this example, having written the numbers that constitute the dividend and the divisor, respectively upon the right and the left hand side of the statement line, we *cut out*, or *cancel*,

the equal factors 7's and 4's in the numbers constituting the dividend and the divisor, and thus obtain 3 as the answer to the problem.

To perform the work without the aid of Cancellation, we would be obliged to make the following figures: $7 \times 3 = 21$, which $\times 4 = 84$, the dividend; then $7 \times 4 = 28$, the divisor; then

$$28)84(3, \text{ Ans.}$$
$$84$$
$$—$$

2. Multiply 25, 48, and 88 together, and divide the product by the product of 10, 36, and 8.

Operation by cancellation.   *Explanation.*—In this example, we write the numbers on the line as above directed and then cancel 10 and 25 by 5; then 36 and 48 by 12; then 8 and 88 by 8; then 4 and 2 by 2. This is all that can be cancelled, and we then multiply together 5, 2, and 11, divide the product by 3, and thus obtain the true result, $36\frac{1}{3}$.

$$\begin{array}{ccc} \cancel{2} & 10|2\cancel{5} & 5 \\ 3 & 3\cancel{6}|4\cancel{8} & \cancel{4} \ 2 \\ & \cancel{8}|8\cancel{8} \ 11 \\ \hline & 110 \\ \hline & 36\tfrac{2}{3}, \text{ Ans.} \end{array}$$

Should the student experience any difficulty in this kind of work, he should be orally drilled on the factors of numbers and on composite numbers.

3. Divide the product of 32×3 by $8 \times 9 \times 16$.

Operation by Cancellation.   *Explanation.*—Having written the numbers on the statement line, we first cancel 8 and 32 by 8; then 9 and 3 by 3; then 16 and 4 by 4. Now having no more numbers on the increasing side of the line to cancel, we multiply together the remaining numbers on the decreasing side of the line and thus produce the correct result $\frac{1}{12}$.

$$\begin{array}{cc} & \cancel{8}|3\cancel{2} \ \cancel{4} \\ 3 & 9| \ \cancel{3} \\ 4 & 1\cancel{6}| \\ \hline 12| & 1 = \tfrac{1}{12}, \text{ Ans.} \end{array}$$

In all cases where, after cancelling, no factor appears on either side of the statement line, the factor 1 is always understood as being there. Its non-appearance is in consequence of not having written it when we cancelled a number by itself.

4.   A merchant sold 25 boxes of candles containing 36 pounds each at 16¢ per pound, and received in payment starch at 6 cents per pound. How many boxes, each containing 30 pounds, did he receive?   Ans. 80 boxes.

Operation by Cancellation.

$$\begin{array}{c|cc} & \cancel{6}|16 & \\ \cancel{5}\ \ \cancel{30} & \cancel{25}\ \ 5 & \\ & \cancel{36}\ \ \cancel{6} & \\ \hline & 80 \ \text{boxes, Aus.} \end{array}$$

————

GENERAL DIRECTIONS FOR CANCELLATION.

————

147.   From the foregoing elucidations, we derive the following general directions for cancellation :

1.   *Cancel all the factors common to both dividend and divisor, or of the increasing and decreasing numbers.*

2.   *Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

NOTE.—When a factor cancelled is *equal* to the number itself, the unit 1 remains, since a number *divided* by *itself* gives 1 as a quotient.   If the 1 is in the *dividend* it must be *retained;* if in the *divisor*, it may be disregarded, since *dividing* by **1** does not change the quotient.

17

148.　Cancel and work the following line state-ments or results:

(1)	(2)	(3)	(4)
2|6	12|45	31|124	9|76
3|9	9|56	5|17	20|91
18|54	7|84	51|10	70|140
27|	16|4	4|	|25
1, Ans.	70, Ans.	$\frac{2}{3}$, Ans.	$1921\frac{1}{9}$, Ans.

5.　Divide the product of 6, 7, 12, and 22 by the product of 11, 3, 14, and 8.　　　　Ans. 3.

6.　What is the quotient of $28 \times 65 \times 7 \times 78 \div 56 \times 130 \times 42 \times 13$?　　　　Ans. $\frac{1}{4}$.

＋ 7.　Multiply 21, 55, and 128 together, and divide the product by $14 \times 25 \times 64$.　　　Ans. $6\frac{3}{5}$.

8.　How many bushels of corn, at 70¢ a bushel, will pay for 140 gallons molasses at 65 cents a gal-lon?　　　　Ans. 130 bushels.

9.　Bought 420 pounds of sugar at 6 cents a pound, and gave in payment 360 pounds of rice. What was the price of the rice?　Ans. 7 cents.

10.　Sold a drayman 64 bushels of oats at 75 cents a bushel, for which he is to pay in drayage at 50 cents a load.　How many loads must he haul?　　　　　Ans. 96 loads.

11.　Paid 65¢ for 5 yards of calico.　What will 27 yards cost at the same rate?

Analytic Solution by Cancellation.

$$\begin{array}{c} \cancel{65}\ 13 \\ \cancel{5}\ 27 \\ \hline \$3.51, \text{ Ans.} \end{array}$$

Explanation.—In all practical problems of this kind, we give a reason for each step of the opera-tion, and make the whole state-ment to indicate the final result without performing any of the intermediate work.　In this prob-lem, we place the 65¢ on the in-creasing side of the statement line as our premise, and reason thus: 5 yards cost 65¢.　Since 5 yards cost 65¢, 1 yard will cost $\frac{1}{5}$ part of it, and 27 yards will cost 27 times as much as 1 yard.

12. How many pounds of butter at 35¢ per pound, will pay for 245 pounds of rice at 5 cents per pound?   Ans. 35 pounds.

13. If 17 barrels of flour cost $110.50, what will 500 barrels cost at the same rate?   Ans. $3250.

SYNOPSIS FOR REVIEW.

Define the following words and phrases:

145. Cancellation. 146. Principles of Cancellation. 147. General Directions for the operation.

PROPERTIES OF NUMBERS.

DEFINITIONS.

149. The **Properties** of numbers are those qualities which belong to them.

150. An **Integer** is a whole number; as 1, 5, 6, 18, etc. Whole numbers are divided into two classes, *Prime* and *Composite*.

151. A **Prime Number** is one that can be divided, without a remainder, only by itself and 1; as 1, 2, 3, 5, 7, 11, 13, 17, etc.

152. A **Composite Number** is one that can be divided, without a remainder, by some other whole number than itself and 1; as 4, 9, 12, 15, 24, etc.

All composite numbers are the product of two or more other numbers.

Numbers are prime to each other when they have no common factor that will divide each without a remainder; as 6, 13, 20, etc.

153. An **Even** number is one that can be divided by 2, without a remainder; as 4, 8, 12, 56, etc.

154. An **Odd** number is one that cannot be divided by 2, without a remainder, as 1, 7, 19, 45, 133, etc.

155. The **Factors** of a number are the numbers which multiplied together will produce it. Thus 4 and 4, or 2 and 8 are factors of 16; 2, 3, and 4, or 2, 2, 2, and 3 are factors of 24.

Every factor of a number is a divisor of it. ·

156. A Prime Factor of a number is a prime number that will divide it without a remainder. Thus, 1, 2, 3, and 5 are the prime factors of 30.

157. A Composite Factor of a number is a composite number that will divide it without a remainder. Thus, 6 and 8 are composite factors of 48.

158. A Perfect Number is one which is equal to the sum of all its divisors, except itself; as $6 = 1 + 2 + 3$.

159. An Imperfect Number is one the sum of whose divisors is more or less than itself. Thus, 14 is greater than $1 + 2 + 7$, its divisors; and 18, is less than $1 + 2 + 3 + 6 + 9$, its divisors.

160. An Aliquot part of a number is such a part as will divide it without a remainder. Thus, 1, 2, 3, 4, 6, and 8 are aliquot parts of 24. All aliquots are factors of the number.

161. The Reciprocal of a number is the quotient of 1 divided by the number. Thus, the reciprocal of 8 is $1 \div 8 = \frac{1}{8}$; and the reciprocal of $\frac{1}{4}$ is $1 \div \frac{1}{4} = 4$.

162. Powers of a number are as follows: The *first Power* is the number expressed by itself; as, 5 is the first power of 5. The *second power* is the product arising by using the number as a factor *twice;* as, $5 \times 5 = 25$, which is the *second power* of 5. The *third power* is the product obtained by using the number as a factor *three* times; as, $5 \times 5 \times 5 = 125$, which is the *third power* of 5. And in like manner higher powers of numbers are obtained.

163. The Multiple of a number is any product, dividend, or number of which a given number is a *factor*, or which is exactly divisible by a given number; as, 25 is a multiple of 5; 24 is the multi-

ple of 2, 3, 4, 6, 8, and 12.   A number may have an indefinite number of multiples; as, 2 will divide 4, 6, 8, 10, 12, 14, etc., indefinitely.

164.   A Multiple of a number is one which is divisible by the given number without a remainder. Thus, 9 is a multiple of 3; 28 of 7.

165.   A Common Multiple *of two or more given numbers* is a number divisible by each of them without a remainder.   Thus, 24 is a common multiple of 1, 2, 3, 4, 6, 12, and 24.

166.   The Least Common Multiple of two or more given numbers is the least number that is divisible by each of them without a remainder. Thus, 12 is the *least* common multiple of 1, 2, 3, 4, 6, and 12.

NOTE.—Since every number is divisible by 1 and itself, the factors 1 and the given number are not usually given when naming the multiples.   We shall not hereafter name them as multiples.

DIVISIBILITY OF NUMBERS.

167.   A Divisor, or measure of a number, is any number that will divide it without a remainder. Thus, 4 is a divisor, or measure, of 12, and 5 is a divisor, or measure of 20.

One number is said to be **Divisible** by another when there is no remainder after dividing.

168.   A Common Divisor of two or more numbers is a number that will divide each of them without a remainder.   Thus, 2 is a common divisor of 12, 18, and 24.

169.   The Greatest Common Divisor of two or more given numbers is the greatest number that will divide each of them without a remainder. Thus, 6 is the *greatest* common divisor of 12, 18, and 24.

170. Every number is divisible by 2 whose unit figure is divisible by 2. Thus, 34, 176, 790 are each divisible by 2.

171. Every number is divisible by 4 when its units and tens figures are divisible by 4. Thus, 156, 264, 34512, 561308, are each divisible by 4.

172. All numbers are divisible by 3, the sum of whose figures are divisible by 3. Thus, 114, 225, 4101, are each divisible by 3.

173. All numbers ending in 0 or 5 are divisible by 5. Thus, 10, 15, and 35, are each divisible by 5.

174. All numbers whose unit figure is divisible by 2, and whose sum is divisible by 3, are divisible by 6. Thus, 36, 102, 678, 15936, are each divisible by 6.

175. Every number is divisible by 8 when the units, tens, and hundreds figures are divisible by 8. Thus, 3824, 12512, 190720 are each divisible by 8.

176. All numbers are divisible by 9, the sum of whose figures are divisible by 9. Thus, 441, 3456, 123453, are each divisible by 9.

177. All numbers ending in naught are divisible by 10. Thus, 20, 380, 11750, are each divisible by 10.

178. All numbers occupying four places, in which the *first* and *fourth* are significant and alike; and the *second* and *third* naughts, are divisible by 7, 11, and 13. Thus, 1001, 3003, 5005, 9009, are each divisible by 7, 11, and 13.

SYNOPSIS FOR REVIEW.

Define the following words and phrases:

149. The Properties of Numbers. 150. An Integer. 151. A Prime Number. 152. A Composite Number. 152. When are numbers *prime* to each other? 153. An Even Number. 154. An Odd Number. 155. Factors of a Number. 156. A Prime Factor. 157. A Composite Factor. 158. A Perfect Number. 159. An Imperfect Number. 160. An Aliquot. 161. The Reciprocal of a Number. 162. The Powers of a Number; 1st. 2nd, 3d, etc. 163. The Multiple of a Number. 164. A Multiple. 165. A Common Multiple. 166. Least Common Multiple. 167. A Divisor. 168. A Common Divisor. 169. Greatest Common Divisor. 170 to 178. What numbers are divisible by 2, 3, 4, 5, 6, 8, 9, 10, 7, 11, 13.

RACTIONS.

179. According to the general methods, prescribed by the text books of our country, for the various operations of fractions, the subject is correctly considered, by both the teacher and the pupil, as the most difficult in the science of numbers. But by our fully evolved, logical, and philosophical system of handling fractional numbers the subject is simplified, rationalized, and rendered pleasing to the student. By our method of work, fully *one-half* of the time and figures required by the ordinary methods are saved, and all of the arbitrary and absurd rules which overload the organ of memory and prevent the expansion of the higher faculties of causality and comparison, are abandoned to the shades of the dead past and entombed with the ingenuous minds which gave them birth.

By our system, all the reasoning faculties of the mind are brought into action, and exercised in a manner to give logical strength and acuteness to work not only in the fields of mathematics but upon all the plains of life.

In behalf of truth, education, and humanity, we lament the non-progressive methods of the general school and college arithmetics.

The present authors of school arithmetics, with but few exceptions, are timidly following in the obscure paths of arithmetical science which were marked out ages ago, when the science was in its infancy—too cowardly, non-progressive, and contracted in their views to seek and explore more direct and comprehensible routes to the fountains of mathematical knowledge.

180. The **Unit** is the universal basis of numbers and the foundation of arithmetic. From *unity* arise two distinct classes of numbers. 1. Integers. 2. Fractions. The first class, *Integers*, have their origin in the multiplication of the Unit; and the second class, *Fractions*, result from the division of the *Unit*. The first is synthetical, the second is analytical.

DEFINITIONS.

181. A **Fraction** is one or more of the equal parts of a unit, or of a collection of units taken together. Or more briefly, it is a part of anything, or a numerical expression of a part of a unit; thus, 1 *half* and 3 *fourths* are fractions.

182. A **Fractional Unit** is one of the equal parts into which any integral unit is divided. If the integral unit is divided into two equal parts, each is called *a half;* if into three, each is called *a third;* if into four, each is called *a fourth;* and so on according to the number of parts into which the integral unit is divided.

183. Fractions are divided into two kinds, **Common,** or **Vulgar,** and **Decimal Fractions.**

184. **Common Fractions** are expressed by two numbers, one written above the other, with a horizontal line between them. The number below the line is called the **Denominator,** and the number above the line is called the **Numerator.** Thus, $\frac{1}{2}$ (*one-half*), $\frac{3}{4}$ (*three-fourths*), $\frac{5}{6}$ (*five-sixths*), $\frac{7}{8}$ (*seven-eights*), and $\frac{13}{17}$ (*thirteen-seventeenths*), are common fractions, the denominators of which are respectively, 2, 4, 6, 8, and 17. The Numerator and Denominator together, are called the *terms* of the fraction.

185. The **Denominator** of a fraction shows the number of equal parts into which the unit is divided.

Thus in the fraction $\frac{5}{8}$, the 8 is the denominator and shows that the unit is divided into 8 equal parts called *eighths.*

186. The **Numerator** of a fraction shows the number of equal parts taken to form the fraction.

Thus in $\frac{5}{8}$, the numerator is 5 and shows that 5 of the 8 equal parts are taken, or expressed, by the fraction.

All fractions arise from division and are expressions of unexecuted division in which the *numerator* is the *dividend,* the *denominator* the *divisor,* and the fraction itself the *quotient.*

187. **Decimal Fractions** are those in which the denominators are not generally expressed, but are always 10, or some power of ten; thus, .5, .75, .821, read respectively *five tenths, seventy-five hundredths,* and *eight hundred twenty-one thousandths,* are decimal fractions. To write these fractions as *common fractions,* they would be written thus, $\frac{5}{10}$, $\frac{75}{100}$, and $\frac{821}{1000}$.

The point (.) placed before the 5, 7, and 8, in the above decimally expressed fractions, is called the **decimal point,** and is used to abbreviate the work.

188. *Classification of Fractions.*

For convenience, fractions are classed under the following heads: *Proper Fractions, Improper Fractions, Simple Fractions, Mixed Numbers, Compound Fractions,* and *Complex Fractions.*

189. A Proper Fraction is one in which the numerator is less than the denominator; as, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$.

190. An Improper Fraction is one in which the numerator is equal to or greater than the denominator; as, $\frac{5}{5}$, $\frac{7}{4}$, $\frac{9}{8}$, and $\frac{16}{2}$.

191. A Simple Fraction is one in which both terms are whole numbers, and may be either a *proper* or an *improper* fraction; as, $\frac{3}{4}$, $\frac{9}{7}$, $\frac{11}{8}$, or $\frac{24}{5}$.

192. A Mixed Number is a number composed of a whole number and a fraction; as, $2\frac{1}{2}$, $5\frac{3}{4}$, and $21\frac{5}{12}$.

193. A Compound Fraction is a fractional part of a fraction or mixed number; as, $\frac{3}{4}$ of $\frac{5}{8}$, $\frac{1}{2}$ of $\frac{7}{16}$ of $12\frac{3}{4}$.

194. A Complex Fraction is one that has one or more of its terms fractional; as,

$$\frac{\frac{3}{4}}{\frac{5}{8}} \text{ of } \frac{6\frac{1}{3}}{5\frac{3}{7}}, \qquad \frac{3}{\frac{2}{8}} \text{ of } \frac{\frac{7}{8}}{1\frac{1}{2}} \text{ of } \frac{6\frac{1}{4}}{8}$$

195. The Reciprocal of a Fraction is the result of 1 divided by the fraction. Thus, the reciprocal of $\frac{4}{5}$ is $1 \div \frac{4}{5} = \frac{5}{4} = 1\frac{1}{4}$.

196. The Value of a Fraction is the result of its numerator divided by its denominator. Thus, $\frac{8}{2} = 4$, $\frac{21}{8} = 2\frac{5}{8}$.

197. *General Principles of Fractions.*

1. *Multiplying* the numerator, or *dividing* the denominator, *multiplies* the fraction.

2. *Dividing* the numerator, or *multiplying* the denominator, *divides* the fraction.

3. *Multiplying* or *dividing* both numerator and denominator by the same number does not change the value of the fraction.

FACTORING.

198. Factoring consists in separating or resolving a composite number into its factors. The operation is performed by division.

WRITTEN EXERCISES.

199. What are the prime factors, or divisors, of **5460?**

OPERATION.

2|5460
2|2730
5|1365
3| 273
    91

Explanation.—In all problems of this kind, we first divide the given number by any prime factor, and the successive quotients by prime factors, or divisors, until we obtain a quotient that is a prime number. In this problem, our last quotient is 91, which not being divisible, is a prime number. Hence the divisors 2, 2, 5, 3, and the quotient 91, are all prime factors, or divisors, of 5460.

200. What are the common prime factors of 28, **64,** and 72?

OPERATION.

2|28 64 72
2|14 32 36
  7 16 18

Explanation.—In all problems of this kind, we divide the given numbers by any common prime factor of all the numbers, and the quotients thus obtained are divided in the same manner, till they have no common factor, or divisor. The several divisors will be the common prime factors of the numbers.

NOTE.—A number that is a factor, or divisor, of two or more numbers is called a Common Factor of these numbers.

201. Find the prime factors of the following numbers:

1.	84	4.	6105	7.	25600
2.	376	5.	1683	8.	10376
3.	864	6.	3560	9.	71460

202. Find the prime factors common to the following numbers:

1.	18, 24, and 36	4.	44 and 280
2.	54, 72, and 84	5.	148, 256, and 320.
3.	506, 436, and 308	6.	325, 635, and 550.

203. *Greatest Common Divisor.*

For the definition of *a divisor, a common divisor,* and *the greatest common divisor,* see page 134.

G. C. D. is the abbreviation for the greatest common divisor.

1. What is the greatest common divisor of 42, 56, and 210?

OPERATION.

$$\begin{array}{r|ccc} 2 & 42. & 56. & 210 \\ \hline 7 & 21. & 28. & 105 \\ \hline & 3 & 4 & 15 \\ \hline \end{array}$$

$2 \times 7 = 14$ Ans.

Explanation.—In all problems of this kind, we first divide by any prime factor that will divide all the numbers; then we divide in like manner the successive quotients thus obtained, until we obtain quotients that have no common factor, or are prime to each other; then we multiply all the divisors together and in the product we have the greatest common divisor.

When there is no number greater than 1, that will divide all the numbers without a remainder, then 1 is the greatest common divisor.

When there are two large numbers, the operation may be more easily performed by first dividing the larger number by the smaller, and if there is a remainder divide the preceding divisor by it, and thus continue until there is no remainder. When there are more than two numbers, proceed as with two, and then with the greatest common divisor of the two and one of the other numbers, and thus continue until through with all the numbers. The last divisor will be the greatest common divisor.

Problems 2 and 3 elucidate this operation:

2. What is the greatest common divisor of 88 and 24?   Ans. 8.

OPERATION.

24)88(3
72
——
16)24(1
16
——
8)16(2
16
——

3. What is the greatest common divisor of 195, 285, and 315?   Ans. 15.

OPERATION.

285)315(1
285
——
30)285(9
270
——
15)30(2
30
——
15)195(13
15
——
45
45
——

GENERAL DIRECTIONS FOR FINDING THE GREATEST COMMON DIVISOR.

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204. From the foregoing elucidations, we derive the following general directions for finding the G. C. D.

1. *Write the numbers on a horizontal line and divide by any prime number that will divide all without a remainder, and write the quotients in a line below.*

2. *Continue this process of dividing the successive quotients, until quotients are obtained which have no common factor, or divisor.*

3. *Multiply together all the divisors, and their product will be the G. C. D.*

NOTE.—When there are two or more large numbers, it is often more convenient to work by successive divisions, as elucidated in problems 2 and 3.

205. What is the greatest common divisor of the following numbers?

4.	Of 441 and 567?	Ans. 63.
5.	Of 90, 315, and 810?	Ans. 45.
6.	Of 654, 216, and 108?	Ans. 6.

7. Robinson has 25, and Blaise 45 dimes. How shall they arrange them in packages, so that each shall have the same number in each package?
<div align="right">Ans. 5 in each package.</div>

8. A planter has 697 bushels of corn and 204 bushels of rough rice, which he wishes to put into the least number of bins containing the same number of bushels, without mixing the two kinds. How many bushels must each bin hold?
<div align="right">Ans. 17 bushels.</div>

9. A Commission Merchant has 2490 bushels of wheat, 1886 bushels of corn, and 8438 bushels of oats, which he wishes to ship in the least number of sacks of equal size, that will exactly hold either kind of grain. How many sacks will he require?
<div align="right">Ans. 6407.</div>

<div align="center">OPERATION INDICATED.</div>

Find the Greatest Common Divisor as above, (it is 2).

$$2)\overline{2490 \quad 1886 \quad 8438}$$
$$1245 + \quad 943 + 4219 = 6407 \text{ Ans.}$$

206. *Least Common Multiple.*

For the definition of *a multiple, a common multiple,* and *the least common multiple,* see page 134.

L. C. M. is the abbreviation for the least common multiple.

1. What is the least common multiple of 5, 6, 8, 21, 28?

OPERATION.

2) 5. 6. 8. 21. 28.
$$\overline{}$$
2) 5 3 4 21 14
$$\overline{}$$
3) 5 3 2 21 7
$$\overline{}$$
7) 5 1 2 7 7
$$\overline{}$$
5 1 2 1 1
$$\overline{}$$
2×2×3×7×5×2=840 Ans.

Explanation.—In all problems of this kind, we first arrange the numbers on a horizontal line, and then divide by the *smallest prime* number that will divide two or more without a remainder, and write the quotients and undivided numbers in a line below; this process of dividing we continue until there are no two numbers that can be divided by the same number without a remainder; then we multiply the divisors and the numbers in the last line together, and the product is the least common multiple.

When there is any number that will divide any of the others without a remainder, it may be cancelled before commencing to divide.

—————

GENERAL DIRECTIONS FOR FINDING THE LEAST COMMON MULTIPLE.

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207. From the foregoing elucidations, we derive the following general directions for finding the L. C. M.

1. *Write the numbers in a line and then divide by*

the smallest prime number that will divide two or more without a remainder, and write the quotients and undivided numbers in a line below.

2. *Continue this process of dividing, until there are no two numbers, with common factors, or that can be divided by any number greater than 1.*

3. *Find the continued product of the divisors and the numbers in the last line, and it will be the L. C. M.*

2. What is the least common multiple of 4, 9, 12, 15, and 24?   Ans. 360.

What is the least common multiple of the following numbers?

3. Of 8, 4, 9, and 30?   Ans. 360.
4. Of 50, 27, 3, 45, and 63?   Ans. 9450.
5. Of 21, 36, 11, and 22?   Ans. 2772.
6. Of 800, 600, 10, 40, and 12?   Ans. 2400.
7. Of 8, 18, 20, and 70?   Ans. 2520.

8. A drayman has 2 drays and 2 floats. On 1 dray he can haul 9 barrels of flour, and on the other 12 barrels; on 1 float he can haul 18 barrels, and on the other 21 barrels. What is the least number of barrels that will make full loads for either of the drays or the floats.   Ans. 252.

9. A fruit dealer desires to invest an equal amount of money in oranges, peaches, and grapes, and to expend as small a sum as possible. The price of oranges is $2.40 per box; peaches, $1.60; and grapes for a medium article, 90¢., and for first quality, $1.20; of these two qualities the fruit dealer took the cheaper. How much more money did he invest than he would have done had he taken the grapes at $1.20 per box?   Ans. $28.80.

PARTIAL OPERATION.

L. C. M. of $2.40, $1.60, .90=$14.40.
$14.40×3 (kinds of fruit)=$43.20 spent by pur-
    chasing grapes @ 90¢.
L. C. M. of $2.40, $1.60, $1.20=$4.80.
    $4.80×3=$14.40, what he would have spent
    by taking grapes @ $1.20.
        $43.20—$14.40=$28.80, Ans.

SYNOPSIS FOR REVIEW.

Define the following words and phrases:

181. A Fraction. 182. A Fractional Unit. 183.
How are Fractions Divided. 184. Common Frac-
tions. 184. Terms of the Fraction. 185. Denomina-
tor. 186. Numerator. 186. Quotient. 187. Decimal
Fractions. 187. Decimal Point. 188. Classification
of Fractions. 189. A Proper Fraction. 190. An
Improper Fraction. 191. A Simple Fraction. 192.
A Mixed Number. 193.. A Compound Fraction.
194. A Complex Fraction. 195. The Reciprocal
of a Fraction. 196. Value of a Fraction. 197.
General Principles of Fractions. 198. What is
Factoring? 200. A Common Factor. 204. Gen-
eral Directions for Finding Greatest Common
Divisor. 207. General Directions for Finding Least
Common Multiple.

REDUCTION OF FRACTIONS.

208. Reduction of Fractions is the process of changing their *form* without altering their *value*.

209. A fraction is reduced to **Higher Terms** when the numerator and the denominator are expressed in larger numbers. Thus, $\frac{1}{2}=\frac{2}{4}$, or $\frac{4}{8}$, or $\frac{8}{16}$; $\frac{2}{3}=\frac{4}{6}$, or $\frac{8}{12}$, or $\frac{16}{24}$, etc.

210. A fraction is reduced to **Lower Terms** when the numerator and the denominator are expressed in smaller numbers. Thus, $\frac{8}{12}=\frac{4}{6}$, or $\frac{2}{3}$; $\frac{12}{18}=\frac{2}{3}$, or $\frac{4}{6}$.

211. A fraction is reduced to its **Lowest Terms** when its numerator and its denominator are prime to each other, or have no common divisor. Thus, $\frac{3}{4}$, $\frac{7}{9}$, and $\frac{23}{54}$ are in their lowest terms.

212. Whole Numbers may be reduced to fractions having any desired denominator.

Whole line.   Half lines.

Third lines.   Fourth lines.

213.   ORAL EXERCISES.

1. If a line, an orange, an apple, or a unit of any kind is divided into two equal parts, what is each part called?   Ans. $\frac{1}{2}$.

2. If divided into three equal parts, what is each part called?   Ans. $\frac{1}{3}$.

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3. If divided into four equal parts, what is each part called? Ans. $\frac{1}{4}$.

4. When divided into four equal parts, what are three of those parts called? Ans. $\frac{3}{4}$.

5. How would you get $\frac{3}{4}$ of an apple?

Ans. Divide it into 4 equal parts and take 3 of the parts.

6. When any number or thing is divided into *five* equal parts, what is one of those parts called? Ans. $\frac{1}{5}$.

7. What are two, three, and four of the parts called respectively? Ans. $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$.

8. 1 unit, abstract, or **denominate of any kind**, equals how many halves? thirds? fourths? fifths? sixths? sevenths? eighths? ninths?

9. 2=how many halves? thirds? fourths? fifths? sixths? sevenths? eighths? ninths?

10. 3=how many halves? thirds? fourths? fifths? sixths? sevenths? eighths? ninths?

11. 4=how many halves? thirds? fourths? fifths? sixths? sevenths? eighths? ninths?

12. 5=how many halves? thirds? fourths? fifths? sixths? sevenths? eighths? ninths?

13. 6=how many halves? thirds? fourths? fifths? sixths? sevenths? eighths? ninths?

What kind of numerical work is the above called?

------

214. $\frac{1}{2}$=how many fourths? sixths? eighths? tenths? twelfths? fourteenths?

$\frac{1}{3}$=how many sixths? ninths? twelfths? fifteenths? eighteenths? twenty-firsts?

$\frac{1}{4}$=how many eighths? twelfths? sixteenths? twentieths? twenty-fourths?

$\frac{1}{5}$=how many tenths? fifteenths? twentieths? twenty-fifths? thirtieths?

$\frac{1}{8}$=how many sixteenths? twenty-fourths? thirty-seconds? fortieths? sixty-fourths?

$\frac{1}{16}$=how many thirty-seconds? forty-eighths? sixty-fourths? eightieths?

What kind of numerical work is the **above** called?

215. Answer the following numerical questions:

$\frac{2}{4}=\frac{1}{2}$	$\frac{6}{9}=?$	$\frac{10}{15}=?$	$\frac{14}{28}=?$	$\frac{6}{36}=?$	$\frac{15}{75}=?$
$\frac{3}{6}=?$	$\frac{7}{21}=?$	$\frac{11}{22}=?$	$\frac{15}{45}=?$	$\frac{7}{49}=?$	$\frac{75}{100}=?$
$\frac{4}{12}=?$	$\frac{8}{24}=?$	$\frac{12}{48}=?$	$\frac{19}{64}=?$	$\frac{8}{98}=?$	$\frac{125}{750}=?$
$\frac{5}{10}=?$	$\frac{9}{27}=?$	$\frac{13}{26}=?$	$\frac{17}{51}=?$	$\frac{8}{108}=?$	$\frac{450}{900}=?$

What kind of numerical work is the above called?

216. How many halves=or make a unit?

  " " thirds= " " "
  " " fourths= " " "
  " " fifths= " " "
  " " sixths= " " "
  " " sevenths= " " "
  " " eighths= " " "
  " " ninths= " " "
  " " tenths= " " "
  " " elevenths= " " "
  " " twelfths= " " "

217. 1 unit equals how many eighths? $\frac{8}{8}=?$
  1 " " " twelfths? $\frac{12}{12}=?$
  3 units equal " thirds? $\frac{3}{3}=?$
  4 " " " fourths? $\frac{16}{4}=?$
  5 " " " halves? $\frac{10}{2}=?$
  1½ " " " halves? $\frac{3}{2}=?$
  1½ " " " fourths? $\frac{6}{4}=?$
  1⅓ " " " thirds? $\frac{4}{3}=?$
  1⅔ " " " sixths? $\frac{10}{6}=?$
  1¾ " " " eighths? $\frac{14}{8}=?$
  3¼ " " " fourths? $\frac{13}{4}=?$
  2¾ " " " sixteenths? $\frac{44}{16}=?$
  4⅝ " " " eighths? $\frac{35}{8}=?$

218. What is the reciprocal of 1, of 2, of 3, of $\frac{1}{2}$, of $\frac{3}{4}$, of $2\frac{5}{8}$?

219. Analyse the fraction $\frac{3}{4}$.

Analysis.— $\frac{3}{4}$ is a proper fraction, since the numerator is less than the denominator; 4 is the denominator, and shows that the unit is divided into 4 equal parts; $\frac{1}{4}$ is the *fractional unit*, since it is ONE of the *four* equal parts into which the unit is divided; 3 is the *numerator* and shows that *three* of these equal parts are taken; 3 and 4 are the terms of the fraction, and its value is less than 1, or unity.

In like manner, analyse the following fractions:

$$\tfrac{3}{8}, \ \tfrac{5}{8}, \ \tfrac{9}{16}, \ \tfrac{12}{4}, \ \tfrac{15}{32}, \ \tfrac{8}{5}, \ \tfrac{41}{15}, \ \tfrac{24}{25}.$$

220. *To Reduce Fractions to Higher Terms.*

1. Change $\frac{13}{16}$ to a fraction whose denominator is 64.

OPERATION.

$64 \div 16 = 4$

$13 \times 4 = 52$

$\overline{} \quad$ — Ans.

$16 \times 4 = 64$

Explanation.—In all problems of this kind, we first divide the required denominator by the denominator of the given fraction. Then with the quotient thus obtained, multiply both terms of the given fraction, and in their products we have the required fraction.

GENERAL DIRECTION FOR REDUCING FRACTIONS FROM LOWER TO HIGHER TERMS.

221. From the foregoing elucidations, we derive the following general direction for reducing fractions from lower to higher terms:

Divide the required denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.

222. Change $1\frac{1}{5}$ to a fraction whose denominator is 75
     " $1\frac{7}{2}$ to   "   "   "   176
     " $3\frac{1}{2}$ to   "   "   "   384
     " $2\frac{6}{109}$ to   "   "   "   2180

223. *To Reduce Fractions to their Lowest Terms.*

Reduce $\frac{56}{64}$ to its lowest terms.

FIRST OPERATION.

$2)\frac{56}{64}=\frac{28}{32}$; $4)\frac{28}{32}=\frac{7}{8}$ Ans.

SECOND OPERATION.

$8)\frac{56}{64}=\frac{7}{8}$ Ans.

Explanation.—In all problems of this kind, we divide both the numerator and the denominator by their common factors. Or as shown in the second operation, we may produce the same result with less figures, by dividing both terms of the fraction by their greatest common divisor.

By this reduction we change the form of the fraction $\frac{56}{64}$, but we do not alter, or change, its value, for the fractional unit of the resulting fraction ($\frac{1}{8}$) is 8 times as great, while the number taken is $\frac{1}{8}$ as great.

When the terms of the fraction have no common factor greater than 1, the fraction is in its lowest terms and is called an *irreducible* fraction.

The object of reducing fractions to their lowest terms is to enable us to understand their value more easily and readily.

GENERAL DIRECTION FOR REDUCING FRACTIONS TO THEIR LOWEST TERMS.

224. From the foregoing elucidations we derive the following general direction for reducing fractions to their lowest terms:

Cancel all the factors common to both the Numerator and the Denominator;

Or, Divide both Numerator and Denominator by their Greatest Common Divisor.

225. Reduce the following fractions to their lowest terms:

2.	$\frac{14}{28}$, $\frac{15}{35}$, $\frac{24}{72}$, $\frac{42}{80}$.	Ans. $\frac{1}{2}$, $\frac{3}{7}$, $\frac{1}{3}$, $\frac{21}{40}$.
3.	$\frac{324}{720}$, $\frac{288}{360}$, $\frac{231}{933}$,	Ans. $\frac{9}{20}$, $\frac{4}{5}$, $\frac{77}{311}$.

4.	$\frac{1965}{2620}$.	Ans. $\frac{3}{4}$.	8.	$\frac{13915}{20030}$.	Ans. $\frac{2783}{4006}$.
5.	$\frac{6240}{15630}$.	Ans. $\frac{208}{521}$.	9.	$\frac{25920}{51840}$.	Ans. $\frac{1}{2}$.
6.	$\frac{224}{490}$..	Ans. $\frac{16}{35}$.	10.	$\frac{3575}{4719}$.	Ans. $\frac{25}{33}$.
7.	$\frac{972}{1368}$.	Ans. $\frac{27}{38}$.	11.	$\frac{16848}{34992}$.	Ans. $\frac{13}{27}$.

226. *To Reduce Whole or Mixed Numbers to Improper Fractions.*

1. Reduce $5\frac{2}{3}$ to an improper fraction, or to *thirds.*

OPERATION.

$5\frac{2}{3}$

——

$\frac{17}{3}$ Ans.

Explanation.—In all problems of this kind, we reason thus: Since there are 3 thirds in every unit or whole number, in 5 units there are 5 times as many, which are $\frac{15}{3}$+the $\frac{2}{3}$ make $\frac{17}{3}$.

2. Reduce 9 to a fraction whose denominator is 6.

OPERATION.

$9 \times 6 = \frac{54}{6}$ Ans.

GENERAL DIRECTION TO REDUCE WHOLE OR MIXED NUMBERS TO IMPROPER FRACTIONS.

227. From the foregoing elucidations, we derive the following general direction for reducing whole or mixed numbers to improper fractions:

Multiply the whole number by the required denominator and to the product add the numerator of the fraction, and write the required denominator under the result.

Reduce the following numerical expressions to improper fractions:

3.	$8\frac{1}{3}$	Ans.	$\frac{25}{3}$.	**8.**	$71\frac{1}{2}$	Ans.	$\frac{143}{2}$.
4.	$16\frac{1}{2}$	Ans.	$\frac{33}{2}$.	**9.**	$68\frac{4}{7}$	Ans.	$\frac{479}{7}$.
5.	$17\frac{3}{4}$	Ans.	$\frac{71}{4}$.	**10.**	$2183\frac{3}{4}$	Ans.	$\frac{8735}{4}$.
6.	$32\frac{5}{8}$	Ans.	$\frac{261}{8}$.	**11.**	$23\frac{8}{27}$	Ans.	$\frac{629}{27}$.
7.	$435\frac{3}{5}$	Ans.	$\frac{2178}{5}$.	**12.**	$108\frac{1}{105}$	Ans.	$\frac{11341}{105}$.

13. Reduce 14 to a fraction whose denominator is 9

14.   "   37   "   "   "   "   24

15.   " $54\frac{3}{4}$ "   "   "   "   16

228. *To Reduce Improper Fractions to Whole or Mixed Numbers.*

Reduce $\frac{17}{4}$ to a mixed number.

OPERATION.

$\frac{17}{4} = 4\frac{1}{4}$ Ans.

or

$17 \div 4 = 4\frac{1}{4}$ Ans.

Explanation.—In all problems of this kind, we reason thus: Since there are 4 *fourths* in 1 unit, or whole number, in 17 *fourths* there are as many units as 17 is equal to 4, which is 4 times with 1 remainder, or altogether $4\frac{1}{4}$ as the proper quotient, or answer.

GENERAL DIRECTION TO REDUCE IMPROPER FRAC-
TIONS TO WHOLE OR MIXED NUMBERS.

229. From the foregoing elucidations, we derive the following general direction for reducing improper tractions to whole or mixed numbers:

Divide the Numerator by the Denominator.

230. Reduce the following improper fractions to whole or mixed numbers:

2.	$\frac{24}{6}$	Ans. 4.	6.	$\frac{29}{8}$	Ans. $3\frac{5}{8}$.
3.	$\frac{47}{9}$	Ans. $5\frac{2}{9}$.	7.	$\frac{72}{13}$	Ans. $5\frac{7}{13}$.
4.	$\frac{144}{3}$	Ans. 48.	8.	$\frac{483}{51}$	Ans. $9\frac{8}{17}$.
5.	$\frac{222}{12}$	Ans. $18\frac{1}{2}$.	9.	$\frac{27010}{800}$	Ans. $34\frac{161}{800}$.

231. *To Reduce Compound Fractions to Simple Fractions.*

1. Reduce $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{7}{8}$ to a simple fraction.

OPERATION.

$\frac{1}{2} \times \frac{3}{5} \times \frac{7}{8} = \frac{21}{80}$ Ans.

Explanation.—In all problems of this kind, we multiply together all the numerators for a new numerator and all the denominators for a new denominator.

When a compound fraction contains whole or mixed numbers, they must first be reduced to improper fractions.

When there are common factors in both terms of a compound fraction they should be cancelled before multiplying. By cancelling the common factors, the work is shortened and the result unchanged for the reason that dividing both terms of a fraction by the same number does not alter its value.

2. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{8}$ to a simple fraction.

OPERATION.

$\frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{5}{8} = \frac{5}{16}$ Ans.

3. Reduce $\frac{3}{5}$ of $7\frac{1}{2}$ of $\frac{2}{9}$ of 4 of $\frac{1}{12}$ to a simple fraction.

<div align="center">OPERATION.</div>

$$\frac{\cancel{3}}{\cancel{5}} \times \frac{\overset{3}{\cancel{15}}}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{9}} \times \frac{4}{1} \times \frac{1}{\underset{3}{\cancel{12}}} = \frac{1}{3} \text{ Ans.}$$

GENERAL DIRECTION TO REDUCE COMPOUND TO SIMPLE FRACTIONS.

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232. From the foregoing elucidations, we derive the following general direction for reducing compound to simple fractions:

Cancel common factors if they occur in both terms of the fractions; then multiply the numerators together for the new numerator and the denominators together for the new denominator of the fraction.

233. Reduce the following compound fractions to simple ones:

4. $\frac{4}{7}$ of $\frac{4}{8}$ of $\frac{5}{11}$.　　　　　　Ans.　$\frac{5}{44}$.

5. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{7}{16}$.　　　　　　Ans.　$\frac{7}{48}$.

6. $\frac{4}{5}$ of $3\frac{1}{3}$ of $\frac{1}{2}$.　　　　　　Ans.　$1\frac{1}{3}$.

7. $\frac{1}{4}$ of $8\frac{1}{4}$.　　　　　　　　　Ans.　$2\frac{1}{16}$.

8. $\frac{9}{16}$ of 96.　　　　　　　　　Ans. 54.

9. $\frac{3}{8}$ of $\frac{1}{13}$ of $17\frac{1}{3}$.　　　　　　Ans.　6.

10. $\frac{2}{3} \times \frac{7}{9} \times \frac{5}{11}$.　　　　　　　Ans.　$\frac{70}{297}$.

11. $5 \times 2 \times \frac{3}{37}$.　　　　　　　　Ans.　$1\frac{3}{8}$.

234. *To Reduce Fractions of Different Denomina-*
tors to Equivalent Fractions of a Common
Denominator or of the Least Common Denomi-
nator.

235.   A Common Denominator is a denomina-
tor common to two or more fractions.

236.   The Least Common Denominator of two
or more fractions is the least number divisible by
each of the denominators.

237.   A Common Denominator of two or more
fractions is a *Common Multiple* of their denomina-
tors; and the *Least Common Denominator* of two or
more fractions is the *Least Common Multiple* of their
denominators, for the reason that all higher terms
of a fraction are multiples of its corresponding
lower or lowest terms.

WRITTEN EXAMPLES.

238.   Reduce $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{7}{8}$, to equivalent fractions
having a common denominator.

OPERATION.

$\frac{1}{3}$, $\frac{3}{4}$, $\frac{7}{8}$,

$3 \times 4 \times 8 =$ 96, Common Denominator.

$\frac{1}{3}$ of $\Big)$ $\quad = 32$; hence $\frac{32}{96}$, equivalent of $\frac{1}{3}$.

$\frac{3}{4}$ of $\Big\}$ $96 = 72$; hence $\frac{72}{96}$, equivalent of $\frac{3}{4}$.

$\frac{7}{8}$ of $\Big)$ $\quad = 84$; hence $\frac{84}{96}$, equivalent of $\frac{7}{8}$.

Explanation.—In all problems of this kind, we obtain the
common denominator by multiplying together the denominators
of all the fractions.   Then to find the respective numerators
we take such a part of the common denominator as the res-
pective fractions are parts of a unit, as shown in the opera-
tion.

Or, divide the common denominator by the denominator of
each fraction, and multiply the quotient by its numerator.

GENERAL DIRECTIONS TO REDUCE FRACTIONS TO A COMMON DENOMINATOR.

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239.   From the foregoing elucidations, we derive the following general directions for reducing fractions to a common denominator:

1.   *Multiply together the denominators of all the fractions for a common denominator.*

2.   *Then to find the respective numerators, take such a part of the common denominator as the respective fractions are parts of a unit.   Or, divide the common denominator by the denominator of each fraction, and multiply the quotient by its numerator.*

240.   Reduce the following fractions to equivalent fractions having a common denominator:

2. $\frac{3}{5}$, $\frac{2}{3}$, and $\frac{5}{7}$.   Ans. $\frac{63}{105}$, $\frac{70}{105}$, and $\frac{75}{105}$.

3. $\frac{9}{10}$, $\frac{1}{2}$, and $\frac{7}{12}$.   Ans. $\frac{216}{240}$, $\frac{120}{240}$, and $\frac{140}{240}$.

4. $\frac{8}{15}$ and $\frac{27}{32}$.   Ans. $\frac{256}{480}$ and $\frac{405}{480}$.

5. $\frac{1}{2}$, $\frac{5}{16}$, $\frac{9}{10}$, $\frac{11}{12}$, and $\frac{1}{6}$.

   Ans. $\frac{11520}{23040}$, $\frac{7200}{23040}$, $\frac{20736}{23040}$, $\frac{21120}{23040}$, and $\frac{3840}{23040}$.

6. $\frac{8}{17}$, $\frac{2}{3}$, $\frac{1}{4}$, and $3\frac{1}{6}$.

          Ans. $\frac{576}{1224}$, $\frac{816}{1224}$, $\frac{306}{1224}$, and $\frac{3876}{1224}$.

241. Reduce $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{7}{8}$ to equivalent fractions having the *least common denominator.*

OPERATION.

$$
\begin{array}{c|ccc}
2 & 3. & 4. & 8 \\
\hline
2 & 3. & 2. & 4 \\
\hline
 & 3 & 1 & 2
\end{array}
$$

$2 \times 2 \times 3 \times 2 = 24$ Least Common Denominator.

$\frac{1}{3}$ of $\left.\begin{array}{c}\\ \\ \\\end{array}\right.$ $= 8$, hence $\frac{8}{24}$ is the equivalent of $\frac{1}{3}$.

$\frac{3}{4}$ of $\left.\begin{array}{c}\\ 24 = \\ \\\end{array}\right.$ 18, hence $\frac{18}{24}$ is the equivalent of $\frac{3}{4}$.

$\frac{7}{8}$ ot $\left.\begin{array}{c}\\ \\ \\\end{array}\right.$ $= 21$, hence $\frac{21}{24}$ is the equivalent of $\frac{7}{8}$.

Explanation.—In all problems of this kind, we first find the *Least Common Multiple* of the denominators of all the fractions as explained in article 206, page 145, which is the *Least Common Denominator.* Then, having the least common denominator, to find the respective numerators we take such a part of the least common denominator as the respective fractions are parts of a unit, as shown in the operation. Or, divide L. C. D. by the denominator of each fraction and multiply the quotient by its numerator. Before finding the L. C. D. reduce mixed numbers to improper fractions, and the fractions to their lowest terms.

GENERAL DIRECTIONS TO REDUCE FRACTIONS TO THEIR LEAST COMMON DENOMINATOR.

242. From the foregoing elucidations, we derive the following general-directions for reducing fractions to their least common denominator.

1. *Find the least common multiple of the denominators of all the given fractions.*

2. *Then to find the respective numerators, take such a part of the least common denominator as the*

respective fractions are parts of a unit.   Or, divide the L. C. D. by the denominator of each fraction and multiply the quotient by its numerator.

NOTE.—Mixed numbers must be reduced to improper fractions, and the fractions to their lowest terms before finding the Least Common Denominator.

243.   Reduce the following fractions to equivalent fractions having a least common denominator:

2. $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$.   Ans. $\frac{16}{24}$, $\frac{20}{24}$, and $\frac{21}{24}$.

3. $\frac{5}{12}$, $\frac{1}{4}$, and $\frac{9}{16}$.   Ans. $\frac{20}{48}$, $\frac{12}{48}$, and $\frac{27}{48}$.

4. $\frac{8}{15}$, $\frac{14}{45}$, and $\frac{21}{60}$.   Ans. $\frac{96}{180}$, $\frac{56}{180}$, and $\frac{63}{180}$.

5. $\frac{1}{10}$, $\frac{30}{65}$, $\frac{19}{52}$, and $\frac{3}{26}$.   Ans. $\frac{26}{260}$, $\frac{120}{260}$, $\frac{95}{260}$, and $\frac{30}{260}$.

6. $5\frac{1}{2}$, $\frac{3}{4}$, $1\frac{5}{16}$, and $\frac{1}{3}$.   Ans. $\frac{264}{48}$, $\frac{36}{48}$, $\frac{63}{48}$, and $\frac{16}{48}$.

7. $\frac{16}{21}$, 8, $\frac{20}{63}$, and 1.   Ans. $\frac{48}{63}$, $\frac{504}{63}$, $\frac{20}{63}$, and $\frac{63}{63}$.

8. $3\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{5}{6}$.   Ans. $\frac{42}{12}$, $\frac{9}{12}$, $\frac{8}{12}$, and $\frac{10}{12}$.

9. $\frac{80}{111}$, 3, $\frac{1}{4}$, and $1\frac{3}{8}$.
      Ans. $\frac{2880}{1776}$, $\frac{5328}{1776}$, $\frac{444}{1776}$, and $\frac{2442}{1776}$.

SYNOPSIS FOR REVIEW.

Define the following words and phrases:

208. Reduction of Fractions. 209. Higher Terms. 210. Lower Terms.   211. Lowest Terms.   221. General Direction to Reduce Fractions from Lower to Higher Terms.   224. To their Lowest Terms. 227. Whole or Mixed Numbers to Improper Fractions.   229. Improper Fractions to Whole or Mixed Numbers.   232. Compound Fractions to Simple. 235. A Common Denominator. 236. Least Common Denominator.   239. General Directions to Reduce Fractions to a Common Denominator. 242. To the Least Common Denominator.

ADDITION OF FRACTIONS.

244. **Addition of Fractions** is the process of adding two or more fractional numbers of the same kind, or of the same denomination.

245. **Like Fractions** are those which express *like* parts of *like* units or things.

Thus, $\frac{1}{4}$ yard and $\frac{3}{4}$ yard, also $\frac{2}{7}$ and $\frac{4}{7}$ are like fractions.

246. **Unlike Fractions** are those which express *unlike* parts of *like* units or things, or parts of *unlike* units or things.

Thus, $\frac{2}{3}$ of a pound and $\frac{3}{4}$ of a pound are *unlike* parts of *like* units or things, and $\frac{1}{3}$ and $\frac{4}{7}$ are *unlike* parts of *unlike* units.

In addition of whole numbers we learned that we could not add apples to **oranges, pounds to boxes,** or *units* to *tens* or *hundreds;* that we could only add things that were of the same unit kind.

And this same principle maintains in the addition of fractional numbers. We can not add *halves* to *thirds, fourths* to *fifths,* etc. We can only add *halves* to *halves, fourths* to *fourths,* etc.

247. ORAL EXERCISES.

1. Add $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$. Ans. $1\frac{1}{2}$.

SOLUTION.—Since the fractional parts are alike, we have but to add the numerators together to obtain the sum of the fractions. Thus, $\frac{1}{4}+\frac{2}{4}+\frac{3}{4}=\frac{6}{4}=1\frac{2}{4}$, or $1\frac{1}{2}$, Ans.

2. Paid $\$\frac{1}{2}$ for a grammar and $\$\frac{3}{4}$ for an arithmetic. What did both cost? Ans. $\$1\frac{1}{4}$.

SOLUTION.—Since the halves and fourths are unlike parts of the unit dollar, we cannot add them in their present form. We must first reduce the $\frac{1}{2}$ to the fractional unit of the fourths; and by the exercise of our reason and knowledge of numbers, we see that $\frac{1}{2}$ is equal to $\frac{2}{4}$, and $\frac{2}{4}$ added to $\frac{3}{4}$ equals $\frac{5}{4}$, and $\frac{5}{4}$ equals $\$1\frac{1}{4}$, Ans.

3. What is the sum of $\frac{3}{4}$ and $\frac{5}{8}$?

SOLUTION.—Since the fractional parts are unlike, we cannot add them in their present form. We must first reduce the $\frac{3}{4}$ to the fractional unit of eighths; and by the exercise of our reason and knowledge of numbers, we see that $\frac{3}{4}$ is equal to $\frac{6}{8}$, and $\frac{6}{8}$ added to $\frac{5}{8}$ equals $\frac{11}{8}$, and $\frac{11}{8}$ equals $1\frac{3}{8}$, Ans.

4. What is the sum of $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{16}$, and $\frac{17}{32}$?
Ans. $2\frac{15}{32}$.

5. Add $\frac{2}{5}$, $\frac{9}{10}$, $\frac{11}{20}$, and $\frac{38}{40}$. Ans. $2\frac{33}{40}$.

Mentally add the following fractions:

$\frac{1}{2}+\frac{1}{2}=?$	$\frac{2}{3}+\frac{5}{6}=?$	$\frac{1}{2}+\frac{1}{4}+\frac{7}{6}=?$
$\frac{1}{4}+\frac{3}{4}=?$	$\frac{3}{4}+\frac{7}{5}=?$	$\frac{2}{3}+\frac{5}{6}+\frac{7}{12}=?$
$\frac{1}{3}+\frac{2}{3}=?$	$\frac{1}{2}+\frac{1}{4}=?$	$\frac{3}{4}+\frac{7}{8}+\frac{9}{16}=?$
$\frac{2}{3}+\frac{3}{3}=?$	$\frac{7}{8}+\frac{7}{16}=?$	$\frac{7}{8}+\frac{1}{2}+\frac{7}{16}=?$
$\frac{3}{4}+\frac{3}{4}=?$	$\frac{7}{12}+\frac{7}{12}=?$	$\frac{3}{5}+\frac{9}{10}+\frac{7}{20}=?$
$\frac{1}{2}+\frac{3}{4}=?$	$\frac{5}{9}+\frac{23}{27}=?$	$\frac{3}{4}+\frac{7}{8}+\frac{15}{16}=?$

248. What is the sum of $\frac{2}{3}$ and $\frac{3}{4}$?

SOLUTION.

OPERATION.

$$\frac{2}{3}\quad\frac{3}{4}$$
$$8\quad 9$$
$$-\frac{17}{12}=1\frac{5}{12}\text{ Ans.}$$

Explanation.—Since the fractional parts are unlike, we cannot add them in their present form. We must first reduce them to like fractional units.

And since we cannot reduce either fraction to the unit of the

other, we must reduce both to a common denominator. This we do by multiplying the denominators together, which gives 12 as the common denominator.

Having the common denominator, we next find the numerical value of $\frac{2}{3}$ and $\frac{1}{4}$ in the unit of *12ths.* We first consider the $\frac{2}{3}$ and see that $\frac{1}{3}$ of 12 is 4 and $\frac{2}{3}$ are 8, which we write under the $\frac{2}{3}$. Then we consider the $\frac{1}{4}$ and see that $\frac{1}{4}$ of 12 is 3, and $\frac{3}{4}$ are 9, which we write under the $\frac{3}{4}$. We now add the 8 twelfths and the 9 twelfths together, and produce $\frac{17}{12}=1\frac{5}{12}$ the answer.

2. What is the sum of $\frac{5}{6}$ and $\frac{3}{5}$?　　Ans. $1\frac{13}{30}$.

3. What is the sum of $\frac{2}{7}$ and $\frac{9}{11}$?　　Ans. $1\frac{8}{77}$.

4. Add $\frac{2}{8}$ and $\frac{7}{13}$; $\frac{4}{5}$ and $\frac{6}{7}$; $\frac{4}{9}$ and $\frac{1}{8}$.

Ans. $\frac{95}{104}$; $1\frac{23}{35}$; $\frac{41}{72}$.

249. Add $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{7}{8}$.

SOLUTION.

OPERATION.

$$\frac{1}{2} \quad \frac{2}{3} \quad \frac{7}{8}$$
$$16$$

UNITS.	FRACTIONS.
1	$\frac{3}{8}$　$\frac{25}{24}$
$1\frac{1}{24}$	$\frac{8}{8}$
	9
$2\frac{1}{24}$ Ans.	

Explanation.—Since there are no two fractions of the same unit value, we cannot add them in their present forms. And, since there are more than two fractional numbers, we reduce and add only two at a time. We select any two that may be the most easily reduced and added. We first select $\frac{1}{2}$ and $\frac{7}{8}$, and we see by the use of our reason, that $\frac{1}{2}$ equals $\frac{4}{8}$ which added to $\frac{7}{8}$ make $\frac{11}{8}$ which equals 1 and $\frac{3}{8}$. We write the 1 in the column of units and the $\frac{3}{8}$ in the column of fractions and then cancel the $\frac{1}{2}$ and $\frac{7}{8}$. We now have $\frac{2}{3}$ and $\frac{3}{8}$ to add, and producing the least common denominator, 24, we reason thus: $\frac{1}{3}$ of 24 is 8, and $\frac{2}{3}$ are 16, which is written under the $\frac{2}{3}$; then $\frac{1}{8}$ of 24 is 3 and $\frac{3}{8}$ are 9, which is written under the $\frac{3}{8}$; then adding we have $\frac{16}{24}$ and $\frac{9}{24}$ make $\frac{25}{24}$ which is 1 and $\frac{1}{24}$ which is written in the column of units. The fractions are now all added, and by adding the column of units we obtain 2, which with the $\frac{1}{24}$ gives $2\frac{1}{24}$ Ans.

Add the following:

2. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{15}$.　　　　Ans. $2\frac{41}{120}$.

3. $\frac{1}{18}$, $\frac{3}{7}$, $\frac{5}{9}$.　　　　　　Ans. $1\frac{5}{126}$.

4. $\frac{3}{4}$, $\frac{5}{7}$, and $1\frac{3}{14}$.　　　　Ans. $2\frac{23}{56}$.

250. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$ together.

OPERATION.

$$\frac{1}{2} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{4}{5} \quad \frac{5}{6} \quad \frac{7}{8}$$

UNITS.	MEMORANDUM FRACTIONS.
1	
1	
1	$\frac{1}{4} \quad \frac{1}{2} \quad \frac{3}{4} \quad \frac{5}{8} \quad \frac{17}{40}$
1	

$4\frac{17}{40}$ Ans.

Explanation.—In this example there are no two fractions alike, hence they cannot be added until we shall have reduced them to fractions of the same kind. To facilitate the operation, we reduce and add only two fractions at a time; and we first select such two as can be the most easily reduced and added. Accordingly, we select $\frac{1}{2}$ and $\frac{3}{4}$ as the fractions to first reduce and add; and by the exercise of our reason we see that $\frac{1}{2}$ is equal to $\frac{2}{4}$ which added to $\frac{3}{4}$, make $\frac{5}{4}$, which, for the reason that $\frac{4}{4}$ make 1, is equal to 1 and $\frac{1}{4}$. We write the 1 in the column of whole numbers, and the $\frac{1}{4}$ in the column of fractions. We then cancel the $\frac{1}{2}$ and $\frac{3}{4}$, and select the $\frac{2}{3}$ and $\frac{5}{6}$ as the next two fractions to reduce and add. Again using our reason, we see that $\frac{1}{2}$ is equal to $\frac{4}{6}$, and $\frac{2}{3}$ are equal to $\frac{4}{6}$, which, added to the $\frac{5}{6}$, make $\frac{9}{6}$, equal to 1 and $\frac{3}{6}$, which, reduced, equals $\frac{1}{2}$. The 1 we write in the column of whole numbers, the $\frac{1}{2}$ in the column of fractions, and cancel the $\frac{2}{3}$ and $\frac{5}{6}$. We next add the $\frac{3}{4}$ and $\frac{1}{2}$; by our reason we see that $\frac{1}{2}$ is equal to $\frac{2}{4}$, which, added to the $\frac{1}{4}$, make $\frac{3}{4}$, which we write in the column of fractions, and then cancel the $\frac{1}{2}$ and $\frac{1}{4}$. We then select the $\frac{4}{5}$ and $\frac{7}{8}$ as the next two fractions to add, and reducing the $\frac{4}{5}$ to 8ths, we see by our reason that $\frac{1}{5}$ is equal to $\frac{8}{40}$, and $\frac{4}{5}$ are equal to 3 times as many, which is $\frac{32}{40}$, which, added to the $\frac{7}{8}$, make $\frac{67}{40}$, which is equal to 1 and $\frac{27}{40}$; we write the 1 in the column of whole numbers, the $\frac{27}{40}$ in the col-

umn of fractions, and cancel the $\frac{1}{2}$ and $\frac{2}{3}$. We then proceed
to reduce and add the two remaining fractions, $\frac{1}{5}$ and $\frac{5}{8}$. By
inspection, the exercise of our reasoning faculties, and the
use of our knowledge of the principles of numbers as con-
tained in the preceding work, we see that the $\frac{1}{5}$ and $\frac{5}{8}$ are not
only unlike, but that we can neither reduce the $\frac{1}{5}$ to 8ths nor
the $\frac{5}{8}$ to 5ths; and, therefore, before we can add them we must
reduce both the $\frac{1}{5}$ and $\frac{5}{8}$ to equivalent fractions of the same
unit, or to the least common denominator. To do this, we
first observe that the denominators are not divisible by the
same number, greater than 1, and hence the product of them
(40) is the least number that both of the fractions are reduc-
ible to, or, in other words their product, 40, is the least com-
mon denominator of the two fractions. Having this, we next
reduce the $\frac{1}{5}$ and $\frac{5}{8}$ to 40ths, and by our reason we see that $\frac{1}{5}$
is equal to $\frac{8}{40}$, and $\frac{1}{5}$ are equal to 4 times as many, which is
$\frac{32}{40}$, then, that $\frac{1}{8}$ is equal to $\frac{5}{40}$, and $\frac{5}{8}$ are equal to 5 times as
many, which is $\frac{25}{40}$, which, added to the $\frac{32}{40}$, make $\frac{57}{40}$, which
for the reason that $\frac{40}{40}$ make a whole one, is equal to 1 and $\frac{17}{40}$,
which we place in their respective columns and cancel the
$\frac{1}{5}$ and $\frac{5}{8}$. The operation of adding the fractions is now com-
pleted, and by adding the whole numbers and annexing the
remaining fraction, we have as the correct result, $4\frac{17}{40}$.

The foregoing problems illustrate the most rational, easy,
and rapid system of adding fractions known, and as fractions
are so indispensable and of so frequent occurrence in practi-
cal life, the principles involved in the system should be thor-
oughly understood.

In practical work, we would very much shorten the opera-
tion by adding several fractions at once, and mentally per-
forming the most, if not all of the reduction and addition
work, without stating the results. Thus, in the above prob-
lem, we would add the $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{2}{3}$ at once. We can instantly
see that their sum is $\frac{17}{8}$, or $2\frac{1}{8}$, and without naming or setting
the $2\frac{1}{8}$, we add to it mentally the result of $\frac{1}{5}$ and $\frac{5}{8}$, which we
mentally see is $\frac{2}{5}$, or $1\frac{1}{4}$, making $3\frac{3}{8}$, which are the only figures
we write. Thus all the fractions, except $\frac{1}{5}$ and $\frac{5}{8}$, are added
at one mental operation. Then we mentally add the sum of
$\frac{1}{5}$ and $\frac{5}{8}$ by the same process of reasoning as given in the illus-
tration of the above example, and obtain the correct result,
$4\frac{17}{40}$.

2.   Add $\frac{1}{2}$ of $\frac{2}{3}$ of $2\frac{1}{4}$, $\frac{3}{8}$, $\frac{5}{6}$, and $\frac{9}{23}$ together.

OPERATION.

Statement showing the reduction of the fractions.

$$\begin{array}{ccccccc} & & 3 & & & & \\ 1 & 2 & 9 & 3 & 5 & 9 \\ - & \times & - & \times & - & - & - & - \\ 2 & 3 & 4 & 8 & 6 & 23 \end{array}$$

Statement showing the result of the reduction and the addition of the fractions.

UNITS.	MEMORANDUM FRACTIONS.				
	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{5}{6}$	$\frac{9}{23}$	$\frac{1}{8}$	$\frac{23}{24}$
1			20	216	3	529
1		$\frac{23}{24}$		$\frac{745}{552}$		

$2\frac{193}{552}$ Ans.

Explanation.— Here we have compound fractions and mixed numbers, and before adding, we reduce the mixed numbers to improper fractions, and the compound fractions to simple ones. Then we add the $\frac{3}{4}$ and $\frac{3}{8}$, which are equal to 1 and $\frac{1}{8}$; then the $\frac{5}{6}$ and $\frac{1}{8}$, which are equal to $\frac{23}{24}$; then the $\frac{23}{24}$ and $\frac{9}{23}$, which are equal to 1 and $\frac{193}{552}$. Then adding the whole numbers, and annexing the fraction, we have $2\frac{193}{552}$ as the correct result.

GENERAL DIRECTIONS FOR THE ADDITION OF FRACTIONS.

251.   From the foregoing elucidations, we derive the following general directions for the addition of fractions:

1.   *Select such two fractions as can be the most*

*easily reduced to the same fractional unit; reduce them
to the same fractional unit, find the equivalent value
of both the fractions in this fractional unit, and add
the numerators together; if the sum equals or exceeds a
unit write the unit in the column of units, and the
fractional remainder in the column of memorandum
fractions. If the sum is less than a unit, write the
fraction in the column of memorandum fractions.
Then cancel the fractions added.*

*2. In like manner select, reduce, and add two more
fractions, and thus proceed until all are added.*

*3. When there are compound fractions, reduce them
to simple ones before adding. When there are mixed
numbers write the whole numbers in the column of
units, cancel them from among the fractions, and then
add the fractional numbers. All fractional expres-
sions should be in their lowest terms before adding*

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252. PROBLEMS IN ADDITION OF FRACTIONS.

1. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{9}{10}$.   Ans. $4\frac{21}{40}$.

2. Add $\frac{5}{8}$, $\frac{7}{6}$, and $1\frac{3}{20}$.   Ans. $2\frac{19}{340}$.

3. Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{9}{16}$.   Ans. $2\frac{7}{16}$.

4. Add $\frac{7}{8}$, $\frac{11}{16}$, $2\frac{3}{2}$, and $\frac{52}{64}$.   Ans. $3\frac{6}{64}$, or $3\frac{3}{32}$.

5. Add $\frac{3}{8}$, $\frac{9}{10}$, $\frac{14}{15}$, and $\frac{17}{20}$.   Ans. $3\frac{17}{60}$.

6. $\frac{2}{3}$ of $1\frac{1}{4}$ and $2\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{1}{15}$.   Ans. $\frac{17}{33}$.

7.   Add $2\frac{1}{2}$, $6\frac{3}{4}$, $5\frac{5}{6}$, and $2\frac{1}{8}$.   Ans. 17.

8. $\frac{3}{8}$, $\frac{5}{6}$, and $\frac{7}{12}$.   Ans. $2\frac{1}{12}$.

9. $\frac{4}{5}$, $\frac{6}{7}$, and $\frac{1}{3}$.   Ans. $1\frac{23}{105}$.

10. $1\frac{1}{10}$, $6\frac{2}{3}$, $18\frac{19}{20}$, and $2\frac{7}{30}$.   Ans. $28\frac{41}{60}$.

11. $\frac{7}{8}$, $\frac{9}{20}$, and $\frac{12}{25}$.   Ans. $1\frac{191}{200}$.

12. $3\frac{1}{2}$, $1\frac{1}{4}$, $2\frac{1}{4}$, and $4\frac{5}{12}$.   Ans. $11\frac{1}{2}$.

13. $\frac{13}{18}$, $\frac{8}{15}$, $\frac{11}{20}$, and $\frac{13}{30}$.   Ans. $2\frac{43}{180}$.

14. $2\frac{1}{2}$, $3\frac{1}{3}$, $4\frac{1}{4}$, and 5.   Ans. $15\frac{1}{12}$.

15. $\frac{3}{4}$, $\frac{1}{8}$, $\frac{2}{3}$, and $\frac{5}{12}$.   Ans. $1\frac{97}{168}$.

16. $7\frac{1}{2}$, $5\frac{2}{3}$, and $10\frac{3}{4}$.   Ans. $23\frac{11}{12}$.

17. $\frac{14}{45}$, $\frac{27}{39}$, and $\frac{2}{1}$.   Ans. $1\frac{404}{4095}$.

18. $14\frac{4}{5}$, $3\frac{9}{10}$, $1\frac{2}{3}$, and $1\frac{9}{20}$.   Ans. $21\frac{19}{60}$.

19. $\frac{7}{8}$, $1\frac{7}{12}$, $10\frac{5}{6}$, and 5.   Ans. $18\frac{7}{24}$.

20. $125\frac{1}{4}$, $327\frac{5}{12}$, and $25\frac{1}{4}$.   Ans. $478\frac{5}{21}$.

21. $\frac{140}{320}$, $\frac{57}{80}$, $1\frac{1}{10}$, $\frac{19}{20}$, and $\frac{105}{160}$.   Ans. $3\frac{127}{160}$.

22.   What is the weight of 10 sacks of wheat which weigh respectively: $154\frac{1}{2}$, $149\frac{1}{4}$, $160\frac{3}{4}$, $157\frac{3}{8}$, $152\frac{1}{2}$, $141\frac{5}{8}$, $163\frac{3}{4}$, $158\frac{1}{2}$, $139\frac{1}{4}$, and $161\frac{3}{4}$ pounds?   Ans. $1539\frac{1}{4}$ lbs.

23.   Add $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{4}{5}$ and $2\frac{1}{3}$ of $\frac{9}{14}$ of 1.   Ans. $1\frac{4}{5}$.

24.   Add $\frac{4}{5}$ of $\frac{5}{8}$ of 4 and $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{4}{5}$.   Ans. $2\frac{29}{30}$.

25.   How many yards in 8 bolts of domestic, measuring as follows: $40\frac{3}{4}$, $39\frac{1}{2}$, $43\frac{1}{4}$, $42\frac{1}{8}$, $43\frac{5}{8}$, $38\frac{1}{2}$, $39\frac{9}{16}$, and $41\frac{1}{2}$ yards?   Ans. $328\frac{13}{16}$.

26.   14 bags of coffee weigh as follows: $162\frac{7}{16}$,

163⅝, 161 5/16, 164½, 165⅛, 164¾, 165½, 162¾, 165 3/16, 164⅝, 165¾, 165¼, 164 11/16, and 165½ pounds. How many pounds in all?   Ans. 2301.

27. A merchant bought 1153½ pounds of rice for $92¼; 871¾ pounds of sugar for $87⅕; 580¾ pounds of coffee for $115⅞; 240¼ pounds of cheese for $43½; and 408¾ pounds of Graham flour for $18¾. What was the total number of pounds, and the total cost of all he purchased?

Ans. 3254⅝ pounds; $357½ cost.

√28. Add ¾, ⅚, ⅞, 9/10, and 3¾ of 7/15 of 1½.

Ans. 5 52/60.

29. Add 5/7, ⅔, ½, 11/13, 1⅕, and ½ of 3.   Ans. 5 583/1365.

SYNOPSIS FOR REVIEW

Define the following words and phrases:

244. Addition of Fractions.   245. Like Fractions. 246. Unlike Fractions.   251. General Directions for the Addition of Fractions.

SUBTRACTION OF FRACTIONS.

253. Subtraction of Fractions is the process of finding the difference between two fractional numbers of like units and of like parts.

254. The **Principle** is, that fractions can be subtracted only when they express *like* parts of *like units* or when they have a common denominator.

255.   ORAL EXERCISES.

1. One unit of any kind equals how many $\frac{1}{2}$'s?
2. $\frac{1}{2}$ taken from $\frac{2}{2}$ leaves how many $\frac{1}{2}$'s?
3. One unit of any kind equals how many $\frac{1}{4}$'s?
4. $\frac{1}{4}$ from $\frac{4}{4}$ leaves how many $\frac{1}{4}$'s?
5. $\frac{2}{4}$ from $\frac{4}{4}$ leaves how many $\frac{1}{4}$'s?
6. $\frac{3}{4}$ from $\frac{4}{4}$ leaves how many $\frac{1}{4}$'s?
7. $\frac{4}{4}$ from $\frac{4}{4}$ leaves how many $\frac{1}{4}$'s?

256. Answer by mental work the following numerical questions:

$\frac{3}{4}-\frac{1}{4}=?$	$\frac{1}{2}-\frac{1}{4}=?$	$3\frac{1}{2}-1\frac{3}{8}=?$
$\frac{4}{5}-\frac{3}{5}=?$	$\frac{3}{4}-\frac{1}{6}=?$	$\frac{1}{2}+\frac{2}{3}-\frac{3}{8}=?$
$\frac{7}{8}-\frac{5}{8}=?$	$\frac{1}{2}-\frac{3}{8}=?$	$\frac{3}{4}+\frac{1}{3}-\frac{7}{12}=?$
$\frac{6}{16}-\frac{5}{8}=?$	$1-\frac{5}{12}=?$	$2\frac{1}{2}-\frac{3}{8}+\frac{9}{16}=?$

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257.   WRITTEN EXAMPLES.

1. **What** is the difference betweeen $\frac{3}{4}$ and $\frac{4}{5}$?

Ans. $\frac{1}{20}$.

OPERATION.

$\frac{3}{4} = 15$ | or $\frac{3}{4}$ $\frac{4}{5}$

$\frac{4}{5} = 16$ | 15   16

——————

$\frac{1}{20}$ **Ans.** | $\frac{1}{20}$ **Ans.**

Explanation.—Here we see that the fractions are not of *like* units, or that they have not the same denominator, and, therefore, before we can subtract, we must reduce the fractions to a common denominator. By inspection, and in accordance with the principles as explained in the first four problems of addition of fractions, we see that the least common denominator is 20 ; then, that $\frac{3}{4}$ are equal to $\frac{15}{20}$, and that $\frac{4}{5}$ are equal to $\frac{16}{20}$, and that the difference is $\frac{1}{20}$.

2.   **From** 28$\frac{5}{8}$ take 7$\frac{1}{2}$.   **Ans. 21**$\frac{1}{8}$.

OPERATION.

28$\frac{5}{8}$

7$\frac{1}{2}$

———

21$\frac{1}{8}$ **Ans.**

Explanation.— In performing the operation of the question before us, we first observe that the fractions which constitute a part of the numbers to be subtracted are not of the same unit or denominator, and hence, before we can perform the work, we must reduce them to fractions of *like units* or of a *common denominator.* We next observe that the $\frac{1}{2}$ may be reduced to 8ths and by the exercise of our reason we see that it is equal to $\frac{4}{8}$, which taken from $\frac{5}{8}$ leaves $\frac{1}{8}$; this completes the work with the fractions, and we have but to find the difference between the whole numbers as in simple subtraction.

3.   **What** is the difference between 37$\frac{3}{8}$ and 12$\frac{7}{9}$?

Ans. 24$\frac{43}{72}$.

OPERATION.

72

$37\frac{3}{8}=27$ } 99

$12\frac{7}{9}=56$

——————

24$\frac{43}{72}$ **Ans.**

Explanation.—By inspection we here see that the fractions belonging to the whole numbers are not of *like units* or of the same denominator, and that neither can be reduced to an equivalent fraction of the same unit as the other, and, therefore, we must reduce both to frac-

tions of *like units* or of a *common denominator*, before we can
subtract; and by multiplying together the denominators we
produce 72 as the least common denominator, which, for con-
venience, we write below the fractions, and by the same
reasoning as given in the preceding examples we see that $\frac{5}{8}$
are equal to $\frac{45}{72}$ and that $\frac{5}{8}$ are equal to $\frac{4}{7}$, which, for conve-
nience, we carry to the right of the respective fractions, and
to economize time we write 'only the numerators. We now
observe that the upper fraction, belonging to the greater
number, is less than the lower fraction, belonging to the lesser
number. Therefore, before we can subtract the fractions, we
must add 1, reduced to 72ds, to $\frac{4}{7}$, which gives us $\frac{98}{72}$. We
now subtract $\frac{45}{72}$ from $\frac{98}{72}$ and have a remainder of $\frac{43}{72}$ as the
fractional part of our answer. We now add 1 to the subtra-
hend, because we previously added 1 to the minuend, making
it 13, which we subtract from 37 and have a remainder of 24,
which we write below the line and complete the operation.

GENERAL   DIRECTIONS   FOR   SUBTRACTING
FRACTIONS.

258.   From the foregoing elucidations, we derive
the following general directions for subtracting
fractions:

1.   *Reduce the given fractions to equivalent frac-
tions of like fractional units, or of the least common
denominator; then take the difference of their nume-
rators and write the same over the common denomi-
nator.*

2.   *When there are mixed numbers, subtract the
fractional parts first and then the whole numbers.*

259.   PROBLEMS IN SUBTRACTION OF FRACTIONS.

1.   What is the difference between $\frac{7}{8}$ and $\frac{3}{8}$? $\vee$
        Ans. $\frac{5}{16}$.
2.   What is the difference between $\frac{3}{5}$ and $\frac{7}{9}$?
        Ans. $\frac{8}{45}$.
3.   What is the difference between $5\frac{1}{2}$ and $3\frac{1}{8}$? $\vee$
        Ans. $2\frac{3}{8}$.
4.   What is the difference between 7 and $3\frac{1}{16}$?
        Ans. $3\frac{15}{16}$.
5.   What is the difference between $23\frac{3}{7}$ and 14? $\vee$
        Ans. $9\frac{3}{7}$.

What is the difference between the following numbers:

6. $\frac{4}{7}$ and $\frac{4}{9}$.   Ans. $\frac{17}{63}$.
7. $\frac{15}{24}$ and $\frac{2}{5}$.   Ans. $\frac{9}{40}$.
8. $\frac{3}{5}$ and $\frac{3}{10}$.   Ans. $\frac{9}{20}$.
9. $\frac{4}{17}$ and $\frac{3}{8}$.   Ans. $\frac{19}{136}$.
10. $\frac{11}{12}$ and $\frac{5}{7}$.   Ans. $\frac{17}{84}$.
11. $\frac{4}{35}$ and $\frac{84}{126}$.   Ans. $\frac{4}{10}$.
12. $2\frac{3}{5}$ and $1\frac{1}{4}$.   Ans. $1\frac{4}{15}$.
13. $9\frac{1}{3}$ and $2\frac{3}{4}$.   Ans. $6\frac{7}{12}$.
14. $\frac{1}{2}$ of $\frac{4}{5}$ and $\frac{1}{3}$ of $\frac{3}{4}$. $\vee$ Ans. $\frac{11}{40}$.
15. $8\frac{1}{2}$ and $3\frac{7}{9}$.   Ans. $4\frac{13}{18}$.
16. $12\frac{5}{9}$ and $9\frac{1}{4}$.   Ans. $2\frac{2}{63}$.
17. $25\frac{4}{5}$ and $9\frac{7}{10}$.   Ans. $16\frac{2}{15}$.
18.   9 and $3\frac{4}{25}$. $\checkmark$ Ans. $5\frac{21}{25}$.
19. $\frac{43}{125}$ and $3\frac{7}{15}$.   Ans. $3\frac{46}{375}$.
20. $7\frac{11}{18}$ and 4.   Ans. $3\frac{13}{18}$.
21. $31\frac{1}{2}$ and $17\frac{5}{8}$. $\checkmark$ Ans. $13\frac{7}{8}$.

22.   From $6\frac{1}{2}$ of $\frac{4}{13}+13\frac{5}{6}$ take $\frac{1}{3}$ of $\frac{3}{4}$ of $15-\frac{2}{5}$ of $\frac{7}{2}$.

Ans. $13\frac{22}{60}$.

OPERATION INDICATED.

$\frac{13}{2}\times\frac{4}{13}=2$; $2+13\frac{5}{6}=15\frac{5}{6}$; $\frac{1}{3}\times\frac{3}{4}\times15=\frac{15}{4}=3\frac{3}{4}$;

$3\frac{3}{4}-(\frac{2}{5}\times\frac{7}{2})=2\frac{7}{20}$.

$15\frac{5}{6}-2\frac{7}{20}=13\frac{22}{60}$ Ans.

23.   From $8\frac{1}{2}+6\frac{3}{8}-\frac{9}{10}$ take $\frac{1}{2}$ of $\frac{2}{3}$ of 3 of $1\frac{3}{4}+2\frac{1}{4}$.

Ans. $10\frac{91}{240}$.

24.   From $17\frac{3}{4}+\frac{5}{8}$ take $6\frac{2}{3}-\frac{3}{4}$.   Ans. $12\frac{11}{24}$.

OPERATION INDICATED.

$17\frac{3}{4}+\frac{5}{8}=18\frac{3}{8}$. $6\frac{2}{3}-\frac{3}{4}=5\frac{11}{12}$.

$18\frac{3}{8}-5\frac{11}{12}=12\frac{11}{24}$ Ans.

25.   E. J. Jacquet had $38\frac{3}{4}$. He gave $2\frac{1}{2}$ for a pair of Indian clubs, $5\frac{3}{4}$ for books, $1\frac{1}{4}$ for a drawing board, and $\frac{5}{8}$ for ink and pencils How much had he left?   Ans. $28\frac{5}{8}$.

26.   V. G. Crena had $7\frac{1}{4}$ and his friend gave him $\frac{1}{2}$ more; R. C. Bush had $16\frac{1}{2}$ and he spent $5\frac{7}{8}$. How much more has R. C. Bush than V. G. Crena?   Ans $2\frac{7}{8}$.

27.   M. Gundersheimer bought 2 bags of coffee each weighing $163\frac{1}{2}$ pounds. He sold $27\frac{1}{4}$ pounds, $50\frac{3}{8}$ pounds, $87\frac{1}{4}$ pounds, and $45\frac{3}{4}$ pounds. How many pounds has he left?   Ans. $116\frac{3}{4}$ lbs.

28.   S. Delerno bought $75\frac{1}{2}$ gallons of molasses. He used $4\frac{1}{4}$ gallons, lost by leakage $2\frac{3}{8}$ gallons, and sold $22\frac{3}{4}$ gallons. How much has he left?

Ans. $46\frac{1}{8}$ gallons.

OPERATION INDICATED.

$4\frac{1}{4}+2\frac{3}{8}+22\frac{3}{4}=29\frac{3}{8}$ gallons.

$75\frac{1}{2}-29\frac{3}{8}=46\frac{1}{8}$ gallons, Ans.

29. What is the difference between a dozen times 6, plus $6\frac{1}{2}$, and 6 times a dozen minus one dozen and a half dozen? Ans. $24\frac{1}{2}$.

30. J. Astredo bought 6 chests of tea weighing $38\frac{1}{2}$, $42\frac{3}{4}$, $41\frac{5}{8}$, $44\frac{1}{4}$, $39\frac{1}{2}$, and $43\frac{3}{4}$ pounds. He sold $120\frac{7}{8}$ pounds and used $5\frac{3}{4}$ pounds. How many pounds has he on hand? Ans. $123\frac{3}{4}$.

31. J. Birba owned the Steamer Isabel, and sold $\frac{3}{8}$ of it. What is $\frac{1}{2}$ of his present interest?
Ans. $\frac{5}{16}$.

32. From the sum of $6\frac{1}{4}$ and $8\frac{5}{8}$, take the difference between $14\frac{2}{7}$ and $9\frac{1}{5}$? Ans. $10\frac{69}{280}$.

33. What number is that to which if $16\frac{1}{2}$ be added, the sum will be $44\frac{3}{8}$? Ans. $27\frac{7}{8}$.

34. P. H. Weiss bought $\frac{1}{2}$ of $\frac{2}{3}$ of a vessel and sold $\frac{2}{3}$ of $\frac{3}{4}$ of his share. How much of the whole vessel has he left. Ans. $\frac{1}{6}$.

35. H. P. Hester bought a barrel of molasses containing $41\frac{1}{4}$ gallons and sold $9\frac{1}{2}$ gallons. How many gallons remain in the barrel?
Ans. $31\frac{3}{4}$ gallons.

36. T. McGinnis bought two sacks of coffee weighing respectively $161\frac{1}{2}$ and $163\frac{3}{4}$ pounds. He sold to J. W. Godberry $186\frac{1}{2}$ pounds. How many has he left? Ans. $138\frac{3}{4}$ pounds.

37. L. J. Godberry sold to A. B. Brand $\frac{1}{2}$ of $\frac{5}{8}$ of his plantation. What part has he left? Ans. $\frac{11}{16}$.

38. What is the difference between $\frac{1}{2}$ of $\frac{1}{2}$ plus $\frac{2}{3}$, and $\frac{2}{3}$ of $\frac{1}{2}$ plus $\frac{1}{2}$? Ans. $\frac{1}{12}$.

39. W. Van Benthuysen owned $\frac{3}{4}$ of the Steamer Natchez. He sold to J. Maier $\frac{1}{4}$ interest in the Steamer, and to Leo. Winner $\frac{1}{4}$ of his remaining

interest. What is the present interest of each in the boat?　　　Ans. Van Benthuysen $\frac{3}{8}$; Maier $\frac{1}{4}$; and Winner $\frac{1}{8}$.

40. Percy Belt and Henry Pike were each $\frac{1}{2}$ owners of a broom and brush factory. P. Belt sold $\frac{1}{2}$ of his interest to G. Wagner, and then $\frac{1}{2}$ of his remaining interest to Henry Pike who subsequently sold $\frac{1}{2}$ of $\frac{3}{4}$ of his whole interest to W. Lacoume. What is the present interest of each owner?　　　Ans. P. Belt $\frac{1}{8}$;　　　G. Wagner $\frac{1}{4}$; H. Pike $\frac{25}{64}$ and Wm. Lacoume $\frac{15}{64}$.

MEMORANDUM SOLUTION.

1. P. B. owned $\frac{1}{2}$, and sold $\frac{1}{2}$ of his share to G. W. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, bought by G. W.; $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ still owned by P. B.

He then sold $\frac{1}{2}$ of his $\frac{1}{4} = \frac{1}{8}$ to H. P.; $\frac{1}{4} - \frac{1}{8} = \frac{1}{8}$ which is P. B.'s remaining interest.

2. G. W. owns $\frac{1}{4}$ interest, which he bought of P. B.

3. H. P. owned $\frac{1}{2}$; he bought $\frac{1}{8}$ interest of P. B. $\frac{1}{2} + \frac{1}{8} = \frac{5}{8}$. He then sold $\frac{1}{2}$ of $\frac{3}{4}$ of his $\frac{5}{8} = \frac{15}{64}$, to W. L.; $\frac{5}{8} - \frac{15}{64} = \frac{25}{64}$ now owned by H. P.

4. W. L. owns $\frac{15}{64}$ bought of H. P.

SYNOPSIS FOR REVIEW.

Define the following words and phrases:

253. Subtraction of Fractions. 254. Principle of Subtraction of Fractions. 258. General Directions for the Operation.

MULTIPLICATION OF FRACTIONS.

260. Multiplication of Fractions is the process of multiplying when one or both of the factors contain fractional numbers.

In the multiplication of simple numbers, we saw that the result of multiplication operations was increasing, but in the multiplication of fractions, when the multiplier is less than a unit, the result is decreasing. This is evident from the fact that multiplication is the process of repeating the multiplicand as many times as there are units in the multiplier, and, therefore, when the multiplier is less than a unit, the multiplicand will be repeated only a part of a time, or such a part of itself as the multiplier is a part of a unit.

To elucidate the principles of the subject and render clear the reasoning, we present our first questions in denominate numbers; and to aid still farther in comprehending the work, we give the following *practical* definition of multiplication.

261. Multiplication is that operation in the practical computation of numbers, of finding the cost of either a *part of one*, or of *many* pounds, yards, barrels, etc., when the cost of *one* pound, yard, barrel, etc., is given. On the principle or fact embraced in this definition, we *found* our reasoning for the solution of every question that can possibly be presented in multiplication, either of simple numbers or of fractions.

Considering the foregoing, we see that in all multiplication questions of a practical nature, we **must**

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necessarily *reason from one, or unity, to a part of one or many.* Thus, if 1 pound cost 50¢, ¼ of a pound will cost ¼ part of it; and if 1 yard costs $2, 3 yards will cost *three* times as much, or 3 times $2.

In the solution of questions in abstract numbers, we apply the same system of reasoning without naming the factors, and thereby avoid all of the arbitrary rules given in other arithmetics of the day.

262. ORAL EXERCISES.

1. What will 6 pounds cost at 5¢ per pound?
Ans. 30¢.

SOLUTION.—According to Article 112, page 69, we reason as follows: 1 pound cost 5¢. Since 1 pound cost 5¢, 6 pounds will cost 6 times as much, which is 30¢.

2. What will 6 pounds cost @ 5½¢ per pound?
Ans. 33¢.

SOLUTION STATEMENT.

¢	
2	11
	6 3
—	———
	33¢.

Reason.—1 pound cost 5½¢. Since 1 pound cost 5½¢ or ¹¹⁄² cents, 6 pounds will cost 6 times as much, which is 33¢.

3. What will 6½ pounds cost at 5¢ per pound?
Ans. 32½¢.

SOLUTION STATEMENT.

¢	
	5̆
2	13
—	———
	32½¢

Reason.—1 pound cost 5¢. Since 1 pound cost 5¢, ½ pound will cost ½ as much, and ¹³⁄² pounds will cost 13 times as much, which is 32½ cents.

263. The Statement Line. In all problems where we have both multiplication and division work to perform, we use a vertical line, which, because we state the problem upon it, we call the *statement line.*

The right hand side is the multiplication side and the left hand side is the division side.

On the top of the right hand side, we first write the premise of problems, or the number to be multiplied or divided, or that which the conditions of the question require the answer to be in.

On the right hand side of this line we place all multiplication numbers or factors, and on the left hand side we place all division numbers or divisors. But, be it remembered, we never place a number on either side, after the premise or nature of the answer is written, without giving a reason therefor. In practice the reason is instantly seen by the mind and directs the placing of the figures. When a problem is fully stated on this line, cancellation is applied and the operation is performed more readily than by any other manner of statement. .

4. One pound cost 5½¢. What will 6½ pounds cost, at the same rate?

SOLUTION STATEMENT.

	¢
2	11
2	13
4	143

35¾¢ Ans.

Explanation and Reason.— Here we see by considering the question, that 5½¢ is the premise and the number to be multiplied. Accordingly, we place the same on the top of the statement line; and to facilitate the work, *to free the operation of fractions*—we reduce the 5½¢ to *halves,* making ¹¹⁄₂¢, the denominator of which we place on the decreasing side and the numerator on the increasing side of the statement line. We then reason as follows: since 1 pound cost ¹¹⁄₂ cents, ½ of a pound will cost ½ part of it, and as this conclusion is a decreasing one, we write the 2 on the decreasing side; then, since ½ costs the result of the statement thus far made, ¹¹⁄₂, 6½ reduced, will cost 13 times as much, which, because the conclusion is an increasing one, we write on the increasing side, and thus complete the reason and the statement.

In working out the statement, there being no common fac-

tors in the increasing and decreasing numbers that can be cancelled, we have but to multiply the increasing numbers together, which produce 143, and the decreasing numbers together, which produce 4; then we divide the 143 by 4 and obtain 35¾¢ as the result of the reasoning and operation.

5. One pound cost 5⅓¢. What will 6¼ pounds cost at the same rate?

SOLUTION STATEMENT.

¢
3   16   4
4   25
—
3   100
—
33⅓¢ Ans.

Reason.—1 pound cost 5⅓ or ¹⁶⁄₃¢. Since 1 pound cost ¹⁶⁄₃¢, ¼ pound will cost the 4th part and ²⁵⁄₄ pounds will cost 25 times as much.

The Reason, Why, and Wherefore, Continued.

Question. How do you know that if 1 yard cost ¹⁶⁄₃ cents, ¼ of a yard will cost the 4th part?

Answer. By the exercise of my judgment—by the use of the reasoning faculties of the mind.

Question. What do you mean in this connection, by judgment?

Answer. The conclusion arrived at by the operations of the mind after duly considering the premise, the facts, and the conditions of the problem.

Question. What do you mean by premise or premises?

Answer. The proposition, declaration, truth, or fact which is asserted as the basis or predicate of a question. In this problem the premise is, *one pound cost 5⅓ or ¹⁶⁄₃ cents.*

Question. Why will 6¼ pounds cost 6¼ times as much as 1 pound?

Answer. Because 6¼ is six and one-fourth times as much as 1.

Question. What kind of reasoning is the foregoing?

Answer. Analogical and axiomatical. Analogical because there is analogy, relationship, or likeness existing between the cost of 1 pound and the cost of 6¼ pounds. Axiomatical because, the premise and question considered, the conclusion is self-evident.

Question. What is reason?

Answer. The faculty or power of the human mind by which truth is distinguished from falsehood, right from wrong, and by which correct conclusions are reached by considering the logical relationship which exists between the premises, the facts, and the conditions of particular statements and questions.

5.   At 16⅔¢ per yard, what will 22½ yards cost?

SOLUTION STATEMENT.

	¢
3	50
2	45
—	—
	\$3.75 Ans.

Reason.—1 yd. cost 16⅔¢. Since 1 yard cost ⁵⁰⁄₃¢, ⅓ of a yard will cost the ⅓ part, and 4⁵ yards will cost 45 times as much.

6.   What will 4¾ bushels cost at 66½¢ per bushel?

SOLUTION STATEMENT.

	¢
2	133
4	19
—	—
	\$3.15⅞ Ans.

Reason.—1 bushel cost 66½¢. Since 1 bushel cost 1⅓³cts., ¼ of a bushel will cost the ¼ part, and ¹⁹⁄? bushels will cost 19 times as much.

Question.   How do you know this?

7.   Chickens are selling at $3¾ per dozen. **What** will 4⅙ dozen cost?

SOLUTION STATEMENT.

$

4	15
6	25
—	
	$15⅝ Ans.

Reason.—1 dozen **cost** $¹⁵⁄₄. Since 1 dozen **cost** ¹⁵⁄₄ dollars, ⅙ of a **dozen** will cost ⅙ part, and ²⁵⁄₆ dozen will cost 25 times **as** much.

Question—1.   How do you know this?   2. **What** do you mean by judgment?

8.   What will 8 gallons cost at 42½ cents per **gal**-lon?

SOLUTION STATEMENT.

¢

2	85
	8
—	
	$3.40 Ans.

Reason.—I gallon **cost** 42½¢. Since 1 gallon **cost** ⁸⁵⁄₂ cents, 8 gallons will **cost** 8 times as much.

9.   What will 14½ pounds cost at 12 cents **per** pound?

SOLUTION STATEMENT.

¢

	12
2	29
—	
	$1.74 Ans.

Reason.—1 pound **cost** 12¢. Since 1 pound **cost** 12 cents, ½ of a pound will cost the ½ part, and ²⁹⁄₂ lbs. will cost 29 times as **much.**

10.   What will ⅔ of a dozen cost at 8¾ per dozen?

SOLUTION STATEMENT.

$

4	3
3	2
—	
	$½ Ans.

Reason.—1 doz. cost $¾. Since 1 dozen cost ¾ of a dollar, ⅓ of a dozen will cost ⅓ part, and ⅔ of a dozen will cost 2 times as **much.**

11. A mechanic works 15½ days and receives \$3¼ per day. How much money is due him?

SOLUTION STATEMENT

$$
\begin{array}{c|c}
 & \$ \\
4 & 13 \\
2 & 31 \\
\hline
8 & 403 \\
\hline
 & \$50\frac{3}{8} \text{ Ans.}
\end{array}
$$

Reason.—1 day's work is worth \$3¼. Since 1 day's work is worth ¹³⁄₄ dollars, ¼ of a day's work is worth ¼ part, and 3¼ days' work 31 times as much.

Question.—1. How do you know this? 2. What do you mean by judgment? 3. What is the use of the statement line? 4. What kind of numbers do you place on the right hand side? 5. What kind on the left? 6. What do you do before you place a number on either side? Answer.—Give the reason for so doing. 7. What do you mean by reason? 8. What is Cancellation? 9. Why do you use Cancellation?

GENERAL DIRECTIONS FOR MULTIPLYING FRACTIONS.

—

264. From the foregoing elucidations, we derive the following general directions for multiplying fractions:

1. *Write on the upper right hand side of the statement line the premise of the problem ; or the number which is to be multiplied or divided ; or the number*

representing the answer. Then, reasoning from ONE, OR UNITY, TO A PART OF ONE *or to many, write the other numbers upon the multiplication or division side of the line, according as the conclusion is increasing or decreasing.*

2. *Mixed numbers should be reduced to fractional expressions, and the reason given for writing both the numerator and the denominator.*

265.   PROBLEMS.

1. What will ⅝ of a yard cost, at ⅘ of a dollar per yard?   Ans. $½.

OPERATION

$
5 | 4
8 | 5   Write the
2 | Reason.
- | ―
  | $ ½ Ans.

2. What will 58½ pounds cost, at 16⅔¢ per pound?   Ans. $9.75.

OPERATION.

¢
3 | 50 25
2 | 117 39   Write the
- | Reason.
  | ――――
  | $9.75 Ans.

3. What will 3¾ dozen cost, at $3⅖ per dozen?
Ans. $12¾.

OPERATION.

$
5 | 17
4 | 15 3
- | Write the Reason.
  | ―――
  | 51
  | ――
  | $12¾ Ans.

4. What will 5¾ bushels cost, at 15½¢ per pint?

OPERATION.

```
  ¢
2 | 31
  |  2
  |  8
  |  4 = 124
½ | 43
  | ---------
  | $53.32 Ans.
```

Explanation.—By inspection and reason, we see that the 15½¢ is the number to be increased; hence we reduce and place the same on the line and proceed to reason as follows: if 1 pint costs ¼¢, 2 pints or a quart will cost 2 times as much; and if 1 quart costs the result of the statement now made, 8 quarts, or a peck, will cost 8 times as much; and if a peck costs the result of this statement, 4 pecks, or a bushel, will cost 4 times as much; and if a bushel costs the result of this statement, ½ of a bushel will cost ½ part of it, and 4¾ will cost 43 times as much.

5. What will 50¼ pounds of coffee cost, at 10½¢ per ounce?

OPERATION.

```
  ¢
2 | 21
  | 16   2
4 | 201
  | ---------
  | $84.42 Ans.
```

Explanation.—Here we reduce and place the 10½¢ on the line, and reason thus: since 1 ounce costs ¼¢, 16 ounces or 1 pound will cost 16 times as much, and since 1 pound costs the result of this statement, ¼ of a pound will cost ¼ part of it, and 2¼¹ will cost 201 times as much. This completes the reasoning and statement, which worked gives $84.42, answer.

Make the solution statement and write the reason for the following problems:

6. What will 24 pounds cost, at 9½¢ per pound?
Ans. $2.28.

7. What will 14⅔ dozen cost, at $5 per dozen?
Ans. $73⅓.

8. What will 6 dozen and 7 chickens cost, at $4.87½ per dozen? Ans. $32.09⅜.

24

9. What will 42 pounds and 11 ounces of butter cost, at 22½¢ per pound? Ans. $9.60¹¹⁵⁄₁₃.

SOLUTION STATEMENT.

```
      ¢
   2 | 45
  16 | 683
  ―――――――
```

$9.60¹⁵⁄₃₂ Ans.

Explanation and Reason.— As usual, we here place the premise, the price of one pound, on the statement line, and reason as follows: 1 pound, or 16 ounces, cost

⁴⁵⁄₂¢. Since 1 pound cost ⁴⁵⁄₂¢, 1 ounce will cost the 16th part and 683 ounces (which is 42 pounds and 11 ounces) will cost 683 times as much.

10. What will 19⅔ pounds cost, at 18⅝¢ per pound?
Ans. $3.60⁵⁵⁄₈₄.

11. What will 25⅔ yards cost, at 17¾¢ per yard?
Ans. $4.55⁷⁄₁₂.

12. What will 11⅓ yards cost, at 12½¢ per yard?
Ans. $1.42³⁄₁₆.

13. What will 21¾ yards cost, at 16½¢ per yard?
Ans. $3.58⅞.

14. What will 14¼ pounds cost, at 12¼¢ per pound?
Ans. $1.74⁹⁄₁₆.

15. What will 31⅛ pounds cost, at 11⅛¢ per pound?
Ans. $3.46¹⁷⁄₆₄.

266. *To Multiply Abstract Fractional Numbers.*

1. Multiply 8⅓ by 3¾.

OPERATION.

```
  3 | 25
  4 | 15  5
  ―――――――
    | 125
```

31¼ Ans.

Explanation and Reason.— In this problem, both factors are abstract numbers. Hence we cannot give the same analogical reasoning as we gave in the foregoing problems where the factors were denominate, or concrete, numbers; although were we to do so, the result, so far as the figures are concerned, would be correct. We therefore reduce and place the 8⅓, the number to be multiplied, on the statement line, and reason as follows: Axiomat-

ically, 1 time 8½ or ²ᵖ is 2ᵖ. Since 1 time ³ᵖ is ²ᵖ, ½ time ²ᵖ
is ½ part of ²ᵖ, and ¹ᵖ are 15 times as many.

Or, by analysis, thus: 1 time ³ᵖ is 2ᵖ. Since 1 time ²ᵖ is ²ᵖ,
½ time ²ᵖ is the ½ part=⅞, and ¹ᵖ are 15 times ⅞=³ᵖ=
31¼ answer.

It will be observed that we found our reasoning upon *one*,
which is the basis of all numbers, as explained in articles 113
and 180. Multiplication is the process of repeating one number
as many times as there are units—*ones*—in another. And it is
self-evident that *one* time any number is equal to the number.
This self-evident conclusion is the premise for all questions
in multiplication of abstract numbers.

2. Multiply ¾ by ⅔.   Ans. ½.

SOLUTION STATEMENT.

$$\begin{array}{c|c} 4 & 3 \\ 3 & 2 \\ \hline & \end{array}$$

     ½ Ans.

Reason.—1 time ⅔ is ⅔. Since
1 time ⅔ is ⅔, ½ time ⅔ is the ½
part, and ¾ times is 2 times as
many, which is ½. Or by analysis, thus: Since
1 time ⅔ is ⅔, ½ time ⅔ is the ½ part of ⅔=³⁄₁₂, and ¾ times ⅔ is 2
times ³⁄₁₂=¹⁄₁₂, or ½, answer.

3. Multiply 1¹⁄₁₂ by ⁷⁄₉, and write the reason.

                   Ans. ⁷⁷⁄₁₀₈.

4. Multiply 6 by ⅝, and write the reason.

SOLUTION STATEMENT.

$$\begin{array}{c|c} & 6 \\ 8 & 5 \\ \hline & \end{array}$$

     Ans. 3¾.

5. Multiply ⅚ by 9, and write the reason.

SOLUTION STATEMENT.

$$\begin{array}{c|c} 6 & 5 \\ & 9 \\ \hline & \end{array}$$

     Ans. 7½.

6. 5½ by ⅞⅔, and write the reason.   Ans. 2⅕.

7. 4⁷⁄₉₃ by ³⅟₉₄, and write the reason.   Ans. ⅙.

267. *Miscellaneous Examples in Multiplication of Fractions.*

1. What will 16 yards cost, at $14\frac{3}{4}$¢ per yard?
Ans. $2.36.

2. What will $23\frac{3}{4}$ pounds cost, at 35¢ per pound?
Ans. 8.31\frac{1}{4}$.

3 What will $\frac{3}{4}$ of a yard cost, at $\frac{5}{8}$ per yard?
Ans. 1\frac{5}{3}$.

4. What will $\frac{1}{2}$ a yard cost, at $\frac{1}{2}$ per yard?
Ans. $\frac{1}{4}$.

5. What will $8\frac{1}{2}$ pounds cost, at $7\frac{1}{2}$¢ per pound?
Ans. $63\frac{3}{4}$¢.

6. What will $10\frac{3}{4}$ pounds cost, at $9\frac{1}{4}$¢ per pound?
Ans. $99\frac{7}{16}$¢.

7. Multiply $\frac{1}{2}\frac{1}{4}$ by 12.   Ans. $5\frac{1}{2}$.

8. Multiply $\frac{1}{1}\frac{3}{3}$ by 13.   Ans. $10\frac{5}{13}$.

9. Multiply $\frac{9}{16}$ by 19.   Ans. $10\frac{11}{16}$.

10. Multiply 13 by $\frac{7}{10}$.   Ans. $9\frac{1}{10}$.

11. Multiply 105 by $\frac{4}{35}$.   Ans. 12.

12. Multiply 136 by $\frac{33}{100}$.   Ans. $44\frac{22}{25}$.

13. Multiply 12 by $31\frac{5}{6}$.   Ans. 382.

14. Multiply 25 by $3\frac{2}{5}$.   Ans. 85.

15. Multiply $\frac{1}{12}$ by $\frac{18}{35}$.   Ans. $\frac{3}{70}$.

16. Multiply $\frac{23}{29}$ by $\frac{1}{7}$.   Ans. $\frac{18}{43}$.

17. Multiply $11\frac{1}{2}$ by $1\frac{3}{5}$.   Ans. $18\frac{2}{5}$.

18. Find the value of $\frac{3}{7}$ of $\frac{1}{3}$ of $\frac{5}{9}$ of $\frac{63}{24}$ of 4.
Ans. $\frac{5}{16}$.

19. Multiply $7\frac{7}{10}$ by $\frac{5}{6}$.   Ans. $6\frac{5}{12}$.

20. What is the product of $\frac{9}{16}$, $\frac{2}{7}$, $\frac{5}{9}$, and $\frac{1}{4}$?
Ans. $\frac{1}{8}$.

21. What is the product of $1\frac{5}{6}$, $\frac{3}{8}$, 2, and $5\frac{1}{3}$?
Ans. $11\frac{11}{12}$.

22. What is the product of $\frac{9}{13}$ of $2\frac{1}{4}$ by $\frac{1}{3}$ of $7\frac{1}{2}$?
Ans. $1\frac{17}{23}$.

23. What is the product of $12\frac{1}{2}$ multiplied by $5\frac{1}{2}$ times $6\frac{3}{4}$? Ans. $464\frac{1}{16}$.

24. At $\frac{10}{11}$ of a dollar a pound, what will $\frac{9}{10}$ of a pound of tea cost? Ans. $\frac{9}{11}$ of a dollar.

25. What will $5\frac{3}{4}$ dozen buttons cost, at $\frac{2}{46}$ of a dollar per dozen? Ans. $\frac{1}{4}$ of a dollar.

26. What will $4\frac{3}{4}$ yards cost, at $4\frac{1}{4}\cancel{c}$ per yard? Ans. $20\frac{3}{16}\cancel{c}$.

27. What will $9\frac{1}{2}$ yards cost, at $9\frac{3}{4}\cancel{c}$ per yard? Ans $92\frac{5}{8}\cancel{c}$.

28. What will $12\frac{7}{8}$ yards cost, at $12\frac{1}{2}\cancel{c}$ per yard? Ans. $1.60\frac{15}{16}$.

29. What will $12\frac{1}{2}$ pounds cost, at $12\frac{1}{2}\cancel{c}$ per pound? Ans. $1.56\frac{1}{4}$.

30. What will $6\frac{1}{4}$ pounds cost, at $6\frac{3}{4}\cancel{c}$ per pound? Ans. $42\frac{3}{16}\cancel{c}$.

31. What will $8\frac{2}{3}$ pounds cost, at $8\frac{1}{2}\cancel{c}$ per pound? Ans. $72\frac{2}{3}\cancel{c}$.

32. What will $19\frac{3}{8}$ pounds cost, at $19\frac{5}{8}\cancel{c}$ per pound? Ans. $3.80\frac{11}{64}$.

33. What will $9\frac{3}{4}$ pounds cost, at $11\frac{3}{4}\cancel{c}$ per pound? Ans. $1.14\frac{9}{16}$.

34. What will $15\frac{1}{2}$ pounds cost, at $10\frac{1}{2}\cancel{c}$ per pound? Ans. $1.62\frac{3}{4}$.

35. What will $40\frac{5}{8}$ pounds cost, at $22\frac{5}{8}\cancel{c}$ per pound? Ans. $9.19\frac{9}{64}$.

36. What will $2812\frac{1}{2}$ gallons cost, at 4.50 per gal.? Ans 12656.25.

37. What cost $471\frac{3}{4}$ gallons, at $3\frac{3}{8}$ per gallon? Ans. $1592\frac{5}{42}$.

38.   Sold 937852½ pounds of cotton at 14$\frac{13}{32}$¢ per pound.   What did it amount to ?

Ans. 135695.53\frac{21}{64}$.

39.   If a man earns $2½ in 1 day, how much will he earn in 16½ days?   Ans. $41¼.

40.   A contractor pays $1¼ per day for labor, and he has 370 men employed for six days.   How much money will it take to pay them?   Ans. $2775.

·41.   P. Machray paid $\frac{9}{10}$ of a dollar for a book, and for paper $\frac{2}{3}$ of the cost of the book.   How much did he pay for the paper ?   Ans. $\frac{3}{5}$·

42.   Distillers of the essence of rose have determined by experience that it requires 48000 pounds of rose leaves to make or distill one pound of the ottar of roses.   How many pounds of rose leaves will it require to distill 50$\frac{7}{8}$ pounds of the ottar of roses ?   Ans. 2442000 pounds.

43.   If one pound cost a cent and a half, what will 25½ pounds cost?   Ans. 38¼ cents.

OPERATION INDICATED.

$$
\begin{array}{c|c}
 & ¢ \\
2 & 3 \\
2 & 51 \\
\hline
- & \\
 & 38\frac{1}{4}¢
\end{array}
$$ Write the Reason.

44.   G. V. Hooper owned $\frac{15}{16}$ of the Steamer Katie and sold $\frac{2}{3}$ of his share to Maschek.   What part of the whole steamer did he sell?   Ans. $\frac{5}{8}$.

OPERATION INDICATED

$$\frac{15}{16} \times \frac{2}{3} = \frac{5}{8} \text{ Ans.}$$

45.   E. Garner can work the problems in this book in $4\frac{3}{4}$ months. How many months would it take him to work $\frac{2}{3}$ of them?   Ans. $3\frac{1}{6}$ months.

OPERATION INDICATED.

MOS.

$$\begin{array}{c|c} 4 & 19 \\ 3 & 2 \\ \hline - & \\ & 3\frac{1}{6} \text{ Ans.} \end{array} \qquad \text{or, } \tfrac{19}{4} \times \tfrac{2}{3} = 3\frac{1}{6}, \text{ Ans.}$$

46.   W. J. Kearney paid $\$\frac{3}{6}$ for 1 gallon of molasses. What is $\frac{3}{4}$ of a gallon worth at the same rate?   Ans. $\$\frac{5}{8}$.

47. What will $7\frac{1}{2}$ boxes of raisins cost, at $\$2\frac{1}{4}$ per box?   Ans. $\$16\frac{7}{8}$.

48. On one occasion at the New Orleans Opera, $\frac{1}{2}$ of the ladies and gentlemen present were French; $\frac{1}{2}$ of the remainder, American; $\frac{1}{3}$ of the remainder, German; and the others were of different nationalities. What part were Americans, what part Germans, and what part were of different nationalities?   Ans. $\frac{1}{4}$ Americans, $\frac{1}{12}$ Germans, and $\frac{1}{6}$ of different nationalities.

49.   C. Reynolds owned $\frac{7}{8}$ of a plantation and sold $\frac{2}{3}$ of his share to D. C. Williams, who sold $\frac{1}{2}$ of what he purchased to Frank Soulé, who sold $\frac{3}{4}$ of what he purchased to L. B. Keiffer. What is Keiffer's share in the plantation?   Ans. $\frac{7}{32}$.

OPERATION INDICATED.

$\frac{7}{8} \times \frac{2}{3} = \frac{7}{12} =$ D. C. William's share.

$\frac{7}{12} \times \frac{1}{2} = \frac{7}{24} =$ F. Soulé's share.

$\frac{7}{24} \times \frac{3}{4} = \frac{7}{32} =$ Keiffer's share.

50.   W. Weiss owned $\frac{4}{5}$ of 2000 acres of land **and** sold $\frac{3}{4}$ of his share to H. Marsden, who sold $\frac{5}{6}$ **of** what he purchased to J. T. Finney.   How many **acres** have each?   Ans. W. Weiss, 400; H. Marsden, 450; and J. T. Finney, 750 acres.

SYNOPSIS FOR REVIEW.

Define the following words and phrases:

260. Multiplication of Fractions.   261. Practical Definition of Multiplication of Fractions.   261. The Basis of Reasoning.   263. Statement Line. 263. Reason.   263. Judgment.   263. Premise.   263. Analogical.   263. Axiomatical.   264. General Directions for Multiplying Fractions.

DIVISION OF FRACTIONS.

268. Division of Fractions is the process of dividing when the divisor or dividend, or both, contain fractional numbers.

In the division of simple numbers, we saw that the result of division operations was decreasing, but in the division of fractions, when the divisor is less than a unit, the result is increasing. This fact is plain, for the reason that the operation of division is the process of finding how many times the dividend is equal to the divisor, and, hence, when the divisor is less than 1, the dividend will be equal to the divisor as many times itself as the divisor is part of 1.

In practical operations, we usually have the three following cases or questions in division of fractional numbers:

1st. To *find* the cost of *one* pound, yard, or article of any kind, when we have the cost of *many* pounds, yards, or articles of any kind given.

2d. To *find* the cost of *one* pound, yard, or article of any kind, when we have the cost of *a part* of a pound, yard, or article of any kind given.

3d. To *find* the *number* of pounds, yards, or articles of any kind that can be bought with a specified sum, when we have the price of *one*, or *a part of one* pound, yard, or article of any kind given.

From these questions we see that division is the converse of multiplication, and that from the nature of the question, we must reason from *many* to *one*

or from *a part of one* to *one.* Thus: 1st, if 5 pounds cost 50¢, 1 pound will cost the ⅕ part of it; in the 2d case, if ¾ of a yard cost $2, ¼ of a yard will cost the ⅛ part of it, and ⁴⁄₄, or a *whole* yard, will cost 4 times as much; and in the third case, if ²⁵⁄₂¢ buy 1 yard, or any other thing, ⁵⁄₂¢ will buy the ¹⁄₂₅ part of it, and ⅖, or a whole cent, will buy 2 times as much.

NOTE.—See introductory remarks and elucidations of division on pages 92 and 93.

269.   ORAL AND WRITTEN PROBLEMS.

1. If $\dfrac{5}{\text{Books}}$ cost x, what will 1 book cost?

Answer. If 5 books cost x, 1 book will cost the 5th part of x.

2. $\dfrac{8}{\text{Hats}}$ cost x. What will 1 hat cost?

Answer. Since 8 hats cost x, 1 hat will cost the 8th part of x.

3. $\dfrac{25}{\text{Pencils}}$ cost x. What will 1 pencil cost?

4. $\dfrac{6}{\text{Oranges}}$ cost x. What will 1 orange cost?

5. $\dfrac{9}{\text{Chickens}}$ cost x. What will 1 chicken cost?

6. ⅞ of a yard cost x. What will ⅛ of a yard cost?

7. If ⅔ of a yard cost 10¢, what will ⅓ of a yard cost?

Analytic Solution.—If $\frac{2}{3}$ of a yard costs 10 cts., $\frac{1}{3}$ of a yard will cost the *half* part of 10 cts., which is 5 cts.

Question. How do you know this?

8. $\frac{3}{4}$ of a pound cost 15 cts. What will $\frac{1}{4}$ of a pound cost?

9. $\frac{5}{8}$ of pound cost 20 cts. What is the cost of $\frac{1}{8}$?

10. $\frac{3}{4}$ of a pound cost 12 cts. What did 1 pound cost?

Analytic Solution.—Since $\frac{3}{4}$ of a pound cost 12 cts., $\frac{1}{4}$ of a pound will cost $\frac{1}{3}$ of 12 cts., which is 4 cts., and $\frac{4}{4}$, or one pound, will cost 4 times as much, which is 16 cts.

11. $\frac{5}{8}$ of a yard cost 40 cts. What will be the cost of 1 yard?

12. $\frac{2}{3}$ of a dozen cost $16. What is the value of $\frac{3}{4}$ of a dozen?

Analytic Solution.—Since $\frac{2}{3}$ of a dozen cost $16, $\frac{1}{3}$ of a dozen will cost $\frac{1}{2}$ part, which is $8, and $\frac{3}{3}$ or a whole dozen will cost 3 times as much, which is $24; and since 1 dozen cost $24, $\frac{1}{4}$ of a dozen will cost $\frac{1}{4}$ part, which is $6, and $\frac{3}{4}$ part will cost 3 times as much, which is $18.

13. $2\frac{1}{2}$ yards cost 6\frac{1}{4}$. What will $3\frac{1}{5}$ yards cost at the same rate?

SOLUTION STATEMENT.

$		
4	25	
5	2	
5	16	
—	—	
	$8 Ans.	

Reason, or the Philosophic Solution. $2\frac{1}{2}$ or $\frac{5}{2}$ yds. cost 6\frac{1}{4}$, or $\frac{25}{4}$. Since $\frac{5}{2}$ yards cost $\frac{25}{4}$ dollars, $\frac{1}{2}$ of a yard will cost the 5th part, and $\frac{2}{2}$ or a whole yard will cost 2 times as much; and since 1 yard cost the result of this statement, $\frac{1}{5}$ of a yard will cost the 5th part, and $\frac{16}{5}$ will cost 16 times as much.

14. 8½ pounds cost 62½ cents. What was the cost of 1 pound?

SOLUTION STATEMENT.

2	125
25	3
—	
	7½¢ Ans.

Reason—²⁵⁄₄ pounds cost 1²⁵⁄₄¢. Since ²⁵⁄₄ pounds cost 1²⁵⁄₄ cts., ¼ of a pound will cost the 25th part, and ¾, or a whole pound, will cost 3 times as much.

Question.—1. How do you know this? 2. What do you mean by judgment? 3. What kind of reasoning is this? 4. What is the premise in this problem? 5. What do you mean by premise?

15. 3¾ yards cost 37½ cents. What did 1 yard cost?

SOLUTION STATEMENT.

2	75
15	4
—	
	10¢ Ans.

Reason.—¹⁵⁄₄ yds. cost ⁷⁵⁄₂¢. Since ¹⁵⁄₄ yards cost ⁷⁵⁄₂ cents, ¼ of a yard will cost the 15th part, and ¼ or a whole yard will cost 4 times as much.

16. A man receives $21½ for 6¼ days' services. What was the rate per day?

SOLUTION STATEMENT.

2	43
25	4
—	
	$3.44 Ans.

Reason.—²⁵⁄₄ days' services cost $⁴³⁄₂. Since he receives ⁴³⁄₂ dollars for ²⁵⁄₄ days' services, for ¼ of a day's service he will receive the 25th part, and for ¼, or a whole day, 4 times as much. Or, since ²⁵⁄₄ days' services are worth ⁴³⁄₂ dollars, ¼ of a day's service is worth the 25th part, and ¼ or a whole day's service is worth 4 times as much.

17. 8⅔ dozen cost $52. What will 1 dozen cost?

SOLUTION STATEMENT.

$$
\begin{array}{c|c}
\$ & \\
26 & 52 \\
 & 3 \\
\hline
 & \$6 \text{ Ans.}
\end{array}
$$

Reason.—²⁶⁄₃ dozen cost $52. Since ²⁶⁄₃ dozen cost $52, ⅓ of a dozen will cost the 26th part, and ⅓ or a whole dozen, will cost 3 times as much.

18. 9 sheep cost $33¾. What will 1 sheep cost?

SOLUTION STATEMENT.

$$
\begin{array}{c|c}
\$ & \\
4 & 135 \\
9 & \\
\hline
 & \$3\frac{3}{4} \text{ Ans.}
\end{array}
$$

Reason.—9 sheep cost $¹³⁵⁄₄. Since 9 sheep cost ¹³⁵⁄₄ dollars, 1 sheep will cost the 9th part.

19. If ¾ of a pound cost $⅞, what will 1 pound cost?

SOLUTION STATEMENT

$$
\begin{array}{c|c}
\$ & \\
8 & 7 \\
3 & 4 \\
\hline
 & \$1\frac{1}{6} \text{ Ans.}
\end{array}
$$

Reason.—¾ of a pound cost $⅞. Since ¾ of a pound cost ⅞ of a dollar, ¼ of a pound will cost the third part, and ¾ or a whole pound, will cost 4 times as much.

20. Bought 12½ dozen for $12½. What was the cost per dozen?

SOLUTION STATEMENT.

$$
\begin{array}{c|c}
\$ & \\
2 & 25 \\
25 & 2 \\
\hline
 & \$1 \text{ Ans.}
\end{array}
$$

Reason.—²⁵⁄₂ dozen cost $²⁵⁄₂. Since ²⁵⁄₂ dozen cost $²⁵⁄₂, ¼ dozen will cost the twenty-fifth part, and ½ or 1 dozen will cost twice as much.

21. At 7½ cents per pound, how many pounds can be bought for 83⅓ cents?

SOLUTION STATEMENT.

	℔
	1
15	2
3	250
—	—
	11¹⁄₉ lbs. Ans.

Reason.—¹⁵⁄₂¢ buy 1 pound. Since ¹⁵⁄₂ cents will buy 1 pound, ½ of a cent will buy the 15th part, and ²⁄₂ or a whole cent, will buy 2 times as much; and since 1 cent will buy this result, ⅓ of a cent will buy the third part and ²⁵⁰⁄₃ cents will buy 250 times as much.

22. Chickens cost $¾ a piece. How many can be bought for $13½?   Ans. 18 chickens.

270. *Reasoning for the Division of Abstract Numbers.*

1. Divide 6 by 2.

STATEMENT.

2	6
—	—
	3 Ans.

Remarks.—The real question in this problem is to find how many times 6 is equal to 2. Or, in other words, we are required to measure 6 by the unit of measure, 2. The basis or unit of all numbers is 1; and hence in our reasoning for division of abstract numbers, we use 1 as our first unit of measure. The following is our premise, reasoning, and conclusion: 6 is equal to 1, 6 times. Since 6 is equal to 1, 6 times, it is equal to 2, instead of 1, ½ as many times, which is 3.

2. Divide ⅞ by ¾.

SOLUTION STATEMENT.

.8	7
3	4
—	—
24	28
	—
	1⅙ Ans.

Explanation and Reason.— The real question to be determined in this problem is to find how many times ⅞ is equal to ¾. Or, in other words, we are required to measure ⅞ by the unit of

measure, $\frac{1}{4}$. The basis or unit of all numbers is 1. Hence, as explained on page 93, in our reasoning for division of whole numbers we use 1 as our first unit of measure. The following is our premise, reasoning, and conclusion: $\frac{1}{4}$ is equal to 1, $\frac{1}{4}$ of a time. Since $\frac{1}{4}$ is equal to 1, $\frac{1}{4}$ of a time, it is equal to $\frac{1}{4}$, instead of 1. 4 times as many times; and to $\frac{1}{4}$, instead of $\frac{1}{4}$, the $\frac{1}{4}$ part of the number of times.

3.   Divide $3\frac{3}{4}$ by $2\frac{2}{3}$.

SOLUTION STATEMENT.

$$\begin{array}{c|c} 4 & 15 \\ 8 & 3 \\ \hline & \\ & 1\frac{13}{32} \text{ Ans.} \end{array}$$

Reason.—$\frac{15}{4}$ is equal to 1 $\frac{15}{4}$ times. Since $\frac{15}{4}$ is equal to 1, $\frac{15}{4}$ times, it is equal to $\frac{1}{4}$. instead of 1, 3 times as many times, and to $\frac{1}{3}$,

instead of $\frac{1}{4}$, the eighth part of the number of times.

4.   Divide 5 by $\frac{6}{11}$.

SOLUTION STATEMENT.

$$\begin{array}{c|c} & 5 \\ 6 & 11 \\ \hline & \\ & 55 \\ \hline & 9\frac{1}{6} \text{ Ans.} \end{array}$$

Reason.—Writing 5 on the statement line, we reason thus: 5 is equal to 1, 5 times. Since 5 is equal to 1, 5 times, it is equal to $\frac{1}{11}$, 11 times as many times, and to $\frac{6}{11}$, the 6th part of the number of times.

5.   Divide $\frac{2}{15}$ by 8.

SOLUTION STATEMENT.

$$\begin{array}{c|c} 15 & 2 \\ 8 & \\ \hline & \\ & \frac{1}{60} \text{ Ans.} \end{array}$$

Explanation and Reason.— In this example, the dividend being less than the divisor, the question is, what part of a time is the

$\frac{2}{15}$ equal to 8   We first write the dividend on the statement line and reason thus: $\frac{2}{15}$ is=to 1, $\frac{2}{15}$ of a time. Since $\frac{2}{15}$ is= to 1, $\frac{2}{15}$ of a time, it is=to 8, the 8th part of the number of times.

NOTE.—The solution of the 5 preceding problems elucidates the only correct reasoning for dividing abstract fractional numbers. But for practical work we would not advise a change from the reasoning given where the numbers are denominate.

6.   Divide 22¾ by 5¼.   Ans. 4$\frac{3}{22}$.

7.   Divide 3 by ⅔.   Ans. 4½.

8.   Divide 14⅖ by 9.   Ans. 1⅗.

9.   Divide 32 by 9.   Ans. 3⅝.

GENERAL DIRECTIONS FOR DIVISION OF FRACTIONS.

271.   From the foregoing elucidations, we derive the following general directions for division of fractions:

1.   *Write on the upper right hand side of the statement line the number which is to be divided or measured.   Then, reasoning from the basis of* MANY *to* ONE, *or from a* PART OF ONE *to* ONE, *write the other numbers on the division or multiplication side of the statement line, according as the conclusion is decreasing or increasing.*

2.   *Mixed numbers should be reduced to fractional expressions, and the reason given for writing both the numerator and the denominator.*

272. MISCELLANEOUS ORAL EXERCISES.

1. If ⅔ of a yard cost $2¼, what will 1 yard cost?
Ans. $6.

2. If ⅔ of a yard cost $2¼, what will ⅛ of a yard cost? Ans. $¾.

3. ¾ of a number is 15. What is the number? Ans. 20.

4. If ⅔ of a number is 8, what is 1¾ times the number? Ans. 21.

5. If ⅔ of a dozen cost $8, what will ¾ of a dozen cost at the same rate? Ans. $9.

6. What part of 4 is 3? Ans. ¾.

Analytic Solution.—Here, by the terms of the question, we have 3 to divide or measure by 4, and by the exercise of our reason, we proceed thus: since 3 is equal to 1, 3 times, it is equal to 4, ¼ of 3 times, which is ¾. Or thus: since 1 is ¼ of 4, 3 is 3 times ¼, which is ¾.

7. What part of 5 is ⅔? Ans. $\frac{2}{15}$.

Analytic Solution.—Since ⅔ is equal to 1, ⅔ of a time, it is equal to 5 the ⅕ part of ⅔ of a time, which is $\frac{2}{15}$.

8. What part of ⅕ is 7? Ans. 8¾.

Analytic Solution.—Since 7 is equal to 1, 7 times, it is equal to ⅕, 5 times 7 which is 35, and to ¼ instead of ⅕, ¼ part of 35, which is 8¾.

9. What part of ⅜ is $\frac{5}{9}$? Ans. 1$\frac{13}{27}$.

Analytic Solution.—Since ⅜ is equal to 1, ⅜ of a time, it is equal to ⅛, 8 times ⅜ which is $\frac{40}{9}$, and to ¾ instead of ⅛, ¾ part of $\frac{40}{9}$, which is $\frac{40}{27}$, or 1$\frac{13}{27}$.

10. What part of ⅞ is ⅘? Ans. $\frac{32}{35}$.

11. What part of 3½ is 2¼? Ans. $\frac{9}{14}$.

12. What part of 5 is ¾ of 2? Ans. $\frac{3}{10}$.

13. What part of ⅕ is ¾ of ⅝? Ans. $\frac{75}{224}$.

14. 9 is $\frac{1}{8}$ of what number? Ans. 72.

Analytic Solution.—Since 9 is $\frac{1}{8}$ of a number, $\frac{8}{8}$ or the whole number is 8 times 9, or 72.

15. 13 is $\frac{1}{7}$ of what number? Ans. 91.

16. $21\frac{3}{10}$ is $\frac{1}{5}$ of what number? Ans. $106\frac{1}{2}$.

17. $\frac{3}{35}$ is $\frac{1}{7}$ of what number? Ans. $\frac{3}{5}$.

18. 24 is $\frac{4}{5}$ of how many times 3? Ans. 10.

Analytic Solution.—Since 24 is $\frac{4}{5}$ of the number, $\frac{1}{5}$ is $\frac{1}{4}$ part of 24 which is 6, and $\frac{5}{5}$ or the whole number is 5 times 6, which is 30; and as 30 is equal to 3, 10 times, therefore, 24 is $\frac{4}{5}$ of 10 times 3.

19. 32 is $\frac{4}{7}$ of how many times 8? Ans. 7.

20. 28 is $\frac{7}{15}$ of how many times 12? Ans. 5.

21. $\frac{3}{4}$ of 48 is $\frac{2}{3}$ of what number? Ans. 54.

Analytic Solution.—Since 48 is the whole of a number, $\frac{1}{4}$ of the number is $\frac{1}{4}$ part of 48, which is 12, and $\frac{3}{4}$ is 3 times 12, which is 36; and since 36 is $\frac{2}{3}$ of an unknown number, $\frac{1}{3}$ of it is $\frac{1}{2}$ of 36, which is 18, and $\frac{3}{3}$ or the whole number is 3 times 18, which is 54.

22. $\frac{8}{9}$ of 63 is $\frac{4}{11}$ of what number? Ans. 154.

23. $\frac{3}{8}$ of $\frac{2}{3}$ of 64 is $\frac{2}{13}$ of what number?

Ans. 104.

24. $\frac{1}{4}$ of $\frac{1}{7}$ of 42 is $\frac{3}{8}$ of what number?

Ans. $7\frac{1}{5}$.

25. $\frac{3}{4}$ of 32 is $\frac{2}{3}$ of 4 times what number?

Ans. 9.

Analytic Solution.—Since 32 is the whole of a number, $\frac{1}{4}$ of the number is $\frac{1}{4}$ part of 32, which is 8, and $\frac{3}{4}$ is 3 times 8, which is 24; and since 24 is $\frac{2}{3}$ of 4 times an unknown number, $\frac{1}{3}$ of 4 times the number is $\frac{1}{2}$ of 24, which is 12, and $\frac{3}{3}$ or the whole of 4 times the number, is 3 times 12, which is 36; and since 36 is 4 times the number, $\frac{1}{4}$ of 36, which is 9, is the required number.

26. ⅞ of 40 is ⅝ of 7 times what number ?
<div align="right">Ans. 6.</div>

27. ⅘ of 56 is ¼ of 6 times what number ?
<div align="right">Ans. 12.</div>

28. ⅝ of ¾ of 66 is 3⅔ of 3 times what number ?
<div align="right">Ans. 3.</div>

29. What is ⅓ and ½ of a ⅓, of ⅔ of 15 ?
<div align="right">Ans. 5.</div>

273. MISCELLANEOUS PROBLEMS IN DIVISION OF
FRACTIONS.

1. Bought 4 yards for $14½. What was the cost
per yard ?
<div align="right">Ans. $3⅝.</div>

2. Sold 8½ pounds for $1.87. What was the
price per pound ?
<div align="right">Ans. 22 cents.</div>

3. Paid 37½ cents for 6¼ yards of calico. What
was the price per yard ?
<div align="right">Ans. 6 cents.</div>

4. At $1⅜ per gallon, how many gallons can be
bought for $148½.
<div align="right">Ans. 108 gallons.</div>

5. Divide $\frac{9}{7}$ by 2. Ans. $\frac{9}{7}$.

6. Divide $\frac{9}{21}$ by 3. Ans. $\frac{1}{7}$.

7. Divide $\frac{14}{15}$ by 5. Ans. $\frac{14}{75}$.

8. Divide $\frac{6}{7}$ by 5. Ans. $\frac{6}{35}$.

9. Divide 7⅕ by 9. Ans. $\frac{4}{5}$.

10. Divide 2 by $\frac{9}{7}$. Ans. 2⅓.

11. Divide 3 by $\frac{9}{21}$. Ans. 7.

12. Divide 5 by $\frac{14}{15}$. Ans. $5\frac{5}{14}$.

13. Divide 21 by $\frac{7}{11}$. Ans. 33.

14. Divide 105 by $\frac{14}{17}$. Ans. 119.

15. Divide $\frac{10}{11}$ by $\frac{5}{22}$. Ans. 4.

16. Divide $2\frac{1}{3}$ by $\frac{7}{52}$. Ans. 12.

17. Divide $2\frac{1}{4}$ by $\frac{3}{4}$.   Ans. 3.
18. Divide $\frac{13}{17}$ by $\frac{4}{11}$.   Ans. $2\frac{7}{68}$.
19. Divide $3\frac{1}{2}$ by $2\frac{1}{5}$.   Ans. $1\frac{13}{22}$.
20. Divide $\frac{4}{9}$ of $\frac{14}{81}$ by $\frac{8}{9}$.   Ans. $\frac{1}{6}$.

21. If one pound of tea cost $\frac{5}{6}$ of a dollar, how many pounds can be bought for $25?
                                   Ans. 30 lbs.

22. 6 barrels of flour were divided among some poor families in such a manner that each received $\frac{2}{3}$ of a barrel. How many families were there?
                                   Ans. 9 families

OPERATION INDICATED.

$2\left|\begin{array}{l} 1=\text{Family.} \\ 3 \\ 6 \\ \hline 9 \text{ families, Ans.} \end{array}\right.$ or $2\left|\begin{array}{l} 6 \\ 3 \\ \hline 9 \text{ families.} \end{array}\right.$

23. If a boy can earn $\frac{7}{11}$ of a dollar in one day, how many days will it take him to earn $21?
                                   Ans. 33 days.

24. Henry walked 25 miles, which was $\frac{5}{6}$ of the distance Robert walked. How many miles did Robert walk?   Ans. 30 miles.

25. At the battle of Germantown, the British lost about 600 men; this was $\frac{3}{5}$ of the number lost by the Americans; and the number lost by the Americans was $\frac{2}{5}$ of the number they received as re-enforcements just before the battle. How many men did the Americans lose, and how many did they receive as re-enforcements?
         Ans. 1000 men lost, 2500 re-enforcements.

OPERATION INDICATED.

$3\left|\begin{array}{l} 600 \\ 5 \\ \hline 1000 \text{ men lost by the} \end{array}\right.$ [Americans. $2\left|\begin{array}{l} 1000 \\ 5 \\ \hline 2500 \text{ re-enforcements.} \end{array}\right.$

26. A man had his store insured for $9000, which was $\frac{8}{9}$ of $\frac{9}{11}$ of its value. What was the store worth? Ans. $12375.

27. Sulphur will fuse at 232° Fahrenheit, which is $7\frac{1}{4}$ times the temperature required to melt ice. At what temperature will ice melt? Ans. 32°.

OPERATION INDICATED.

$$29 \,\Big|\, \begin{matrix} 232° \\ 4 \\ \hline 32° \text{ Ans.} \end{matrix}$$

28. A quantity of mercury weighed $32062\frac{1}{2}$ lbs., which is $13\frac{1}{2}$ times the weight of an equal bulk of water. What would an equal bulk of water weigh? Ans. 2375 lbs.

29. A pound of water at 212° F. was mixed with a pound of powdered ice at 32°. The united temperature of the two was $4\frac{9}{13}$ times the temperature of the mixture when the ice became melted. What was the temperature of the two pounds after the ice became melted? Ans. 52°.

OPERATION INDICATED.

$$212+32=244. \qquad 61 \,\Big|\, \begin{matrix} 244 \\ 13 \\ \hline 52° \text{ Ans.} \end{matrix}$$

30. When the air was at the freezing point, a cannon $27613\frac{1}{2}$ feet distant from New Orleans was discharged. $25\frac{1}{3}$ seconds elapsed after the discharge before the sound reached New Orleans. How many feet per second did the sound travel? Ans. 1090 feet.

31. Divide $287\frac{3}{4}$ by 5. Ans. $57\frac{11}{20}$.

Operation without the Statement Line.

$$5)\,287\frac{3}{4}$$

$57\frac{11}{20}$ Ans.

Explanation.—We first divide the 287 by the process of short division and obtain a quotient of 57, and a remainder of 2; this remainder we reduce to a fraction whose denominator is the same as that of the fraction to be divided, add it to this fraction, and then divide the sum by 5 and annex the result to the quotient 57. Thus $2=\frac{8}{4}+\frac{3}{4}=\frac{11}{4}$, and $\frac{11}{4}\div5=\frac{11}{20}$.

32. Divide $1471\frac{3}{16}$ by 9. Ans. $163\frac{67}{144}$.

33. Divide $1044\frac{2}{3}$ by 12. Ans. $87\frac{1}{18}$.

34. E. T. Churchill divided $14\frac{7}{12}$ dozen apples among 3 boys and 2 girls; he gave each girl twice as many as each boy. How many did each boy and each girl receive?

Ans. $2\frac{1}{12}$ doz. each boy, $4\frac{1}{6}$ doz. each girl.

OPERATION INDICATED.

3 boys, each receives 1 apple, which makes 3 apples.
2 girls, each receives 2 apples, which makes 4 apples.
$3+4=7$, the sum of the proportion of the apples due the 3 boys and 2 girls.

Statement to obtain the amount due each boy.

DOZ.

12	175
7	3
3	

$2\frac{1}{12}$ doz. each boy.

Statement to obtain the amount due each girl.

DOZ.

12	175
7	4
2	

$4\frac{1}{6}$ doz. each girl.

35. Divide 1 by $\frac{1}{5}$. Ans. 5.

36. Divide $\frac{1}{5}$ by 1. Ans. $\frac{1}{5}$.

37. Divide $\frac{8\frac{1}{4}}{6\frac{1}{4}}$ of $\frac{\frac{3}{16}}{11}$ by $\frac{9\frac{1}{4}}{4\frac{4}{9}}$ of $\frac{1}{3}$ Ans. $\frac{5}{7}$.

38. If 4½ pounds of coffee cost 90 cents, what will 22¾ pounds cost? Ans. $4.55.

39. R. E. L. Flemming owns ⅜ of the capital stock of a factory valued at $24000; he gives ½ of ⅛ to educational societies, and the remainder he divides equally between his four children. How much does he give to educational societies and how much does each child receive?

Ans. $1500 to educational societies.
$1875 each child receives.

OPERATION INDICATED.

⅜ of $24000=$9000, Flemming's stock.
½ of ⅛ of $9000=$1500, given to educational societies.
$9000—1500=$7500, amt. divided between 4 children
$7500÷4=$1875, each child's share.

40. S. J. Weis has 65½ yards of cloth, 2 yards wide. How many yards of lining ⅔ of a yard wide will be required to line it? Ans. 196½ yards.

$$65\tfrac{1}{2}\times 2 = 131 \text{ yds.}$$

	131
2	3
—	—
	196½ yds. Ans.

41. Divide 18 oranges between A. and B. so that A. will have ¼ more than B. What number will each have? Ans. A. 10; B. 8.

OPERATION.

		Statement showing what B. rec'd.		Statement showing what A. rec'd.	
B. receives	1		18		18
A. receives	1¼	9	4	9	4
	—		—	4	5
A. and B. receive	2¼		8	—	—
					10

42. Divide 18 oranges between A. and B. so that A. will have $\frac{1}{4}$ less than B. What number will each have? Ans. A. 7$\frac{5}{7}$; B. 10$\frac{2}{7}$.

43. A., B., and C. are to receive $26 in proportion to $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. What will each receive?
Ans. A. $12; B. $8; C. $6.

44. Frank can work 100 problems in 4 hours, and Lillie can work the same problems in 5 hours. How many hours will it require for both to work the problems? Ans. 2$\frac{2}{9}$ hours.

Solution.—Since Frank can work the problems in 4 hours, in 1 hour he can work $\frac{1}{4}$ of them; and since Lillie can work the problems in 5 hours, in 1 hour she can work $\frac{1}{5}$ of them. Hence, $\frac{1}{4} + \frac{1}{5} = \frac{9}{20}$ of the problems, worked by both in 1 hour: and since $\frac{9}{20}$ of the work required 1 hour, $\frac{1}{20}$ of the work will require $\frac{1}{9}$ of an hour, and $\frac{20}{20}$, or the whole work, will require 20 times as many hours, or $\frac{20}{9}$ hours, which is 2$\frac{2}{9}$ hours.

45. A. and B. can do a piece of work in 14 days. A. can do $\frac{3}{4}$ as much as B. How many days will it take each to do it, working alone?
Ans. 24$\frac{1}{2}$ days for B.
32$\frac{2}{3}$ days for A.

OPERATION INDICATED.

1 equals the work done by B.
$\frac{3}{4}$ " " " " A.

1$\frac{3}{4}$, or $\frac{7}{4}$, equals the work done by B. and A.

Hence B. does $\frac{4}{7}$ of the work in 1 day
and A. " $\frac{3}{7}$ " " 1 "

14 ds. $\div \frac{4}{7} = 24\frac{1}{2}$ days, for B. to do the work alone
14 ds. $\div \frac{3}{7} = 32\frac{2}{3}$ " " A. " " " "

46. Three persons, A., B., and C., do a piece of work; A. and B. together do $\frac{7}{9}$ of it, and B. and C. do $\frac{7}{11}$ of it. What part of the work is done by B.?   Ans. $\frac{41}{99}$.

Solution.—As A. and B. do $\frac{7}{9}$ of it, it is clear that C. does the remaining $\frac{2}{9}$; and as B. and C. do $\frac{7}{11}$ of it, it is clear that A. does the remaining $\frac{4}{11}$. Then, as A. does $\frac{4}{11}$ and C. $\frac{2}{9}$, they, together, do $\frac{4}{11} + \frac{2}{9} = \frac{58}{99}$; and if A. and C. do $\frac{58}{99}$ of a piece of work done by A., B., and C., it is clear that B. does the difference between $\frac{99}{99}$ and $\frac{58}{99}$, which is $\frac{41}{99}$.

47. If 6 oranges and 7 lemons cost 33¢, and 12 oranges and 10 lemons cost 54¢, what was the cost of 1 orange and 1 lemon each?

Ans. 1 orange, 2¢.
1 lemon, 3¢.

OPERATION.

6 oranges and   7 lemons cost 33¢.
12   "   " 10   "   "   54¢.

12   "   " 14   "   "   66¢.
12   "   " 10   "   "   54¢.

                4   12¢.

12¢ ÷ 4 = 3¢, cost of 1 lemon.
3¢ × 7 (lemons) = 21¢, cost of 7 lemons.

33¢ — 21¢ = 12¢, cost of 6 oranges.
12¢ ÷ 6 = 2¢, cost of 1 orange.

Reason.— Since 6 oranges and 7 lemons cost 33 cents, twice as many, or 12 oranges and 14 lemons will cost twice as much which is 66 cents; and since by the second condition of the problem, 12 oranges and 10 lemons cost 54 cents, the difference between 66 cents and 54 cents, which is 12 cents, will be the cost of the difference between (12 oranges and 14 lemons) and (12 oranges and 10 lemons) which is 4 lemons. And since 4 lemons cost 12 cents, *one lemon* will cost the fourth part which is 3 cents. And since 6 oranges and 7 lemons cost 33 cents, by subtracting the cost of 7 lemons which is 21 cents, we have 12 cents, the cost of 6 oranges. And since 6 oranges cost 12 cents, *one orange* will cost the sixth part which is 2 cents.

   27

48. A miller invested $54 in grain of which $\frac{3}{10}$ was barley at $62\frac{1}{2}$¢ per bushel; $\frac{3}{5}$ was wheat at 1.87\frac{1}{2}$ per bu.; and the balance oats @ $37\frac{1}{2}$¢ per bu. How many bushels of grain did the miller buy?

<div align="center">OPERATION.</div>

Bushels.

$\frac{3}{10}$ @ $1\frac{25}{4}$¢ $= \frac{75}{4}$¢ $= .18\frac{3}{4}$ proportionate cost of the barley.

$\frac{3}{5}$ @ $3\frac{15}{4}$¢ $= \frac{45}{4}$ $= 1.12\frac{1}{2}$ "   "   wheat.

$\frac{1}{10}$ @ $\frac{75}{2}$¢ $= \frac{15}{4}$¢ $= .03\frac{3}{4}$ "   "   oats.

$1.35 Proportionate cost of 1 bu. of grain.

BU.

$$1.35 \mid \begin{array}{l} 1 \\ 54.00 \end{array}$$

40 bus. purchased, Ans.

49. A. and B. can do a piece of work in 10 days; A. alone can do it in 15 days. How many days will it take B. to do it? Ans. 30 days.

———◆———

SYNOPSIS FOR REVIEW.

———

Define the following words and phrases:

268. Division of Fractions.   268. Three Questions in Division.   268. How does Division of Fractions Compare with Multiplication of Fractions. 270. Reasoning for the Division of Abstract Numbers.   271. General Directions for Division of Fractions.

ISCELLANEOUS PROBLEMS,

INVOLVING THE PRINCIPLES OF ADDITION, SUB
TRACTION, MULTIPLICATION, AND DIVISION
OF FRACTIONS.

274. Find the difference between $\frac{5}{6}$ and $\frac{2}{3}$; $\frac{4}{7}$ and
$\frac{5}{6}$; $\frac{2}{3}$ and $\frac{7}{11}$; $3\frac{7}{9}$ and $2\frac{3}{7}$; $4\frac{2}{3}$ and $\frac{1}{2}$ of $3\frac{1}{4}$?
$\qquad$ Ans. To last, 3.

2. Find the sum of $\frac{4}{7}$ of $\frac{9}{20}$ and $\frac{3}{4}$ of $\frac{8}{21}$.
$\qquad$ Ans. $\frac{19}{35}$.

3. To the quotient of $2\frac{3}{5}$ divided by $5\frac{1}{5}$, add the
quotient of $3\frac{2}{3}$ divided by $\frac{1}{2}\frac{1}{1}$. $\qquad$ Ans. $7\frac{1}{2}$.

4. A number was divided by $\frac{3}{4}$, and gave a quo
tient of 20. What was the number? $\qquad$ Ans. 15.

5. What number is that, which being multiplied
by $\frac{7}{11}$, gives as a product $\frac{14}{33}$? $\qquad$ Ans. $\frac{2}{3}$.

OPERATION INDICATED.

$$\tfrac{14}{33} \div \tfrac{7}{11} = \tfrac{2}{3} \text{ Ans.}$$

6. What number is that, from which, if you take
$\frac{2}{5}$ of itself, the remainder will be 12? $\qquad$ Ans. 30.

OPERATION INDICATED.

$1 = \frac{5}{5} - \frac{2}{5} = \frac{3}{5}$; if $\frac{2}{5} = 12$, $\frac{1}{5} = 6$, and $\frac{5}{5}$ equals 30.

7. What number is that, to which, if you add $\frac{3}{5}$
of itself, the sum will be 40? $\qquad$ Ans. 25.

OPERATION INDICATED.

$1 = \frac{5}{5} + \frac{3}{5} = \frac{8}{5}$; if $\frac{8}{5} = 40$, $\frac{1}{5} = 5$, and $\frac{5}{5} = 25$,

8. A. owns $\frac{2}{3}$ of a store which is worth $25000, and sells $\frac{1}{3}$ of his share. What part does he still own, and what is it worth?

Ans. A. owns $\frac{1}{10}$, worth $2500.

9. Smith owns $\frac{5}{11}$ of a cotton mill and sells $\frac{3}{10}$ of his share to Jones for $33000. What is the mill worth at that rate? Ans. $242000.

10. John has 5 cents, and James $\frac{3}{4}$ of 8 cents. What part of James' money is John's? Ans. $\frac{5}{6}$.

OPERATION INDICATED.

$\frac{3}{4}$ of $8\cancel{c}=6\cancel{c}$; $5 \div 6 = \frac{5}{6}$, Ans.

11. One planter raised 500 bales of cotton, and another raised 250. What part of the first one's crop is the second? Ans. $\frac{1}{2}$.

12. The sum of four fractions is $1\frac{5}{6}$. Three of the fractions are $\frac{3}{7}$, $\frac{1}{2}$, and $\frac{2}{3}$. What is the fourth?

Ans. $\frac{5}{21}$.

13. What number is that, to which if $\frac{2}{7}$ of $\frac{25}{36}$ of $1\frac{1}{5}$ be added, the sum will be $1\frac{1}{2}$? Ans. 1.

14. Two boys bought a bushel of oranges, one paying $2\frac{1}{6}$ dollars and the other $4\frac{2}{3}$ dollars. What part of it should each have?

Ans. First, $\frac{13}{41}$; second, $\frac{28}{41}$.

15. A farmer sold $\frac{5}{7}$ of his mules on Monday; on Tuesday he bought $\frac{2}{5}$ as many as he sold, and then had 40. How many mules had he at first?

Ans. 56 mules.

OPERATION INDICATED.

$\frac{5}{7}$ from $\frac{7}{7}=\frac{2}{7}$. $\frac{2}{5}$ of $\frac{5}{7}=\frac{2}{7}$. $\frac{2}{7}+\frac{2}{7}=\frac{4}{7}$.

$\frac{4}{7}=40$; $\frac{1}{7}=8$, and $\frac{7}{7}=56$, Ans.

16. A planter gave 50 bales of cotton at 50\frac{1}{10}$ per bale for flour at $7\frac{1}{2}$ per barrel. How many barrels of flour did he receive? Ans. 334 bbls,

17. A. W. McLellan gave $\frac{1}{6}$, $\frac{1}{2}$, and $\frac{1}{4}$ of his money to different benevolent institutions, and had $1000 left. How much had he at first?

Ans. $20000.

18. C. Manson owning $\frac{6}{11}$ of a rice mill, sold $\frac{2}{3}$ of his share for $8800. What was the value of the mill? Ans. $24200.

19. A book-keeper worked $91\frac{1}{4}$ days, and after paying $\frac{2}{3}$ of $\frac{2}{5}$ of his earnings for board and washing, had $438 remaining. How much money did he receive in all, and how much per day?

Ans. $730 in all, $8 per day.

20. Prophet can do a piece of work in 6 days, and Fisher can do the same work in 8 days. How many days will it take both together to do the work? Ans. $3\frac{3}{7}$ days.

OPERATION INDICATED.

$$\frac{1}{6}+\frac{1}{8}=\frac{7}{24}. \quad 1 \text{ day} \div \frac{7}{24}=3\frac{3}{7} \text{ days.}$$

21. Myers, Levy, and Hoffman can do a piece of work in 10 days; Myers and Levy can do it in 15 days. In what time can Hoffman do it, working alone? Ans. 30 days.

22. A man died and left his wife $14400, which was $\frac{3}{4}$ of $\frac{3}{5}$ of his estate. At her death she left $\frac{5}{6}$ of her share to her daughter. How much money did the daughter receive, and what part was it of her father's estate?

Ans. $12000, $\frac{25}{33}$ of her father's estate.

23. A man engaging in trade lost $\frac{3}{7}$ of the money he invested; he then gained $1000, when he had $3800. What did he have at first, and what was his loss? Ans. $4900 at first, $2100 loss.

OPERATION INDICATED.

$$\$3800-\$1000=\$2800. \quad \$2800\div\frac{4}{7}=\$4900.$$
$$\frac{3}{7} \text{ of } \$4900=\$2100,$$

24. A mule and a dray cost $240; the mule cost
$1\frac{2}{3}$ times as much as the dray. What did each cost?
<div style="text-align:center">Ans. $90 dray, $150 mule.</div>

<div style="text-align:center">OPERATION INDICATED.</div>

1, (cost of dray)$+1\frac{2}{3}$ (cost of mule)$=2\frac{2}{3}=$240.

$				240
	240		8	3
8	3		3	5
—	———		—	———
	$90 cost of dray.			$150 cost of mule.

25. How many bushels of apples at $\frac{1}{4}$ a bushel,
will pay for $\frac{8}{10}$ of a barrel of oranges at $6\frac{2}{3}$ a barrel ?
<div style="text-align:center">Ans. 7½ bushels.</div>

26. Sweeney paid $\frac{4}{7}$ of his year's wages for board,
$\frac{2}{3}$ of the remainder for clothes, and had $80 left.
How many dollars did he receive for labor?
<div style="text-align:center">Ans. $560.</div>

27. Forcheimer lost $\frac{3}{8}$ of his fish-line, and then
added 25½ feet, when it was just $\frac{3}{4}$ of its original
length. What was its original length?
<div style="text-align:center">Ans. 204 feet.</div>

<div style="text-align:center">OPERATION INDICATED.</div>

<div style="text-align:center">$1=\frac{5}{5}-\frac{3}{8}=\frac{5}{8}$. $\frac{3}{4}-\frac{5}{8}=\frac{1}{8}=25\frac{1}{2}$ feet.
$25\frac{1}{2}\times8=204$ feet Ans.</div>

28. Purcell, having a certain number of cents,
gave one-half of them and half a cent over to one
beggar; one-half of what he had remaining and
half a cent over to a second beggar; and to a third,
one-half of what he then had and half a cent over,
and had left 3 cents. How many cents had he at
first ?   Ans. 31 cents.

<div style="text-align:center">OPERATION INDICATED.</div>

(3+½¢ over)×2= 7¢, had before making 3rd gift.
(7+½¢ over)×2=15¢,   "   "   2nd "
(15+½¢ over)×2=31¢,   "   .   "   1st "

29. John lives with his parents, but works for Mr. Smith who pays him $210 per year. His parents board him, but he has his clothes to buy. He spends $\frac{2}{7}$ of his wages for cigars, $\frac{2}{5}$ of the remainder for theater tickets, $\frac{1}{3}$ of the remainder for wine, and $\frac{1}{2}$ of what he then has for novels. How much has he remaining at the end of the year to pay for his clothes?   Ans. $30.

30. Joseph worked on the same conditions as John, in the problem above. He gave $\frac{1}{14}$ of his wages to the cause of charity, $\frac{1}{13}$ of the remainder for useful books, $\frac{1}{6}$ of the remainder for evening tuition, paid $100 for clothes, and deposited the balance in the bank. How many dollars did he put in the bank?   Ans. $50.

31. A. G. Niehues and R. G. Jones have $1899, and Jones has $3\frac{1}{2}$ times as much as Niehues. How much has each?

Ans. Niehues $422, and Jones $1477.

NOTE.—For the operation, see problem 24, above.

32. J. C. Beals can solve 25 problems in 50 minutes and H. H. Barlow can solve them in 30 minutes. In what time can both solve them?

Ans. $18\frac{3}{4}$ minutes.

33. U. Burke purchased 200 barrels of flour for $1450, and sold $\frac{3}{4}$ of it at a profit of $\frac{1}{2}$ per barrel, and the remainder at $7\frac{1}{10}$ per barrel. How much did he gain?   Ans. $67.50.

34. What is the numerical value of

$$\frac{4\frac{1}{2}-\frac{3}{4}}{2\frac{1}{3}+1\frac{1}{3}}?$$ Ans. $1\frac{1}{14}$.

35. M. Ernst bought $3841\frac{1}{2}$ pounds of cotton at $7\frac{3}{4}$ pence per pound. What did it cost?

Ans. £124, $11\frac{5}{8}$ d.

36. R. J. Kennedy has 3 dozen oranges which he wishes to divide between Miss Kate and Miss Tillie, so that Miss Kate shall receive $\frac{1}{4}$ more than Miss Tillie. How many will each receive?

Ans. Miss K. 20, and Miss Tillie 16.

OPERATION INDICATED.

1	=Miss Tillie's proportional share.		36	36
$1\frac{1}{4}$	=Miss Katie's proportional share.	9	4 9	4
—		-	— 4	5
$2\frac{1}{4}$	=the sum of the proportional shares.		16 -	—
				20

37. A tree 110 feet high, had $\frac{3}{5}$ of it broken off in a storm. How much of it was left standing?

Ans. 44 feet.

38. What cost $22\frac{3}{4}$ pounds of coffee at $21\frac{3}{4}$¢ per pound?

Ans. 4.94\frac{11}{16}$.

39. If $18\frac{3}{4}$ yards cost $3.37\frac{1}{2}$, what will $3\frac{1}{2}$ yards cost?

Ans. 63 cents.

40. Miss Cora has $600 of which she wishes to give to A. $\frac{1}{3}$, B. $\frac{1}{4}$, C. $\frac{1}{5}$, and D. $\frac{1}{6}$. How much will each receive?

Ans. A. $200, B. $150, C. $120, and D. $100.

41. Miss Mamie has $600 which she wishes to give to A., B., C., and D. in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$. How much will each receive?

Ans. A. 210\frac{10}{19}$, B. 157\frac{17}{19}$, C. 126\frac{6}{19}$, and D. 105\frac{5}{19}$.

OPERATION INDICATED.

$$\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}=\frac{19}{20}. \quad 19 \left|\begin{array}{l} \$ \\ 600 \\ 20 \end{array}\right. =\$210\frac{10}{19} \text{ A's share.}$$

42. If a yard and a half cost a dollar and a half, what will twelve and a half yards cost?

Ans. 12\frac{1}{2}$.

43. If ⅓ of 6 be 3, what will ¼ of 20 be?
Ans. 7½.

44. If 3 is the third of 6, what will the fourth of 20 be?
Ans. 3¼.

45. E. L. Hunt owned a quantity of rice, of which he sold ⅘ for $99.60. What is ⅔ of the remainder worth at the same rate?
Ans. $16.60.

46. M. Landman paid $60 for ¾ of an acre of land. What is the value of ⅝ of an acre.
Ans. $50.

47. J. J. Hauler bought 937852½ pounds of cotton at 14 15/32¢ per pound. What was the cost?
Ans. $135695.53 23/64.

48. E. T. Berwick invested ¼ of his money in sugar, ⅓ in rice, ⅜ in coffee, and deposited in bank $2645. How much money had he at first?
Ans. $63480.

49. L. Meyer spends ⅓ of his time in study, ¼ in labor, ⅛ in rest and recreation, and the remainder in sleep. How many of the 24 hours of a day does he sleep?
Ans. 7 hours.

50. An industrious young lady spends ¼ of her time in the performance of household affairs, ⅙ in reading good books, 1/12 in physical exercise in the open air and sunlight, ⅛ in the practice of music, singing and parlor amusements, or social intercourse, 2 hours per day in eating, and the remainder of the day in sleeping. How many hours per day does she devote to each?

Ans. 6 hours to household affairs; 4 hours to reading; 2 hours to exercise; 3 hours to music, etc.; 2 hours to eating, and 7 hours to sleeping.

51.   A loafer spends 4 hours per day sauntering on street corners, 3 hours smoking and drinking, $\frac{1}{4}$ of the day in sleep, $\frac{1}{8}$ of the day in drunkenness, $\frac{1}{12}$ in eating, $\frac{1}{12}$ in quarreling, and the remainder of the day in gaming. How many hours does he spend in gaming ?   Ans. 3 hours.

52.   A fashionable young lady spends $\frac{1}{8}$ of her time in dressing, painting, and making her toilet, $\frac{1}{8}$ in reading novels and papers of senseless fiction, $\frac{1}{8}$ in making calls and gossiping, $\frac{1}{12}$ in street promenading, $\frac{1}{24}$ in criticising industrious young men and speculating upon the qualities and fortune of an anticipated husband, $\frac{1}{24}$ in making remarks derogatory to the character of those who labor, while her own mother is perhaps cooking or washing, $\frac{1}{12}$ in entertaining young men, and the remainder in eating and sleeping. How many hours does she devote to useful service, and how many to eating and sleeping ?   Ans. 0 hours to useful service; 8 hours to eating and sleeping.

53.   A man willed $\frac{1}{4}$ of his property to his wife, $\frac{3}{4}$ of the remainder to his daughter, and the remainder to his son; the difference between his wife's and daughter's share was $8000. How much did he give his son *   Ans. $4800.

OPERATION INDICATED.

$\frac{1}{4}$=wife's interest; $\frac{9}{16}$=daughter's interest; $\frac{3}{16}$= son's interest; $\frac{9}{16}-\frac{1}{4}=\frac{5}{16}$=$8000; then 8000\div\frac{5}{16}$ =$25600 the whole estate; $\frac{3}{16}$ of which is $4800, answer.

54.   R. W. Tyler owned a $\frac{3}{4}$ interest in a factory, and sold to C. Modinger $\frac{1}{2}$ of his interest for $15000. What interest does he still own, and how much is it worth at the rate received for the part sold ?   Ans. He still owns $\frac{3}{8}$, worth $15000.

55. J. Cassidy owned $\frac{7}{8}$ of the Steamer R. E.
Lee. He sold to G. Buesing $\frac{1}{8}$ interest in the
Steamer for $20000; and to J. C. Beals $\frac{1}{4}$ of his
remaining interest at the same rate. What did he
receive for the last sale, and what is his remaining
interest in the boat?   Ans. He received $30000;
$\frac{9}{16}$ remaining interest.

56. N. Puech and A. Palacio bought on joint
account, each $\frac{1}{2}$, the New Orleans Cotton Factory.
N. Puech sold $\frac{1}{2}$ of his interest to R. Krone, and
subsequently $\frac{1}{2}$ of his remaining interest to A.
Palacio, who subsequently sold $\frac{1}{2}$ of $\frac{3}{4}$ of his whole
interest to R. Lynd for $7500. What is the factory
worth at the same rate, and what is each owner's
interest?   Ans. $32000 value of Factory; Puech
owns $\frac{1}{4}$; Krone $\frac{1}{4}$; Palacio $\frac{25}{64}$; and Lynd $\frac{15}{64}$.

57. W. D. Maxwell gives $\frac{1}{8}$ of his annual income
to aid meritorious young men in obtaining an edu-
cation; $\frac{1}{2}$ of the remainder for the publication and
free distribution of books treating of the awful
injury to the human race by the use of tobacco, tea,
coffee, and wine; $\frac{1}{2}$ of the second remainder for
various benevolent purposes. The balance $5490
he retains for his own personal use; how much does
he give for each object named?

  Ans. $8235 for meritorious young men; $8235
      for the publication and distribution of
      books; and $2745 for various benevolent
      purposes.

58. C. M. Huber and A. J. Hohensee bought on
speculation $800 worth of merchandise, of which
Huber paid $500 and Hohensee $300; they sold to
W. A. Tomlinson $\frac{1}{8}$ of the whole for $400. How
much of the $400 must Huber and Hohensee receive
respectively, in order to constitute each $\frac{1}{2}$ owner in
the remainder of the goods?

      Ans. Huber $350 and Hohensee $50.

59. Multiply $\dfrac{7\frac{1}{3}}{5\frac{1}{2}}$ by $\dfrac{6\frac{1}{4}}{3\frac{1}{3}}$

OPERATION INDICATED.

3	22										
11	2		3	22		4	25	3	4	
4	25	or	11	2		10	3	8	15	
10	3		—	—		—	—	—	—	
—				$1\frac{1}{3}$			$1\frac{7}{8}$			$2\frac{1}{2}$ Ans.
	Ans. $2\frac{1}{2}$.									

60. Divide $\dfrac{7\frac{1}{3}}{5\frac{1}{2}}$ by $\dfrac{6\frac{1}{4}}{3\frac{1}{3}}$

OPERATION INDICATED.

3	22										
11	2		3	22		4	25	3	4	
25	4	or	11	2		10	3	15	8	
3	10		—	—		—	—	—	—	
—				$1\frac{1}{3}$			$1\frac{7}{8}$			$\frac{32}{45}$ Ans.
	Ans. $\frac{32}{45}$.									

61. Reduce $\dfrac{2\frac{1}{2}}{5}$ of $\dfrac{3\frac{1}{3}}{12\frac{1}{2}}$ of $8\frac{1}{3} \div 7\frac{1}{2}$ to a simple frac-

tion. Ans. $\frac{4}{27}$.

NOTE.—In simplifying fractions, and in all operations indicated by the $+$, $-$, $\times$, or $\div$ signs, it must be remembered that either of these signs affects only the number which immediately follows it, and that the operations indicated by the $\times$ and $-$ signs must always be performed before adding or subtracting. Whenever the parentheses are used, the operation indicated between them must be performed before connecting it to any other expression.

62. What is the result of $\frac{1}{2} + \frac{2}{3} \times \frac{3}{4} + \frac{4}{5} \div \frac{6}{5} + \frac{7}{8}$?
  Ans. $2\frac{13}{24}$.

63. Find the value of $2\frac{1}{3} \times \frac{15}{49} + 7\frac{1}{2} - 6\frac{2}{3}$?
  Ans. $1\frac{23}{42}$.

64. $\frac{2}{3} + \frac{7}{15} \div \frac{14}{25} - \frac{3}{5} \times \frac{10}{21}$ equals what number?
  Ans. $1\frac{3}{14}$.

65. $\frac{3}{4}-\frac{1}{9}\times\frac{18}{35}\div\frac{4}{7}$ = what number?   Ans. $\frac{13}{20}$.

66. What is the value of $\frac{4}{15}+\frac{2}{3}-\frac{3}{8}\times(\frac{14}{39}+\frac{7}{13})\div\frac{7}{16}$?
Ans. $\frac{32}{195}$.

67. Add $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{5}{6}$, and $7\frac{1}{4}-5\frac{2}{3}$.
Ans. $4\frac{71}{120}$.

68. What is the product of $\frac{3}{4}$ of $12\frac{1}{2}$ by $\frac{2}{3}$ of $6\frac{2}{3}-\frac{3}{4}$?
Ans. $34\frac{61}{96}$.

69. What is the product of $\frac{5}{6}$ of $(13\frac{1}{2}+1\frac{2}{3})$ by $\frac{6}{7}$
of $(1\frac{5}{16}-\frac{1}{2})$?   Ans. $4\frac{1}{16}$.

70. Divide the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and 3, by the difference between $3\frac{3}{4}$ and $\frac{3}{4}$ of 3.   Ans. $3\frac{5}{18}$.

71. Multiply [60 divided by $(\frac{8\frac{1}{3}}{7\frac{1}{7}}$ of $7\frac{1}{2}$ of 3)] by
($\frac{3}{4}$ of 7).   Ans. 12.

72. Divide $(\frac{8\frac{1}{3}}{6\frac{1}{4}}$ of $\frac{3}{16}}{1\frac{3}{13}})$ by $(\frac{9\frac{3}{4}}{4\frac{5}{7}}$ of $\frac{1}{3}}{2})$ Ans. $\frac{6}{7}$.

73. Simplify the fraction $\dfrac{\frac{3}{5}+\frac{2}{7}-\frac{2}{3}\times\frac{9}{10}+\frac{8}{15}\div\frac{6}{25}}{\frac{14}{9}-\frac{1}{6}\times\frac{24}{35}\div\frac{2}{7}-\frac{17}{20}}$
Ans. $8\frac{16}{77}$.

OPERATION INDICATED.

$\frac{3}{5}+\frac{2}{7}=\frac{31}{35}$; $\frac{2}{3}\times\frac{9}{10}=\frac{3}{5}$; $\frac{31}{35}-\frac{3}{5}=\frac{2}{7}$; $\frac{8}{15}\div\frac{6}{25}=\frac{20}{9}$; $\frac{2}{7}+$
$\frac{20}{9}=\frac{158}{63}$; $\frac{1}{6}\times\frac{24}{35}=\frac{4}{35}$; $\frac{4}{35}\div\frac{2}{7}=\frac{2}{5}$; $\frac{14}{9}-\frac{2}{5}=\frac{52}{45}$; $\frac{52}{45}-\frac{17}{20}$
$=\frac{11}{36}$. $\frac{158}{63}\div\frac{11}{36}=8\frac{16}{77}$, Ans.

74. L. Kaiser bought $\frac{2}{3}$ of $\frac{3}{4}$ of $28\frac{1}{2}$ barrels of apples, and sold to S. L. Crawford $\frac{3}{4}$ of 9 barrels for $20\frac{1}{4}$, which was \$1.50 more than the same cost. What was the cost of the whole, and how many barrels has he unsold?
Ans. $39\frac{7}{12}$ cost; $7\frac{1}{2}$ barrels unsold.

Greatest Common Divisor of Fractions.

276. The **Greatest Common Divisor** of two or more fractions is the greatest fraction that will divide each of them, without a remainder.

277. One **fraction** is **divisible** by another when the *numerator* of the *divisor* is a factor of the *numerator* of the *dividend*, or when the *denominator* of the *divisor* is a multiple of the *denominator* of the *dividend*.

Thus $\frac{10}{13}$ is divisible by $\frac{5}{26}$; for $\frac{10}{13} = \frac{20}{26}$; and $\frac{20}{26} \div \frac{5}{26} = 4$.

278. The **Greatest Common Divisor** of two or more fractions, is that fraction whose *numerator* is the Greatest Common Divisor of all the numerators, and whose *denominator* is the Least Common Multiple of all the denominators.

Thus the G. C. D. of $\frac{6}{45}$ and $\frac{22}{63}$ is $\frac{2}{315}$.

1. What is the greatest common divisor of $\frac{3}{4}$, $4\frac{1}{2}$, and $\frac{18}{25}$? Ans. $\frac{3}{100}$.

Operation to find the G. C. D. of the numerators :

$$3 \,|\, 3 \quad 9 \quad 18$$
$$\overline{1 \quad\ 3 \quad\ \ 6}$$
$$3 = \text{G. C. D.}$$

Operation to find the L. C. M. of the denominators :

$$2 \,|\, 4 \quad 2 \quad 25$$
$$\overline{2 \quad\ 1 \quad\ 25}$$
$$2 \times 2 \times 25 = 100 \text{ L. C. M.}$$

GENERAL DIRECTION FOR FINDING THE GREATEST COMMON DIVISOR OF FRACTIONS.

279. From the foregoing elucidations, we derive the following general direction for finding the Greatest Common Divisor of Fractions:

Find the G. C. D. of the numerators of all the fractions and write it over the L. C. M. of their denominators.

NOTE.—All the fractions must be in their simplest form before commencing the operation.

2. What is the greatest common divisor of $\frac{3}{3}$, $\frac{4}{5}$, $\frac{4}{7}$, and $\frac{10}{21}$? Ans. $\frac{2}{105}$.

3. What is the greatest common divisor of $8\frac{1}{4}$ and $\frac{17}{18}$? Ans. $\frac{1}{12}$.

4. What is the greatest common divisor of 5, $3\frac{1}{4}$, $6\frac{2}{3}$, and $\frac{8}{30}$? Ans. $\frac{1}{15}$.

5. What is the greatest common divisor of $\frac{1}{4}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{5}$, and $\frac{5}{6}$? Ans. $\frac{1}{60}$.

6. A grocer has three kinds of molasses, which he wishes to ship in the least number of full kegs. Of the 1st quality, he has $115\frac{1}{2}$ gallons; of the 2d quality, $128\frac{1}{2}$ gallons; and of the 3d quality, $134\frac{3}{4}$ gallons. How many gallons will there be in each keg, and how many kegs will be required?

Ans. $6\frac{3}{17}$ gals. in each keg; 59 kegs required.

SYNOPSIS FOR REVIEW.

Define the following words and phrases:

276. Greatest Common Divisor of Fractions. 277. The Divisibility of Fractions. 278. Greatest Common Divisor of two or more Fractions. 279. General Direction for the Operation.

Least Common Multiple of Fractions.

280. The **Least Common Multiple** of two or more fractions is the least number that each fraction will divide without a remainder.

NOTE 1.—The G. C. D. of several fractions is always a fraction; but the L. C. M. of several fractions may be a fraction or a whole number.

281. A fraction is a *multiple* of a given fraction when its numerator is a multiple of the given numerator and its denominator is a divisor of the given denominator.

Thus $\frac{4}{5}$ is a multiple of $\frac{2}{15}$; for 4 is a multiple of 2, and 5 is a divisor of 15. Hence $\frac{4}{5} \div \frac{2}{15} = 6$; or thus $\frac{4}{5} = \frac{12}{15}$; and $\frac{12}{15} \div \frac{2}{15} = 6$.

282. A fraction is a *common multiple* of two or more given fractions when its numerator is a common multiple of the numerators of the given fractions, and its denominator is a common divisor of the denominators of the given fractions.

283. A fraction is the *Least Common Multiple* of two or more given fractions when its numerator is the least common multiple of the given numerators, and its denominator is the greatest common divisor of the given denominators.

1. What is the L. C. M. of $\frac{2}{3}$, $\frac{5}{12}$, and $\frac{4}{15}$?

Ans. $6\frac{2}{3}$.

Operation to find the L. C. M. of the numerators:	Operation to find the G. C. D. of the denominators:
$2 \mid 2 \quad\; 5 \quad\; 4$	$\cdot\; 3 \mid 3 \quad\; 12 \quad\; 15$
$\overline{\quad\; 1 \quad\; 5 \quad\; 2}$	$\overline{\quad\; 1 \quad\; 4 \quad\; 5}$
L. C. M is, $2 \times 5 \times 2 = 20$.	G. C. D. is 3.

Hence the L. C. M. of the fractions is $\frac{20}{3} = 6\frac{2}{3}$.

GENERAL DIRECTION FOR FINDING THE LEAST COMMON MULTIPLE OF FRACTIONS.

284. From the foregoing elucidations, we derive the following general direction for finding the Least Common Multiple of Fractions:

Find the Least Common Multiple of the numerators and the Greatest Common Divisor of the denominators, and then divide the L. C. M. of the numerators by the G. C. D. of the denominators.

Note.—The fractions must be in their simplest form before commencing the operation.

2. What is the L. C. M. of $\frac{6}{7}$, $\frac{15}{23}$, and $\frac{10}{23}$?

Ans. $4\frac{2}{7}$.

3. What is the L. C. M. of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{6}{7}$?

Ans. 60.

4. What is the L. C. M. of $5\frac{1}{2}$, $7\frac{1}{3}$, $\frac{44}{63}$, and $\frac{11}{24}$?

Ans. 44.

5.   There is an island 15 miles in circuit, **around**
which A. can travel in $\frac{3}{4}$ of a day, B. in $\frac{7}{8}$ of a **day,**
and a horse car in $\frac{3}{10}$ of a day.   Supposing all to
start together from the same point to travel around
it in the same direction, how long must they travel
before coming together again at the place of depart-
ure, and how many miles will each have traveled ?
Ans. 10$\frac{1}{2}$ days; A., 210 miles; B., 180 miles;
Horse Car, 525 miles.

PARTIAL OPERATION.

3) 3   7   3 Numerators.   4   8   10 Denominators.

1   7   1   2 the Greatest Com-
mon Divisor.

$3 \times 7 = 21 \div 2 = 10\frac{1}{2}$ days before they all meet;
then the following proportional statements give the
miles traveled by each:

A.	B.	H. Car.
15	15	15
3 | 4	7 | 8	3 | 10
2 | 21	2 | 21	2 | 21
210 m. Ans.	180 m. Ans	525 m. Ans.

or thus,

$15 \div \frac{3}{4} \times 10\frac{1}{2} = 210$ miles traveled by A.
$15 \div \frac{7}{8} \times 10\frac{1}{2} = 180$ miles traveled by B.
$15 \div \frac{3}{10} \times 10\frac{1}{2} = 525$ miles traveled by Horse Car.

6.   What is the smallest sum of money for which
I could purchase a number of bushels of oats, at
$\$1\frac{5}{6}$ a bushel; a number of bushels of corn, at $\$\frac{5}{8}$ a
bushel; a number of bushels of rye, at $\$1\frac{1}{2}$ a bushel;
or a number of bushels of wheat, at $\$2\frac{1}{4}$ a bushel;
and how many bushels of each could I purchase
for that sum ?
Ans. $\$22\frac{1}{2}$; 72 bushels of oats; 36 bushels of
corn; 15 bushels of rye; 10 bushels of wheat.

PARTIAL OPERATION.

```
5) 5   5   3   9 Numerators.
   ─────────────         16  8  2  4 Denominators.
3) 1   1   3   9                 ─────────
   ─────────────               2 the Greatest Com-
     1   1   1   3                  mon Divisor.
```

$5 \times 3 \times 3 = 45$ the *least common multiple* of the numerators, which divided by 2, the *greatest common dirisor* of the denominators, gives $\$22\frac{1}{2}$, *the smallest sum ;* then $\$22\frac{1}{2}$ divided by $\$_{16}^{5}$, $\$\frac{5}{8}$, $\$\frac{3}{2}$, and $\$\frac{2}{4}$ gives respectively the number of bushels of each article represented by the different prices.

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SYNOPSIS FOR REVIEW.

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Define the following words and phrases :

280. Least Common Multiple of Fractions. 280. What is always the G. C. D. of Fractions? 280. What may the L. C. M. of Fractions be? 281. When is a Fraction a Multiple of a Given Fraction? 282. When is a Fraction a Common Multiple of two or more Fractions? 283. When is a Fraction the Least Common Multiple of two or more Fractions? 284. General Direction for the Operation.

ECIMAL FRACTIONS.

285. A Decimal Fraction is one or more of the equal parts of a unit, which is divided into *tenths, hundredths, thousandths,* etc., according to the decimal scale; hence the denominator of decimal fractions is always 10 or some power of 10. The word *decimal* is derived from the Latin word *decem,* which means ten.

286. The Decimal Point (**.**) is used to distinguish decimals from whole numbers. When there are mixed numbers, it also separates the whole numbers from the decimals.

The following are decimal fractions: $\frac{7}{10}$, $\frac{15}{100}$, $\frac{137}{1000}$, and $\frac{423}{10000}$. They are here written as common fractions, but generally the denominator of decimal fractions is omitted and the value is indicated by writing the *decimal point* before the numerator.

To write the above fractions according to the *decimal notation,* they would be written thus:

$\frac{7}{10}$ decimally expressed is .7.

$\frac{15}{100}$ decimally expressed is .15.

$\frac{137}{1000}$ decimally expressed is .137.

$\frac{423}{10000}$ decimally expressed is .0423.

287. Notation of Decimals. Whenever decimal fractions are expressed decimally, the numerator must have as many decimal places as there are naughts in the denominator. Thus $\frac{4}{10} = .4$;

$\frac{16}{100}$=.16; $\frac{1456}{10000}$=.1456. When the number of naughts in the denominator is greater than the number of figures in the numerator, naughts must be prefixed to the numerator until the number of *places* is equal to the naughts in the denominator. Thus $\frac{4}{100}$=.04; $\frac{7}{1000}$=.007; $\frac{125}{100000}$=.00125; etc.

When the number of naughts in the denominator is less than the figures in the numerator, the result or value of the fraction will embrace a whole number and a fraction.

288. A Pure or Simple Decimal consists of a decimal fraction, decimally expressed or written. Thus .5, .42, .875, and .1256 are pure decimals, and are read respectively 5 tenths; 42 hundredths; 875 thousandths· and 1256 ten thousandths.

289. A Mixed Decimal consists of a whole number and a decimal. Thus 24.5 and 41.25 are mixed decimals. They are read respectively, 24 and 5 tenths; 41 and 25 hundredths.

290. A Complex Decimal consists of a decimal with a common fraction annexed. Thus .15$\frac{3}{4}$ and .005$\frac{1}{6}$ are complex decimals. They are read respectively, 15$\frac{3}{4}$ hundredths; 5$\frac{1}{6}$ thousandths.

291. A Circulating Decimal is one in which a figure or set of figures constantly repeats itself. Thus $\frac{1}{3}$=.3333+, $\frac{1}{7}$=.142857+, $\frac{11}{15}$=.73333+. The figure or set of figures which is repeated is called a *Repetend*. If the repetend consists of only one figure, a dot is placed over it; if of a set of figures, a dot is placed over the first and last figures, as $\frac{1}{3}$=.$\dot{3}$, $\frac{2}{3}$=.$\dot{6}$, $\frac{1}{11}$=.$\dot{0}\dot{9}$, $\frac{1}{7}$=.$\dot{1}42857\dot{}$.

292.　A Pure Circulating Decimal is one which contains only the repetend; as $\frac{2}{3}=.\dot{6}$, $\frac{1}{7}=.\dot{1}4285\dot{7}$, $\frac{1}{9}=.\dot{1}$.

293.　A Mixed Circulating Decimal is one which contains other figures than the repetend; as $\frac{1}{6}=.1\dot{6}$, $\frac{583}{900}=.64\dot{7}$.

There are still other kinds of circulating decimals, , but as they are of very little practical importance, we will not consider them in this work.

294.　Decimal fractions, like whole numbers, decrease toward the right and increase toward the left in a *ten-fold* ratio, and hence the prefixing of naughts between the decimal figures and the decimal point, or the removal of the decimal point towards the left diminishes their value *ten-fold*, or divides the decimal by ten for each order or place removed.　Thus: $.5 = \frac{5}{10}$; $.05 = \frac{5}{100}$; $.005 = \frac{5}{1000}$; etc.

The removal of the decimal point to the right, increases the value *ten-fold* or multiplies the decimal by ten for each place removed.　Thus: $.005 = \frac{5}{1000}$; $.05 = \frac{5}{100}$; $.5 = \frac{5}{10}$, etc.

Annexing naughts to decimals does not change their value, because the significant figures are not thereby removed nearer to nor farther from the decimal point.　Thus: $.5 = \frac{5}{10}$; also $.50 = \frac{50}{100}$; $.500 = \frac{500}{1000}$, all of which are equal.

295.　Decimal *orders* are also called decimal *places*, each order being counted as one place.　Thus in .0043 there are *four decimal places*, although the 3 is of the *fifth decimal place* from *unity*, the base of the system.

The following table will illustrate more fully the relation of whole numbers and decimals, with their

increasing and decreasing orders to the left and right of the decimal point:

TABLE.

WHOLE NUMBERS.									DECIMALS.								
Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal Point.	Tenths.	Hundredths.	Thousandths.	Ten-Thousandths.	Hundred-Thousandths.	Millionths.	Ten-Millionths.	Hundred-Millionths.
9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9

Orders of ascending scale.　　Orders of descending scale.

This number is read 987 *million* 654 *thousand* 321, and 23 million 456 thousand 789 *hundred-millionths*.

In order to clearly understand decimals, we must bear in mind that *one* is the basis of all numbers, integral and fractional, abstract and denominate, and that all mathematical operations have this fundamental principle for their origin, and every number is but a multiple, either ascending or descending of *unity* or *one.*

The names of the decimal orders are derived from the names of the orders of whole numbers. Thus the names of the orders in the ascending scale, are, after units, *tens, hundreds,* etc., and the orders in decending scale, are, after units, *tenths, hundredths,* etc., the decimal orders being the reciprocal of the orders of whole numbers equally distant with themselves from the units.

296. Numeration of Decimals. In reading decimal fractions the entire decimal is regarded as reduced to units of the lowest order expressed, and the name of this order is given to the entire number of decimal units. Thus .25 is read *twenty-five hundredths.*

Before reading a decimal, we must determine 1st. How many units are expressed. To do this, we numerate and read the significant figures of the decimal as in whole numbers. 2nd. We must determine the name of the lowest order in the decimal. To do this, we numerate the number decimally. Thus to read .001073, we commence at the 3 and numerate to the 1 thousand, and thus find that 1073 units are expressed; then we commence at the decimal point and numerate decimally to the 3 and thus find that millionths is the lowest order· we then read 1073 millionths.

EXERCISES

297. Read the following numbers:

1. 16.008; reads thus. sixteen units and eight thousandths.
2. .94⅜; reads thus. ninety-four and three-eights hundredths.
3. 5067.4005; reads thus 5067 units and 4005 ten-thousandths.
4. Write and read 197.8: 4.68907: .00073: 48.769146.
5. Write and read 2.491: 10.0101089167; 582.400410905.
6. Write and read 5841.291⅖; 8000.0000000217; 9876541.1000001.

298. Writing Decimals. In writing decimals we write down the given number as if it were a

whole number; then, to facilitate the opration, we numerate from right to left, beginning the numeration with *tenths*, and continue until we come to the required place or order, always writing 0's to fill the places not occupied by significant figures. Thus, to write 25 ten thousandths, we first write the 25; then we begin at the right and numerate thus, tenths, hundreths, thousandths, ten-thousandths; by this we find that *four* places are required and as there are but two figures in the number we prefix two 0's and obtain the correct result .0025.

1. Write 104 hundred-thousandths.

Ans. .00104.

OPERATION.
.00104.

Explanation.—According to the above directions, we write the 104 and then commence on the right and numerate thus: tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths This numeration shows that *five* places are required, and as we have but *three* we therefore prefix two 0's.

299. EXERCISES.

1. Write 10101 hundred-billionths.

Ans. .00000010101.

Write decimally, numerate, and read the following:

2. 314 millionths.
3. 12 thousandths.
4. 107 billionths.
5. 1 trillionth.

6. 1205 ten-millionths.
7. 897 hundred-billionths.
8. 1 sextillionth.
9. 21001 ten-vigintillionths.

10. $\frac{5}{10}$

11. $\frac{25}{100}$

12. $\frac{742}{1000}$

13. $\frac{1042\frac{1}{2}}{100000}$

14. $\frac{50400}{1000000000}$

15. $\frac{748\frac{9}{34}}{1000000}$

16. $\frac{1}{1000000000}$

17. $\frac{87}{10000}$

18. $\frac{1\frac{3}{7}}{100}$

19. $\frac{990099}{1000000000000}$

30

PRINCIPLES.

300. From the foregoing work we recapitulate the following principles:

1. Decimals are governed by the same laws of notation as whole numbers; hence the value of any decimal figure depends upon the place it occupies.

2. Each removal of the decimal point one place to the *right* is equivalent to multiplying the decimal by 10.

3. Each removal of the decimal point one place to the *left* is equivalent to dividing the decimal by 10.

4. Annexing or rejecting naughts at the right of any decimal does not change its value.

REDUCTION OF DECIMALS.

301. *To Reduce Decimal Fractions to a Common Denominator.*

1. Reduce .7, .18, .2581, and .045 to a common denominator.

OPERATION.

.7000
.1800
.2581
.0450

Explanation—To reduce decimals to a common denominator, we have but to annex a sufficient number of 0's to give each decimal the same number of places.

302. *To Reduce a Decimal to a Common Fraction.*

1. Reduce .25 to a common fraction.

OPERATION.

$\frac{25}{100} = \frac{1}{4}$ Ans.

Explanation.—In all problems of this kind, we simply write the decimal as a common fraction and then reduce it to its lowest terms.

2. Reduce .125 to a common fraction.

OPERATION.

$$\tfrac{125}{1000} = \tfrac{1}{8}\ \text{Ans.}$$

3. Reduce .59$\tfrac{3}{8}$ to a common fraction.

FIRST OPERATION.

$$\frac{59\tfrac{3}{8}}{.100} = \frac{4\tfrac{7}{8}\tfrac{5}{8}}{8\tfrac{0}{8}\tfrac{0}{8}},\ \text{and}\ \tfrac{475}{8} \div \tfrac{800}{8} = \tfrac{475}{800} = \tfrac{19}{32}\ \text{Ans.}$$

Explanation —To reduce complex decimals to simple frac-
tions, we first write the decimal as a common fraction; then
we reduce both the numerator and the denominator to the frac-
tional unit of the denominator contained in the numerator
term of the fraction, and thus obtain a complex fraction,
which we reduce to a simple fraction.

SECOND OPERATION.

$$\frac{59\tfrac{3}{8}}{100} = \tfrac{475}{800} = \tfrac{19}{32}\ \text{Ans.}$$

Explanation.—Here, when re-
ducing the fraction to the frac-
tional unit of the denominator
contained in the numerator
term of the fraction, we shorten
the work by omitting the denominator (8) in both terms of
the complex fraction, and writing the result as a simple frac-
tion. By this process, we save the operation of division, the
result of which is the cancelling of the denominator in both
terms of the complex fraction.

GENERAL DIRECTION FOR REDUCING DECIMAL
FRACTIONS TO COMMON FRACTIONS.

303. From the foregoing elucidations, we derive
the following general direction for reducing decimal
fractions to common fractions:

*Write the decimal as a fraction; then reduce the
fraction to its lowest terms.*

Reduce the following decimals to common fractions:

4.	.8.	Ans.	$\frac{4}{5}$.	13.	.88.	Ans.	
5.	.05.	Ans.	$\frac{1}{20}$.	14.	.909.	Ans.	
6.	.25.	Ans.		15.	.00025.	Ans.	
7.	.125.	Ans.		16.	.48$\frac{1}{2}$.	Ans.	$\frac{97}{200}$.
8.	.675.	Ans.		17.	.055$\frac{3}{4}$.	Ans.	$\frac{223}{4000}$.
9.	.105.	Ans.		18.	.008$\frac{2}{7}$.	Ans.	
10.	.07.	Ans.	$\frac{7}{100}$.	19.	.00054$\frac{7}{18}$.	Ans.	
11.	.005.	Ans.	$\frac{1}{200}$.	20.	.999.	Ans.	
12.	.1045.	Ans.		21.	.4007$\frac{1}{11}$.	Ans.	

304. *To Reduce Common Fractions to Equivalent Decimals.*

1. Reduce $\frac{3}{8}$ to a decimal.

OPERATION.

8) 3.000
——————
.375 Ans.

Explanation.—To reduce common fractions to decimals, we annex naughts to the numerator and divide by the denominator, and then point off as many places for decimals as there were 0's annexed. When a remainder continues beyond four or six places, we discontinue dividing and write the sign + to the right of the last figure obtained, which indicates that the quotient is not complete. The annexing of 0's to the numerator is equivalent to multiplying it by 10 for each naught annexed, consequently the quotient obtained is as many times 10 too great as there were 0's annexed; and hence the reason for pointing off as many places in the quotient as there were 0's annexed to the numerator.

2. Reduce $\frac{5}{7}$ to an equivalent decimal.

OPERATION.

7) 5.000000
——————
.714285+ Ans.

3. Reduce $\frac{3}{725}$ to an equivalent decimal.

FIRST OPERATION.   SECOND OPERATION.

725) 3.000000(4137+   725) 3. (.004137+ Ans.
     2900   .004137+ Ans.   30 tenths.
     ————
     1000   300 hundredths.
      725   ————
     ————   3000 thousandths.
     2750   2900
     2175   ————
     ————   1000 ten-thousandths.
     5750   725
                      ————
                      2750 hundred-thousandths.
                      2175
                      ————
                      5750 millionths.
                      5075
                      ————
                       675 Remainder.

Explanation.—Here, in the first operation, we annex six 0's and obtain but 4 figures in the quotient. Therefore, in order to point off as many decimal places as we annexed 0's, we prefix two 0's and thus obtain the correct result. The reason for this will appear clear if we consider each step of the work as performed in the second operation. We are to divide or measure 3 by 725, and we first see that 3 is not equal to 725 any whole or unit number of times; we, therefore, write the decimal point in the quotient, annex a 0 to the 3 units and thus reduce it to 30 tenths, which we also see is not equal to 725 any tenth times, and hence we write 0 in the tenths place of the quotient; we then annex another 0 and thereby reduce the 30 tenths to 300 hundredths, which we see is not equal to 725 any hundredths times, and hence we write 0 in the hundredths place of the quotient; we then annex another 0 and thereby reduce the 300 hundredths to 3000 thousandths, which we see is equal to 725, 4 times, with a remainder. We have now obtained the first significant figure of the decimal, and we continue the division in the usual manner to the sixth decimal place and annex the + sign to indicate that there is still a remainder. The sign — is sometimes used to indicate that the last figure is too great. Thus $\frac{1}{6}$=.1666+; or, by abbreviating, $\frac{1}{6}$=.167 —.

4. **Reduce 6¾ to a decimal.**

FIRST OPERATION.   SECOND OPERATION.

$6\frac{3}{4}=\frac{27}{4}$ and $\frac{27}{4}=4)\ 27.00$ $6\frac{3}{4}=6$ and $\frac{3}{4}$; and

$\frac{3}{4}=4)\ 3.00$

6.75 Ans.   ————

.75+6=6.75 Ans.

———

GENERAL DIRECTION FOR REDUCING COMMON FRACTIONS TO EQUIVALENT DECIMALS.

305. From the foregoing elucidations, we derive the following general direction for reducing common fractions to equivalent decimals:

Annex naughts to the numerator and divide by the denominator. Then point off, from the right of the quotient, as many decimal figures as there are naughts annexed.

Reduce the following fractions to equivalent decimals not exceeding 6 places:

5. $\frac{23}{32}$. Ans. .71875 | 9. $\frac{5}{8}$. Ans. .625

6. $\frac{210}{625}$. Ans. .336 | 10. $\frac{1}{13}$. Ans. .076923+

7. $\frac{4}{125}$. Ans. .032 | 11. $.37\frac{1}{16}$. Ans. .370625

8. $\frac{3}{4}$ of $\frac{1}{7}$ Ans. .107142+ | 12. $47.18\frac{3}{4}$. Ans. 47.1875

Reduce $\frac{2}{3}$ to a complex decimal of 3 places.

OPERATION.

3) 2.000

————

.666$\frac{2}{3}$ Ans.

13. Reduce $\frac{3}{7}$ to a complex decimal of 4 places.
Ans. .4285$\frac{5}{7}$.

14. Reduce $\frac{2}{9}$ to a complex decimal of 6 places.
Ans. .222222$\frac{2}{9}$.

ADDITION OF DECIMALS.

306. Addition of Decimals is finding the sum of two or more decimals.

Since decimals increase from right to left, and decrease from left to right in a tenfold ratio as do simple whole numbers, they may be added, sub tracted, multiplied, and divided in the same manner.

1. Add .785, .93, 166.8, 72.5487, and 4.17.

OPERATION.

```
  .785
  .93
166.8
 72.5487
  4.17
─────────
245.2337 Ans.
```

Explanation. — In all problems of this kind, we write the numbers so that units of the same order stand in the same column, and the decimal point be in a vertical line; then we add as in simple whole numbers.

When the addition is completed we point off in the sum, from the right hand, as many places for decimals as equal the greatest number of decimal places in any of the numbers added.

If there are complex decimals they must be reduced to pure decimals, as far, at least as the decimal places extend in the other numbers.

GENERAL DIRECTIONS FOR ADDITION OF DEC-IMALS.

307. From the foregoing elucidations, we derive the following general directions for addition of decimals:

1. *Write the numbers so that units or figures of the same order stand in the same column.*

2. *Then add as in simple numbers and point off in the sum, from the right, as many decimal places as equal the greatest number of decimal places in any of the numbers added.*

Add the following numbers:

2. 3.25, 42.348, 748.4, and 29.32.
<div align="right">Ans. 823.318.</div>

3. .0049, 47.0426, 37.041, and 360.0039.
<div align="right">Ans. 444.0924.</div>

4. 1121.6116, 61.87, 46.67, 165.13, and 676.167895.
<div align="right">Ans. 2071.449495.</div>

5. .8, .09, 34.275, 562.0785, and 1.01.
<div align="right">Ans. 598.2535.</div>

6. 81.61356, 6716.31, 413.1678956, 35.14671, 3.1671, and 314.6.
<div align="right">Ans. 7564.0052656.</div>

7. 1.01¾, 240.06½, 999.9, 80.6051, and .17.
<div align="right">Ans. 1321.7576.</div>

8. What is the sum of the following numbers: twenty-five, and seven millionths; one hundred forty-five, and six hundred forty-three thousandths; one hundred seventy-five, and eighty-nine hundredths; seventeen, and three hundred forty-eight hundred-thousandths.   Ans. 363.536487.

9. A farmer sold at one time 3 tons and 75 hundredths of a ton of hay; at another time, 11 tons and 7 tenths of a ton; and at a third time, 16 tons and 125 thousandths of a ton. How much did he sell in all?   Ans. 31.575 tons.

SUBTRACTION OF DECIMALS.

308. Subtraction of Decimals is finding the difference between two decimals.

1. From 345.3046 subtract 92.1435847.

OPERATION.

345.3046
92.1435847
———————
253.1610153 Ans.

Explanation.— In all problems of this kind, we write the numbers so that units of the same order stand in the same column, and the decimal points be in a vertical line; then we subtract as in simple whole numbers, and point off in the difference, from the right hand, as many places for decimals as equal the greatest number of decimal places in either the minuend or subtrahend.

When the decimal places in the subtrahend exceed those in the minuend, naughts are understood to occupy the vacant places, and may be filled in if it is desired.

2. From 142.6⅔ subtract 51.1111⅑.

OPERATION.

142.6666⅔
51.1111⅑
———————
91.5555⅝ Ans.

Explanation.—In all problems of this kind, we reduce the complex decimals to pure decimals of equal places and then subtract as in subtraction of fractions.

GENERAL DIRECTIONS FOR SUBTRACTION OF DECIMALS.

309. From the foregoing elucidations, we derive the following general directions for subtraction of decimals.

1. *Write the numbers so that units of the same order stand in the same column.*

31

2.   *Then subtract as in simple numbers, and write the decimal point as in addition of decimals.*

NOTES.—1.   If there are complex decimals of unequal places in either or both of the given decimals, reduce them to pure decimals of equal places and then subtract as in subtraction of fractions.

2.   If there are not as many decimal places in the minuend as there are in the subtrahend, annex decimal naughts to it, until the decimal places are equal.

PROBLEMS.

(3.)	(4.)	(5.)
81.04089	121.25	532.8
14.587	109.05438	9.00451681
66 45389 Ans.	12.19562 Ans.	523.79548319 Ans.

6   From 461.072 take 427.125.   Ans. 33.947.

7.   From   17.5   take   4.19.   Ans.   13.31.

8.   From 4000.0004 take 4.3.   Ans. 3995.7004.

9.   From three million take three millionths.
Ans. 2999999.999997.

10.   From 11 take 1 and 9 trillionths.
Ans. 9.999999999991.

11.   From 24000 subtract 2.078.
Ans. 23997.922.

12.   From 886.333 subtract 98.5427.
Ans. 787.7903,

MULTIPLICATION OF DECIMALS.

310. Multiplication of Decimals is finding the product, when either or both of the factors contain decimals.

1. Multiply 26.58 by 4.3.

FIRST OPERATION.

$$26.58$$
$$4.3$$

$$7974$$
$$10632$$

$$114.294 \text{ Ans.}$$

Explanation.—In all problems of this kind, we multiply as in whole numbers, and point off on the right of the product as many places for decimals as there are decimal places in both the multiplicand and multiplier. The reason for thus pointing off the 3 decimal places in this problem is obvious from the fact that in the multiplicand we have 2 decimal places or hundredths, which we used as whole numbers and thereby produced a product 100 times too great; and in the multiplier we have 1 decimal place or tenths which we also used as a whole number and thereby produced a product 10 times too great; and both together give a product 1000 times too great; hence to obtain the correct product we divide by 1000, or point off 3 decimal places.

SECOND OPERATION.

$$\frac{100}{10}\Big|\frac{2658}{43}$$

$$\frac{114294}{1000} \text{ Ans.}$$

or decimally written
114.294 Ans.

Explanation.—In this operation, we reduce the factors to common fractions, and then multiplying them together, we obtain a product of $\frac{114294}{1000}$, which written decimally is 114.294. This process shows in another way why we point off on the right of the product as many places for decimals as there are decimal places in both factors.

2. Multiply 4.024 by .0056.

OPERATION.
4.024
.0056
———
24144
20120
———
.0225344 Ans.

Explanation.—In all problems of this kind, where the number of figures in the product is not equal to the number of decimal places in the two factors, we must prefix a sufficient number of 0's to supply the deficiency. In this example, we prefix one 0. The reason of this will appear evident by working the example as a common fraction as shown in the second operation of the first problem.

GENERAL DIRECTIONS FOR MULTIPLICATION OF DECIMALS.

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311. From the foregoing elucidations, we derive the following general directions for the multiplication of decimals:

1. *Multiply as in whole numbers and from the right of the product point off as many figures for decimals as there are decimal places in the multiplicand and multiplier.*

2. *If the product does not contain as many decimal places as both factors, supply the deficiency by prefixing naughts.*

PROBLEMS.

3. Multiply 27 by .9.   Ans. 24.3.
4. Multiply .38 by 8.   Ans. 3.04.
5. Multiply .75 by .42.   Ans. .3150.
6. Multiply .006 by .0103,   Ans. .0000618.

7. Multiply 340.012 by 61.23.

Ans. 20818.93476.

8. Multiply .1234 by 1234.   Ans. 152.2756.

9. Multiply 1500 by .00014.   Ans. .21.

10. What is the product of one thousand twenty-five, multiplied by three hundred twenty-seven ten-thousandths?   Ans. 33.5175.

11. What is the product of seventy-eight million two hundred five thousand two, multiplied by fifty-three hundredths?   Ans. 41448651.06.

12. Multiply one hundred fifty-three thousandths by one hundred twenty-nine millionths.

Ans. .000019737

13. Multiply 1 thousand by 1 thousandth.

Ans. 1

14. Multiply 2 million by 2 billionths.

Ans. .004.

15. What will 37.23 tons of hay cost at $20.75 per ton?   Ans. $772.52+.

16. What will 428.431 bushels cost at $1.125 per bushel?   Ans. $481.98+.

312. *To Multiply a Decimal or Mixed Number by* 10, 100, 1000, *etc.*

1. Multiply 428.375 by 100.

OPERATION.

42837.5 Ans.

Explanation.—In all problems where the multiplier is 10, 100, etc., we simply remove the decimal point as many places to the right as there are naughts in the multiplier, annexing naughts if required, as shown in articles 294 and 300.

2. Multiply 271.32 by 1000.   Ans. 271320.

3. Multiply .756 by 100.   Ans.   75.6.

4. Multiply .025 by 10.   Ans.   .25.

5. Multiply 61.052 by 10000.   Ans. 610520.

DIVISION OF DECIMALS.

313. Division of Decimals is the process of finding the quotient when the divisor or dividend, or both, contain decimals.

1. Divide 17.094 by 8.14.

FIRST OPERATION.

8.14) 17.094 (2.1 Ans.
16 28
————
814
814
——

Explanation.—In all problems of this kind, we divide as in whole numbers, and then *point off as many places for decimals from the right of the quotient as the decimal places in the dividend exceed those in the divisor, observing to supply any deficiency by prefixing naughts.* In this problem the excess is one, and we therefore point off one decimal place in the quotient. The reason for thus pointing off is obvious from the fact that in the dividend we had 3 decimals or THOUSANDTHS, and in the divisor we had 2 decimals or *hundredths*, and *thousandths* divided by *hundredths* give *tenths* as a quotient.

The reason will also appear plain if we observe that the dividend is the product of the divisor and quotient multiplied together, and hence we point off enough decimal places in the quotient to make the number in the two factors equal to the number in the product or dividend, according to the principles shown in the first problem of multiplication of decimals.

SECOND OPERATION.

1000 | 17094
814 | 100
—— | ———
| 2$\frac{1}{10}$ Ans.
Decimally written 2.1 Ans.

Explanation.—In this operation, we reduce the decimals to common fractions and then proceed as in the division of mixed numbers. The reduction of the dividend and divisor to common fractions and then the mixed numbers to improper fractions, is performed thus: the dividend 17.094 = 17$\frac{94}{1000}$ = $\frac{17094}{1000}$; the divisor 8.14 = 8$\frac{14}{100}$ = $\frac{814}{100}$. This method also shows the reason for pointing off and may be used for all problems in decimal fractions.

PRINCIPLES OF DIVISION OF DECIMALS.

314.   From the foregoing elucidations, we derive the following principles:

1°. The dividend must contain at least as many decimal places as the divisor; and when both contain, the same the quotient is a whole number.

2°. The dividend is the product of the divisor and quotient, and hence contains as many decimal places as both the divisor and quotient.

3°. The quotient must contain as many decimal places as the number of decimal places in the dividend exceeds the number in the divisor.

2.   Divide 7898.56 by 2.4683.

OPERATION.

2.4683) 7898.5600 (3200. Ans.
     74049

    49366
    49366

      00

Explanation.—Here we have an excess of decimals in the divisor, and in all cases of this kind, we first make them equal by annexing naughts to the dividend, and the quotient will be a whole number. The reason for annexing the naughts will appear more obvious by solving the problem in the form of a common fraction.

3.   Divide 7.0761 by 687.

OPERATION.

687) 7.0761 (103
   687   .0103 Ans.

   2061
   2061

Explanation. – In this problem, there are 4 decimal places in the dividend and none in the divisor; hence according to the foregoing instruction we must point off 4 decimal places in the quotient, and as there are but 3 figures in the quotient, we prefix 1 naught. In all problems of this kind, 0's are prefixed to supply any deficiency of figures that may occur.

4. Divide 47.789 by 39.27.

OPERATION.

39.27) 47.789 (1.2168+ Ans.
39 27
‑‑‑‑‑‑
8519
7854
‑‑‑‑‑‑
6650
3927
‑‑‑‑‑‑
27230
23562
‑‑‑‑‑‑
36680
31416
‑‑‑‑‑‑

Explanation.—In this problem we have a remainder, after dividing the dividend, of 665; to this and the 2 successive remainders we annex 0's and continue the division until we have produced 4 decimal places. The annexing of 0's reduces the successive remainders to the next lower order of tenths and hence all quotient figures produced by annexing 0's are decimals. We therefore point off from the right of the quotient as many places for decimals as the number of decimals in the dividend exceed those of the divisor, plus the number of 0's annexed. This is done in all division problems where 0's are annexed, and a sufficient number of 0's should be annexed to produce 4 or 6 decimal places. When there is a remainder after the last division, the plus (+) sign should be annexed to the answer to indicate that the quotient is incomplete.

GENERAL DIRECTIONS FOR DIVISION OF DECIMALS.

315. From the foregoing elucidations, we derive the following general directions for dividing decimals:

1. *Divide as in whole numbers and point off as many decimal places in the quotient as those in the dividend exceed those in the divisor.*

2. *When there is a remainder, annex naughts to the dividend and carry the work as far as may be desired.*

5. Divide .112233 by 12.
OPERATION.

12).112233

9352+=.009352+ Ans.

6. Divide 1.12233 by 12.
OPERATION.

12)1.12233

9352+=.09352+ Ans.

7. Divide 11.2233 by 12.
OPERATION.

12)11.2233

.9352+ Ans.

8 Divide 112.233 by 12.
OPERATION.

12)112.233.

9.3527+ Ans.

9. Divide .0004869 by 396.
OPERATION.

396).0004869(12+ Ans.
396 =.0000012+.

909
792

117

10. Divide .0004869 by 3.96.
OPERATION.

3.96).0004869(12+ Ans.
396 =.00012+.

909
792

117

11. Divide .0004869 by .0396.

FIRST OPERATION.

.0396).0004869(122+ Ans.
396 =.0122+.

909
792

1170
792

378

SECOND OPERATION.

396)4869(122+=.0122+ Ans.
396

909
792

1170
792

378

12.	Divide 67.8632 by 32.8.	Ans. 2.069.
13.	Divide 983 by 6.6.	Ans. 148.939+.
14.	Divide 13192.2 by 10.47.	Ans. 1260.
15.	Divide 67.56785 by .035.	Ans. 1930.51.
16.	Divide .00125 by .5.	Ans. .0025.
17.	Divide 7.482 by .0006.	Ans. 12470.
18.	Divide 1 by 999.	Ans. .001001+.

22

19. Divide 84375 by 3.75.   Ans. 22500.

20. Divide 1081 by 39.56.   Ans. 27.3255+.

21. Divide 35.7 by 485.   Ans. .0736+.

22. If rice cost $.0775 per pound, how many pounds can be bought for $40.64875?

                    Ans. 524.5 pounds.

23. Sold 14.75 acres of land for $191.75. What was the price per acre?   Ans. $13.

24. Divide four thousand three hundred twenty-two and four thousand five hundred seventy-three ten-thousandths, by eight thousand and nine thousandths.   Ans. .5403+.

316. *To Divide Decimal Fractions by* 10, 100, 1000, *etc., etc.*

1. Divide 48.76 by 10.

OPERATION.

4.876 Ans.

Explanation.—In all problems of this kind, we simply remove the decimal point as many places to the left as there are 0's in the divisor. The reason for this was fully shown in articles 294 and 300. When there are not a sufficient number of figures in the dividend to allow this to be done, naughts must be prefixed to supply the deficiency.

2. Divide 875.25 by 100.   Ans. 8.7525.

3. Divide .5231 by 1000.   Ans. .0005231.

4. Divide 72 by 10000.   Ans. .0072.

5. Divide 9.85 by 100.   Ans. .0985.

6. Divide .025 by 200.   Ans. .000125.

7. Divide 412.99 by 10.   Ans. 41.299.

SYNOPSIS FOR REVIEW.

Define the following words and phrases:

285. A Decimal Fraction.   286. The Decimal Point.   286. How are Decimal Fractions Generally Indicated?   287. Notation of Decimals.   288. A Pure, or Simple Decimal.   289. A Mixed Decimal.   290. A Complex Decimal.   291. A Circulating Decimal.   292. A Pure Circulating Decimal.   293. A Mixed Circulating Decimal.   294. What is the effect of removing the Decimal Point from *left* to *right*, or from *right* to *left?*   295. Decimal Orders.   296. Numeration of Decimals.   298. Writing Decimals.   300. The Four Principles of Decimals.   301. Reduction of Decimals to a Common Denominator.   303. To a Common Fraction.   305. Common Fractions to Equivalent Decimals.   306. Addition of Decimals.   307. General Directions for the Operation.   308. Subtraction of Decimals.   309. General Directions for the Operation.   310. Multiplication of Decimals.   311. General Directions for the Operation.   312. To Multiply Decimals by 10, 100, 1000, etc.   313. Division of Decimals.   314. The Three Principles of Division of Decimals.   315. General Directions for the Operation.   316. To Divide Decimals by 10, 100, 1000, etc.

Compound Denominate Numbers.

DEFINITIONS.

317. A Denominate Number is a concrete number which expresses a particular kind of unit or quantity, either simple or compound.

318. A Simple Denominate Number is a number which expresses a unit or units or quantities of but *one* kind or denomination: as 5 dollars, 15 boxes, 8 pounds, 4 days, etc.

319. A Compound Denominate Number is a number which expresses units or quantities of *two* or *more* denominations, under *one kind* of measure: as 8 dollars, 20 cents; 12 pounds, 10 ounces; 7 days, 18 hours, 21 minutes, etc.

In *whole numbers* and in *decimals*, the law of increase and decrease, between units of lower and of higher orders, is by the uniform scale of 10; but in *compound numbers* the scale varies according to the kind of measure employed.

MEASURES.

320. A Measure is a *standard unit* established by law or custom, by which *quantity*, such as extent, dimension, capacity, amount, or value, is measured or estimated.

There are seven kinds of measure:

1st. Length. 2d. Surface, or Area. 3d. Solidity, or Capacity. 4th. Weight, or Force of Gravity. 5th. Time. 6th. Angles. 7th. Money, or Value.

WEIGHT.

321. Weight is that property of bodies by virtue of which they tend toward the centre of the earth; and the resistance required to overcome this centralizing pressure, or gravitating tendency of bodies, is what is named weight. Weight varies according to the quantity of matter a body contains, and its distance from the centre of the earth.

VALUE.

322. Value is the ratio or unit of measure of wealth existing between different commodities with reference to an exchange. It is the sole condition of wealth and the universal name given to the *inherent quality or power of one thing to command another in exchange.*

Briefly expressed, value is the worth of one thing as compared with some other thing.

MONEY.

323. Money is stamped metal called coin, or printed bills or notes called paper money. It is issued by the general government of States or Nations, and supplied to the people to facilitate trade and commerce, and it is the standard of value and the almost universal medium of exchange among all civilized peoples.

324. Currency is a term applied to the money of a nation, whether it be coin or paper money.

MEASURE TABLES.

325. A Measure Table is a regularly arranged statement showing how many times a higher denomination of a system of measurement equals

the next lower denomination of the same system. Or, in other words, it shows how many units of each lower denomination are required to equal one unit of the next higher denomination

326. 1. TABLE OF UNITED STATES MONEY.

				E.	$	d.	ct.	m.
10 Mills (m).	= 1 Cent,	ʄ.		1 =	10 =	100 =	1000 =	10000
10 Cents	= 1 Dime,	d.			1 =	10 =	100 =	1000
10 Dimes	= 1 Dollar,	$				1 =	10 =	100
10 Dollars	= 1 Eagle,	E.					1 =	100
20 Dollars	= Double Eagle.						1 =	10

NOTE—The mill is not coined. It is used only in computations.

327. United States Money is the legal monetary measure of value in the United States of America. The *unit* of the measure is the *Gold* and the *Silver Dollar.*

328. The U. S. monetary unit, the Dollar was established by the Continental Congress, August 8, 1786, with the proviso that it should be decimally divided.

329. The coin of the United States consists of gold, silver, nickel, and bronze. The following table shows the name, value, composition, and weight of each coin, as now issued (1886) by the Mints:

TABLE.

330.

COIN.	VALUE.	COMPOSITION.	WEIGHT.
GOLD.			
Dollar............	100 cents....	90 parts gold, 10 parts alloy...	25.8 grains Troy.
Quarter Eagle......	2½ dollars...	90 " " " "	64.5 " "
Three Dollars.....	3 dollars....	90 " " " "	77.4 " "
Half Eagle.........	5 dollars....	90 " " " "	129 " "
Eagle.............	10 dollars...	90 " " " "	258 " "
Double Eagle......	20 dollars...	90 " " " "	516 " "
SILVER.			
Dime.............	10 cents.....	90 parts silver, 10 parts alloy...	38.58 grains Troy.
Quarter dollar.....	25 cents.....	90 " " " "	96.45 " "
Half dollar........	50 cents.....	90 " " " "	192.9 " "
Dollar............	100 cents....	90 " " " "	412.5 " "
NICKEL.			
3-cent piece	3 cents......	75 parts copper, 25 parts nickel	30 grains Troy.
5-cent piece.......	5 cents......	75 " 25 " "	77.16 " "
BRONZE.			
One cent..........	1 cent......	95 parts copper, 5 parts tin and [zinc.]	48 grains Troy.

331. In the coinage of the United States Money the following allowance is made by law for a deviation in weight:
½ grain for the Double Eagles and Eagles;
¼ grain for all other gold pieces;
1½ grains in all silver pieces;
3 grains in the nickel 5-cent piece, and 2 grains in the nickel 3-cent piece, and bronze 1-cent piece.

332. The old silver half dime and 3-cent pieces, the bronze 2-cent pieces, and the nickel 1-cent pieces are not now coined.

333. The Trade Dollar was coined for Asiatic Commerce, and not for Currency. The weight is 420 grains, $\frac{9}{10}$ pure.

334. The ALLOY of a coin is some harder metal mixed with the gold or silver to harden it moderately and thus lessen the wear or *abrasion*. Gold and silver, in a pure state, being very soft, would wear away, were they not alloyed.

The alloy for American gold coin is composed of about ½ silver and ¾ copper. The quantity of silver may be increased, not exceeding ½. The difference in color of our gold coins is because of the different quantity of silver in the alloy.

The alloy for silver coin is pure copper. Coin thus alloyed is called *standard*.

335. WEIGHT OF COIN.

$10000 Gold=258000 gr.=44 lbs. 9 oz. 10 pwt. 0 gr. Troy.
$1000 Silver dollars=412.00 gr.=71 lbs. 7 oz. 7 pwt. 12 gr.
$1000000 Gold weigh 53750 ounces Troy or 3685.71 Avoirdupois pounds.
$1000000 Silver Trade dollars weigh 875000 ounces Troy or 60000 pounds Avoirdupois.
$1000000 Silver, half and quarter dollars, 20-cent pieces and dimes, weigh 803750 ounces Troy or 55114.28 Avoirdupois pounds.

336. VALUE OF GOLD AND SILVER.

1 ounce Troy of pure gold is worth..............$20.67+
1 pennyweight Troy of pure gold is worth....... 1.03+
1 ounce Troy of pure silver is worth............ 1.29+
1 pennyweight Troy of pure silver is worth...... .0645+

CANADA MONEY.

337. Canada Money is the legal currency of the *Dominion* of Canada. It consists of gold, silver, and bronze coin and of paper money.

338. The silver coins are the 50¢, 25¢, 10¢, and 5¢ pieces. The bronze coin is the 1¢ piece. The gold coins in use are the *Sovereign* and *Half Sovereign.*

ENGLISH MONEY.

339. English, or Sterling Money, is the legal currency of Great Britain.

340. The Monetary Unit of Great Britain is the *Pound Sterling* which is a gold coin weighing 123.274 grains, $\frac{11}{12}$ pure. It is equivalent to $4.8665 U. S. money.

NOTE.—For exchange purposes between the United States and England, the Pound Sterling is valued by Bankers at $4.86¼ and the rate of Exchange is quoted in dollars and cents, $4.86½, more or less, according as premium is charged or discount is allowed.

See Soulé's Philosophic Work on Practical Mathematics for a full elucidation of English Exchange.

TABLE OF ENGLISH MONEY.

			£	s.	d.	far.
4 Farthings (far.)=	1 Penny........d.					
12 Pence	= 1 Shilling .....s.		1=	20=	240=	960
20 Shillings	$= \begin{cases} 1 \text{ Sovereign ..sov.} \\ \text{or} \\ 1 \text{ Pound.......£.} \end{cases}$			1=	12=	48
					1=	4
21 Shillings	= 1 Guinea.					

The Guinea is not coined; the term is only used in trade.

341. The money of Great Britain consists of gold, silver, copper, and Bank of England notes, or bills.

FRENCH MONEY.

342. French Money is the legal currency of France. It is based on the decimal system and the **Unit** is the Silver Franc, which equals 19.3 cents, U. S. money.

NOTE.—In Exchange transactions between the United States and France, the rate of exchange is the variable number of francs and centimes allowed for 1 dollar. The basis for the rate is 5.20 francs for $1. This rate is the *par of Exchange,* and is quoted more or less as premium is declared or discount is allowed.

3 3

TABLE OF FRENCH MONEY.

			fr.	dc.	ct.	m.
10 Millimes (m.)	=1 Centime......ct.		1	=10	=100	=1000
10 Centimes	=1 Decime ......dc.			1	= 10	= 100
10 Decimes	=1 Franc ....fr.				1	= 10

The money of France consists of gold, silver, bronze, and National Bank notes.

The Franc is used in Switzerland and Belgium, and under different names, in Spain, Italy, Greece, and Venezuela.

GERMAN MONEY.

343. German Money is the legal currency of the German Empire.

In 1871 the German Empire established a new and uniform system of money of which the "*Mark*" (Reichsmark), is the **Unit**. The *Mark* is equal to 23.8 cents United States money.

344. The coin of the Empire consists of gold, silver, and nickel.

TABLE OF GERMAN MONEY.

100 pfennige, marked Pf., make 1 mark, marked RM.

In exchange transactions with the German Empire, for convenience, bankers base the rate of exchange upon the equivalent value of 4 marks expressed in dollars and cents.

The exchange par of 4 marks is 95¼ cents. The rate of exchange is 95¼, more or less, according as premium is charged or discount is allowed.

For a full discussion and elucidation of Exchange computations for many foreign countries, see Soulé's Philosophic Work on Practical Mathematics.

MEASURE OF TIME.

345. 1.—**Time** is a measured portion of duration.

346. 2.—The **Unit** of measure is the mean solar day.

347. 3.—A **Year** is the time of the revolution of the earth around the sun.

348. 4.—A **Day** is the time of the revolution of the earth on its axis.

349. 5.—The **Solar Day** is the interval of time between two successive passages of the sun across the same meridian of any place, and they are of unequal length on account of the unequal orbital motion of the earth and the obliquity of the ecliptic.

350. 6.—The **Mean Solar Day** is the mean, or average length of all the solar days in the year. Its duration is twenty-four hours.

351. 7.—The **Civil, or Legal Day** used for ordinary purposes, and which corresponds with the Mean Solar Day, commences at midnight and closes at the next midnight.

352. 8.—The **Astronomical Day** commences at noon, and closes at the next noon.

353. 9.—The **Solar Year** is 365 days, 5 hours, 48 minutes, 49.7 seconds.

354. 10.—The **Common, or Civil Year** consists of 365 days for 3 successive years, every fourth year containing 366 days, one day being added for the excess of the Solar Year over 365 days. This intercalary day is added to the month of February, which then has 29 days, and the year is called Leap Year.

355. 11.—To determine what years are Leap Years, the following regulation has been adopted:
Every year that is divisible by 4 is a leap year,

unless it ends with two naughts, in which case it must be divisible by 400 to be a leap year.

Thus, 1884, 1776, 1600, and 2000 are leap years; but 1885, 1794, 1800, and 2100 are not.

For a condensed history of time measure and the units of measure in use in the early ages of civilization, see Soulé's Philosophic Work on Practical Mathematics.

TABLE OF TIME MEASURE.

60 Seconds (sec.)	=	1 Minute .... min.
60 Minutes	=	1 Hour .... hr.
24 Hours	=	1 Day................. d.
7 Days	=	1 Week .... wk.
365 Days	=	1 Common Year...... yr.
366 Days	=	1 Leap Year.......... yr.
12 Calendar Months =		1 Civil Year.......... yr.
100 Years	=	1 Century............. c.

yr.	mos.	wk.	ds.		hrs.		min.		sec.
1 =	12		$\begin{cases} 365 \\ 366 \end{cases}$	=	8760	=	525600	=	31536000
					8784	=	527040	=	31622400
		1 =	7	=	168	=	10080	=	604800
			1 =		24	=	1440	=	86400
					1 =		60	=	3600
							1 =		60

The names and orders of the months, and the number of days contained in each, are now as follows:

Names.	No.	No. ds.	Names.	No.	No. ds.
January,	1st,	31	July,	7th,	31
February,	2d,	28	August,	8th,	31
March,	3d,	31	September	9th,	30
April,	4th,	30	October,	10th,	31
May,	5th,	31	November,	11th,	30
June,	6th,	30	December,	12th,	31

The number of days in each, may be readily remembered by committing to memory the follow· ing lines:

> "Thirty days hath September,
> April, June, and November;
> And all the rest have thirty-one,
> Save February, which alone
> Hath twenty-eight; and this, in fine,
> One year in four hath twenty-nine."

MEASURES OF EXTENSION.

356. Extension is that property of matter by which it occupies space. It may have one or more of the three dimensions—length, breadth, and thickness.

357. The American and English **Unit** of measure of extension, whether a line, surface, or solid, is the **yard.**

358. A **Yard** was formerly, in England, the length of the King's arm, from the sternum bone to the end of the longest finger. A yard is now $\frac{360000}{391393}$ of the length of a "pendulum vibrating seconds of mean time in the latitude of London in a vacuum at the level of the sea."

359. A **Line** has only one dimension—length.

360. A **Surface** has two dimensions—length and breadth.

361. A **Solid,** or volume has three dimensions —length, breadth, and thickness.

LINE, OR LINEAR MEASURE.

362. Line, or Linear Measure is used to meas ure distances, or length, in any direction.

One inch.	Two inches.

Three inches

TABLE.

12	Inches (in.)	=	1 Foot.............ft.
3	Feet	=	1 Yard.............yd.
5½	Yards, or 16½ feet	=	1 Rod, or Pole ......rd., or P.
40	Rods	=	1 Furlong ....fur.
8	Furlongs (320 rds.)	=	1 Mile (Statute Mile).mi.
3	Miles	=	1 League.............L.

L.		m.		fur.		rd.		yd.		ft.		in.
1	=	3	=	24	=	960	=	5280	=	15840	=	190080
		1	=	8	=	320	=	1760	=	5280	=	63360
				1	=	40	=	220	=	660	=	7920
						1	=	5½	=	16½	=	198
								1	=	3	=	36
										1	=	12

Chain Measure, or Surveyors' and Engineers' Measure.

363.   Chain Measure is used by surveyors and topographical engineers, in measuring land, laying out roads, etc.

TABLE.

7.92	Inches	=	1 Link.............l.
25	Links	=	1 Rod, or Pole ......rd., or P.
4	Rods, or ⎰	=	1 Chain...........ch.
66	Feet ⎱		
80	Chains	=	1 Mile.............mi.

Engineers commonly use another chain, or tape line which consists of 100 links, each 1 foot long.

mi.		ch.		rd.		l.		in.
1	=	80	=	320	=	8000	=	63360
		1	=	4	=	100	=	792
				1	=	25	=	198
						1	=	7.92

MARINERS' MEASURE

364.   Mariners' Measure is used to measure distances at sea and also to measure the depths of seas.

TABLE.

6 Feet	=	1 Fathom.
120 Fathoms	=	1 Cable-length.
880 Fathoms, or 7⅓ Cable-lengths	=	1 mile.

365.   A Minute of the earth's circumference is called a **Geographical,** or *Nautical* mile, or *Knot*, which is $\frac{1}{60}$ of $\frac{1}{360}$ = $\frac{1}{21600}$ of the circumference of the earth. The circumference

of the earth at the Equator is 24899 miles, which, divided by 21600, gives 1.15273+ statute miles.

The length of a degree at the Equator is 1.15273×60 equals 69.1638 statute miles.

SHOEMAKERS' MEASURE.

366. **Shoemakers' Measure** is used by shoemakers to measure the human feet and in the manufacture of boots and shoes.

367. The Unit of measure is $\frac{1}{3}$ of an inch which is the same as the former unit of 1 barley-corn when 3 barley-corns made 1 inch.

No. 1 small size is $4\frac{1}{4}$ inches, and every succeeding No. increases $\frac{1}{3}$ of an inch to 13.

No. 1 large size is $8\frac{1}{4}$ inches, and every succeeding No. increases $\frac{1}{3}$ of an inch to 15.

MISCELLANEOUS UNITS OF LINEAR MEASURE.

TABLE.

$\frac{1}{12}$ of an Inch	= A Line (American).
$\frac{1}{10}$ of an Inch	= A Line (French).
4 Inches	= A Hand.
3 Inches	= A Palm.
9 Inches	= A Span.
3 Feet	= A Pace.
$2\frac{1}{3}$ Feet (28 in.)	= A Military Pace.
18 Inches	= A Cubit.

CLOTH MEASURE.

368. **Cloth Measure** is used to measure all kinds of goods sold by the yard.

TABLE.

			yd.	qr.	na.	in.
$2\frac{1}{4}$ Inches (in.)	= 1 Nail,...... na.		1 = 4 = 16 = 36
4 Nails (9 inches)	= 1 Quarter,... qr.		1 = 4 = 9
4 Quarters	= 1 Yard, ...yd.		1 = $2\frac{1}{4}$

This table formerly contained:

The Flemish Ell, or yard, which equaled 3 quarters or 27 inches;

The English Ell, or yard, which equaled 5 quarters or 45 inches;

The French Ell, or yard, which equaled 6 quarters or 54 inches.

All of the above units of measure are now out of use except the yard, which is divided into *halves, quarters, eighths, sixteenths,* etc., in place of feet and inches. At the Custom-house the yard is decimally divided.

SQUARE, OR SURFACE MEASURE
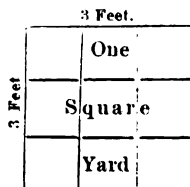
3 ft. × 3, ft. = 9 sq. ft. = 1 sq. yd.

369. Square, or Surface Measure is used in computing surfaces or areas.

370. A **Surface** has length and breadth, but not thickness.

371. The **Area** of a surface is the quantity of surface it contains, and is expressed by the product of the length by the breadth.

372. A **Square** is a plane figure bounded by four equal sides, and having four-right angles.

TABLE.

144 Square Inches (sq. in.) =	1 Square Foot.......sq. ft.
9 Square Feet =	1 Square Yard,.......sq. yd.
30¼ Square Yards 〈 =	1 Square Rod, ...sq. rd.
	or Perch ....P.
160 Square Rods =	1 Acre ....A.
640 Acres =	1 Sq. Mile, or Section..sq. mi., or sec.

sq. mi.	A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
1 =	640 =	102400 =	3097600 =	27878400 =	4014489600
	1 =	160 =	4840 =	43560 =	6272640
		1 =	30¼ =	272¼ =	39204
			1 =	9 =	1296
				1 =	144

373. Architects, Carpenters, and some other mechanics frequently measure their work by the *square*, which is a space 10 ft. by 10 ft., equaling 100 square feet.

374. A square foot, yard, or mile, is a square each side of which is one foot, yard, or mile.

3 Feet.

3 Feet.
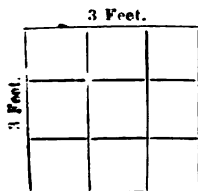
Fig. 2.

The number of small squares contained in any large square is equal to the product of the number of units in one side, multiplied by the number of units in the other side. Thus, in figure 2, each side of which is three feet, there are 9 square feet,

The difference between *square feet* and *feet square*, *square miles* and *miles square*, etc., for any unit of measure, is not generally understood, and because of its practical importance we solicit special attention to it. By 3 *yards square* is meant a square figure, each side of which is 3 yards; but by 3 *square yards* is meant 3 small squares, each a yard long and a yard wide.

Figure No. 2 is 3 feet square, and contains 9 square feet.

The difference between 500 rods square and 500 square rods is 249500 square rods.

There is no difference between 1 yard square and 1 square yard, 1 foot square and 1 square foot, etc., of any unit of measure, but increase the measure above 1 unit and the difference is very great

Formerly, but now obsolete,

40 Square Rods or Perches	=	1 Rood.
4 Roods	=	1 Acre.

375. Surveyors' Square Measure is used by surveyors in measuring the area or surface of land. The acre is the measuring unit for land

TABLE.

625 Square Links (sq. l.)	= 1 Square Rod, or Pole.	sq. rd., or P
16 Poles	= 1 Square Chain........	sq. ch.
10 Square Chaines	= 1 Acre	A.
640 Acres	= 1 Square Mile, or Sec.	sq. mi., or sec.
36 Square Miles (6 miles square)	= 1 Township	Tp

Tp.		sq. ml.		A.		sq. ch.		sq. rd.		sq. l.
1	=	36	=	23040	=	230400	—	3686400	—	230400000

3 4

SOLID, OR CUBIC MEASURE.

376. **Solid or Cubic Measure** is used in measuring the contents, or volume, of solids.

377. A **Solid, or Body** has length, breadth, and thickness.

378. A **Cube** is a solid, bounded by six equal square sides, or faces; hence its three dimensions are equal to each other.

379. The **Contents, or Volume,** of a body is expressed by the product of the length, breadth, and thickness.

TABLE.

1728 Cubic Inches (cu. in.)	= 1 Cubic Foot	cu. ft.
27 Cubic Feet	= 1 Cubic Yard	cu. yd.
16 Cubic Feet	= 1 Cord Foot	cd. ft.
8 Cord Feet, or 128 Cubic Feet	= 1 Cord of Wood	cd.
24¾ Cubic Feet, or 16½ feet long, 1½ ft. high, and 1 foot wide.	= 1 Perch	Pch.

cu. yd.		cu. ft.		cu. in.		cd.		cd ft.		cu. ft.		cu. in.
1	=	27	=	46656		1	=	8	=	128	=	221184

380. A **Square** of earth is a cube $6 \times 6 \times 6 = 216$ cubic feet.

381. In civil engineering, the cubic yard is the unit for measuring excavations, embankments, and levees.

382. In commerce, the cubic foot is often the unit for computing freight charges.

LIQUID MEASURE.

383. **Liquid Measure** is used in measuring molasses, wine, oil, etc., and in estimating the capacities of cisterns, reservoirs, etc.

384. The **Unit** of measure for liquids is the gallon, which contains 231 cubic inches.

TABLE.

4 Gills (gi.)	= 1 Pint ....pt.
2 Pints	= 1 Quart ..... .qt.
4 Quarts	= 1 Gallon .....gal. = 231 cubic in.
31½ Gallons	= 1 Barrel ......bbl.
2 Barrels, or 63 gallons	= 1 Hogshead ..hhd.

$$
\begin{array}{cccccccc}
\text{gal.} & & \text{qt.} & & \text{pt.} & & \text{gi.} \\
1 & = & 4 & = & 8 & = & 32 \\
& & 1 & = & 2 & = & 8 \\
& & & & 1 & = & 4
\end{array}
$$

In the old tables, 2 Hogsheads made 1 Pipe; and 2 Pipes made 1 Tun. But these measures are no longer used. The old table for measuring beer is not now used, beer being measured by the units in the above table.

385. In commerce, the barrel and the hogshead are not used as units of measure. The contents of all barrels, etc., containing liquids are gauged separately.

386. The **Imperial Gallon** of England contains 277.274 cubic inches.

387. Apothecaries' Fluid Measure is used in compounding *liquid* medicines.

TABLE.

60 Minims (m.)	= 1 Fluidram...............f ʒ
8 Fluidrams	= 1 Fluidounce............f ʒ
16 Fluidounces	= 1 Pint....................O.
8 Pints	= 1 Gallon ....Cong.

$$
\begin{array}{ccccccccc}
\text{Cong.} & & \text{O.} & & f\ʒ. & & f\ʒ. & & \text{m.} \\
1 & = & 8 & = & 128 & = & 1024 & = & 61440 \\
& & 1 & = & 16 & = & 128 & = & 7680 \\
& & & & 1 & = & 8 & = & 480 \\
& & & & & & 1 & = & 60
\end{array}
$$

One M. = about 1 drop of water.

388. *O.* is an abbreviation of *octans*, the Latin for one-eighth; *Cong.* for *congiarium,* the Latin for gallon.

389. A single common teaspoonful, or 45 drops, makes about one fluidram. A common teacup holds about 4 fluidounces; a common tablespoon about half a fluidounce; a pint of water weighs a pound.

390. ℞. is an abbreviation for *recipe*, or take; ā., aa., for equal quantities; j. for 1; ij. for 2; ss. for *semi,* or half; gr. for grain; P. for *particula,* or little part; P. æq. for equal parts; q. p., as much as you please.

DRY MEASURE.

391. Dry Measure is used to measure grain, fruit, vegetables, etc.

392. The ·Unit of Dry Measure is the *bushel,* which contains 2150.42 (practically 2150.4) cubic inches.

393. The Dry Gallon, or half peck, contains 268.8 cubic inches.

394. The Imperial Bushel of Great Britain contains 2218.192 cubic inches. One Imperial Quarter of England is 480 pounds.

TABLE.

				bu.	pk.	qt.	pt.
2	Pints (pt.)	= 1 Quart....qt.					
8	Quarts	= 1 Peck ....pk.	1	= 4	= 32	= 64	
4	Pecks	= 1 Bushel ..bu.		1 =	8	= 16	
8	Bushels (480 pounds)	= 1 Quarter ..qr.			1 =	2	
36	Bushels	= 1 Chaldron.ch.					

MEASURES OF WEIGHT

TROY, OR MINT WEIGHT.

395. Troy Weight is used in weighing gold and silver, and in philosophical experiments.

396. The **Standard Unit** of weight in the United States is the *Troy pound,* which contains 5760 grains.

TABLE.

		lb.	oz.	pwt.	gr.
24 Grains (gr.)	= 1 Pennyweight..pwt.	1=	12=	240=	5760
20 Pennyweights	= 1 Ounce .... oz.		1=	20=	480
12 Ounces	= 1 Pound........lb.			1=	24

AVOIRDUPOIS, OR COMMERCIAL WEIGHT.

397. Avoirdupois or Commercial Weight, is used in weighing all coarse articles; as groceries, cotton, iron, etc.

TABLE.

27$\frac{11}{10}$	Grains	= 1 Dram...........dr.
16	Drams	= 1 Ounce ....oz.
16	Ounces	= 1 Pound..........lb.
25	Pounds	= 1 Quarter.........qr.
4	Quarters, or 100 pounds	= 1 Hundredweight. cwt.
20	Hundredweight, or 2000 pounds	= 1 Ton.............T.
480	Pounds	= 1 Imperial Quarter.
100	Pounds is also called	1 Cental.......... c.

T.		cwt.		lb.		oz.		dr.		
1	=	20	=	2000	=	32000	=	512000		
		1	=	100	=	1600	=	25600		
				1	=	16	=	256	=	7000 gr.
						1	=	16		

The cwt. in England is 112 pounds, or 4 quarters of 28 pounds. The ton English is 2240 pounds. This is called the *long ton*, and 2000 pounds, the *short ton*.

398. The *long ton* is used in estimating duties at the U. S. Customhouse, and also at the mines in weighing coal, ores, etc.

APOTHECARIES' WEIGHT.

399. Apothecaries' Weight is used by physicians and apothecaries in weighing and compounding dry medicines.

TABLE.

					lb.	oz.	dr.	scr.	gr.
20 Grains (gr.)	= 1 Scruple..scr. or	϶	1	=12	=96	=288	=5760
3 Scruples	= 1 Dram....dr.	"	3		1=	8=	24=	480
8 Drams	= 1 Ounce...oz.	"	ℨ			1=	3=	60
12 Ounces	= 1 Pound...lb	"	℔				1=	20

The grain, the ounce, and the pound of this weight are the same as those of Troy weight.

DIAMOND WEIGHT.

400.   Diamond Weight is used in weighing diamonds and other precious stones.

TABLE.

16 Parts	=	1 Carat Grain	=	.792 Troy grain.
4 Grains	=	1 Carat	=	3.168 " "

ASSAYERS' WEIGHT.

401.   Assayers' Weight is used by assayers in determining the quantity of any particular metal in ores, or metallic compounds.

TABLE.

1 Carat grain	=	2 Pwts. 12 grains, or 60 grains Troy.
1 Carat	=	10 Pwts. Troy.
24 Carats	=	1 Pound Troy.

This assay carat is entirely different from the carat in Diamond Weight.

The term carat is also used to express the fineness of gold, each carat meaning a twenty-fourth part.

CIRCULAR, OR ANGULAR MEASURE.

402.   Circular, or Angular Measure is used in measuring angles, latitude and longitude, the location of vessels at sea, of planets, stars, etc.

403.   The Standard Unit for measuring angles is the *Degree* which varies with the size of the circle.

404.   A Degree is the angle measured by the arc of $\frac{1}{360}$ part of the circumference of a circle.

405. A **Circle** is a plane figure bounded by a curved line, every part of which is equally distant from a certain point within called the center.
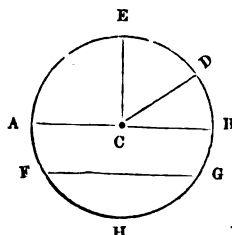
406. The **Circumference** of a circle is the curved line by which it is bounded; as, A. E. D. B. G. H. F.

407. The **Radius** of a circle is a line extending from its center to any point in its circumference; as, C. E. and C. D.

408. The **Diameter** of a circle is a straight line passing through its center and terminating at each end in the circumference; as, A. B.

409. An **Arc** of a circle is any portion of the circumference; as, B. D., D. E., etc.

410. A **Chord** of a circle is a straight line drawn within a circle and terminating in the circumference, but not passing through the center; as, F. G.

411. A **Segment** of a circle is any part cut off by a chord; as, F. H. G.

412. A **Sector** of a circle is any part of a circle bounded by two radii and the arc included between them; as, the space between C. D., C. B., and B. D.

413. An **Angle** is the opening or space between two lines or surfaces which meet in a common point, called the vertex. Thus, A. C. E., E. C. D., and D. C. B. are angles, and C. is their vertex.

414. A **Semi-Circumference** is *one-half* of a circumference, or 180°.

415. A **Quadrant** is *one-fourth* of a circumference, or 90°.

416. A **Sextant** is *one-sixth* of a circumference, or 60°.

417. A **Sign** is *one-twelfth* of a circumference, or 30°.

TABLE.

60 Seconds (marked ″)	= 1 Minute............ ′
60 Minutes	= 1 Degree............ °
30 Degrees	= 1 Sign .... s.
12 Signs, or 360°	= 1 Circle............ c.

c.	s.	°	′	″
1 =	12 =	360 =	21600 =	1296000
	1 =	30 =	1800 =	108000
		1 =	60 =	3600
			1 =	60

THE OLD FRENCH AND SPANISH MEASURES OF LENGTH, SURFACE, AND SOLID.

418. Louisiana having been both a French and Spanish Province, the old French and Spanish units of measure are often met with in private and public records; and to aid in understanding such units, the following table is presented:

TABLE.

419.

Old French System.		English or American Measure.
1 Point	=	.0074 English inches.
1 Line = 12 points	=	.08884 English inches.
1 Inch = 12 lines	=	1.06577 English inches.
1 Foot = 12 inches	=	12.7892 English inches.
1 Ell = 43 inches 10 lines	=	46.716 English inches.
1 Toise= 6 feet	=	76.735 English inches.
1 Perch, or Rod (Paris) = 18 feet	=	19.1838 English feet.
1 Perch, or Rod (Royal) = 22 feet	=	23.447 English feet.

TABLE—Continued.

| **Old French System.** | **English or American Measure.** |

1 League (common)=25 to a degree=2280 toises=14579.688 English feet=2.761 miles.
1 League (post) = 2000 toises = 12789.2 English feet = 2.422 miles.
1 Fathom (brass)=5 feet French=63.946 English inches.
1 Cable length = 100 toises = 639.46 English feet = 106.58 English fathoms.

420. **Old Spanish System.**

1 Foot = 11.1284 English inches.
1 Vara = 3 feet=0.9274 English yd.=33.3864 Eng. inches.
1 Common League = 19800 Spanish feet.
1 Judicial League = 15000 Spanish feet.

421. **Old French Square and Cubic Measure.**

1 Square inch = 1.13587 English square inches.
*1 Arpent (Paris) = 100 sq. perches, = 36804.120336 sq. feet, English.
1 Arpent (Woodland) = 100 sq. perches (Royal) = 54978.994576 sq. feet, English.
1 Cubic inch = 1.2106 cubic inches, English.
1 Cubic foot = 2091.85 cubic inches, English.

*Arpent is the old French name for acre.

422. TABLE OF COMPARATIVE WEIGHTS, MEASURES, AND VALUES.

Avoirdupois.		Troy.		Apothecaries.
7000 gr. = 1 lb.		5760 gr. = 1 lb.		5760 gr. = 1 lb.
1 lb.	=	$1\frac{31}{44}$ lbs.	=	$1\frac{31}{44}$ lbs.
or 144 lbs.	=	175 lbs.	=	175 lbs.
1 oz.	=	$1\frac{7}{24}$ oz.	=	$1\frac{7}{24}$ oz.
or 192 oz.	=	175 oz.	=	175 oz.

A Wine Gallon	=	231	cubic inches.
The Old Beer Gallon	=	282.	" "
A Dry Gallon	=	268.8	" "
An Imperial Gallon	=	277.274	" "
A U. S. Bushel	=	2150.42	" "
A U. S. Bushel heaped	=	2688.	" "
An English Bushel	=	2218.192	" "

Diameter of circle = 1, Circumference = 3.1416
Area of a square = 1, Area of circle
  diameter of which=one side of the square = .7854
Solidity of cube = 1, Solidity of Sphere,
  diameter of which = one side of cube.. = .5236

1 oz. pure gold........................... = $20.67+
1 oz. pure silver .... = $ 1.29+
1 pwt. pure gold .... = 1.03
1 pwt. pure silver........................ = .0645
1 Pint of water weighs 1.0431 lbs.
1 Gallon   "   "   8.3450 lbs.
1 Cubic foot of water weighs 62.425 lbs. at 39.2° F.
The Common year has 365 days
The Leap year has 366 days.
The Solar year has 365 days, 5 hrs., 48 min., 49.7 sec.

A Yard......	=	36 in.	1 Peseta of Spain	=19.3	cts
A Vara	=	33.3864 "	1 Crown of Sweden	=26.8	"
A Meter	=	39.37 "	1 Rupee of India	=38.6	"
$1....		=100 cts.	1 Drachma of		
1 Franc of France	=19.3	"	Greece	=19.3	"
1 Mark of Germany	=23.8	"	1 Peso of Cuba	=93.2	"
1 £ of England	=	$4.8665	1 Peso, or dollar		
1 Florin of Austria	=40.1	"	of Mexico	=88.2	"
1 Milreis of Brazil	=54.6	"	1 Piaster of Egypt	= 4.9	"
1 Rouble of Russia	=65	"	1 Piaster of		
1 Yen of Japan	=87.6	"	Turkey	= 4.4	"
1 Lira of Italy	=19.3	"	1 Sol of Peru	=81 2	"

A Statute mile = 5280 ft.
A Geographical, or Nautical mile, or knot, = 6086.41 ft.
A Statute mile being 1, a Geographical mile is 1.15273+

423. An Acre contains 160 sq. rds., or 43560 sq. ft., and is 208.7103+ ft. on each side.

An Arpent (Paris) contains 100 sq. rds. (Paris), or 36804.120336 sq. ft. (English), and is 191.1844 ft. (English) on each side.

An Arpent (Woodland) contains 100 sq. rds. (Royal), or 54978.994576 sq. ft. English, and is 234.476 English ft. on each side.

424.   MISCELLANEOUS TABLES.

BOOKS AND PAPER.

SIZE OF PAPER.

	Inches.		Inches.
Demy	17 by 22	Letter	10 by 15
Medium	19 " 24	Folio Post	16 " 21
Double Medium	24 " 38	Foolscap	14 " 17
Super-Royal	21 " 27	Crown	15 " 20
Imperial	22 " 32	Double Elephant	26 " 40

A sheet (medium) folded in 2 leaves is called folio.

"	"	"	4	"	"	quarto or 4to.
"	"	"	8	"	"	octavo or 8vo.
"	"	"	12	"	"	duodecimo or 12mo
"	"	"	16	"	"	16mo.
"	"	"	18	"	"	18mo.
"	"	"	24	"	"	24mo.
"	"	"	32	"	"	32mo.

24 Sheets	=	1 Quire.
480 Sheets	=	20 Quires = 1 Ream.
2 Reams	=	1 Bundle; 5 Bundles = 1 Bale.
12 Units	=	1 Dozen.
144 Units	=	12 Dozen = 1 Gross.
12 Gross	=	1 Great Gross.
20 Units	=	1 Score.
56 lbs.	=	1 Firkin of Butter.
100 lbs.	=	1 Quintal of Dried Fish.
196 lbs.	=	1 Barrel of Flour.
200 lbs.	=	1 Barrel of Flour in California.
200 lbs.	=	1 Barrel of Beef, Pork, or Fish.
280 lbs.	=	1 Barrel of Salt.
100 lbs.	=	1 Cask of Raisins.

14 lbs. Iron or Lead = 1 Stone.

12 Barrels of Wheat = 7 English Quarters.

31½ Stone = 1 Pig; 8 Pigs = 1 Fother.

256 Pounds of Soap = 1 Barrel.

25 Pounds of Powder = 1 Keg.

425. *WEIGHT OF GRAIN AND PRODUCE PER BUSHEL,

As used in New Orleans when there is no agreement to the contrary.

Wheat........ ....bush.	60 lbs.	Flaxseed ....bush.	56 lbs.
Corn............ "	56 "	Hempseed ...... "	44 "
Rye .... "	56 "	Buckwheat ..... "	52 "
Oats........... "	32 "	Castor Beans... "	46 "
Barley .... "	48 "	Dried Peaches.. "	33 "
Irish Potatoes... "	60 "	Dried Apples.... "	24 "
Sweet Potatoes . "	60 "	Onions.......... "	57 "
Beans .... "	62 "	Coarse Salt ..... "	50 "
Bran........... "	24 "	Fine Salt .... "	50 "
Clover Seed..... "	60 "	Stone Coal ...... "	80 "
Timothy Seed... "	45 "	Corn Meal ...... "	44 "
Barley Malt..... "	34 "	Plastering Hair. "	7 "
Peas, split ...... "	60 "	Blue Grass Seed. "	10 "
Small Hominy .. "	50 "		

* In several States the weight of some of these articles is different from the figures here given.

426. In copying legal papers, recording deeds, etc., clerks are usually paid by the folio. Thus:

    100 words make 1 folio in New York.
    72 words make 1 folio in Com. Law in England.
    90 words make 1 folio in Chancery in England.

In printing books, 240 impressions, or 120 sheets printed on both sides, make 1 token.

SYNOPSIS FOR REVIEW.

Define the following words and phrases:

317. A Denominate Number. **318.** A Simple Denominate Number. **319.** A Compound Denominate. Law of *Increase* and *Decrease*. **320.** A Measure. *Seven* kinds of Measure. **321.** Weight. **322.** Value. **323.** Money. **324.** Currency. **325.** A

Measure Table. 326. Table of Money. 327. Money; the *Unit* of the Measure. 328. When and by whom established? 329. Of what does the Coin of the U. S. consist? 330. The Value, Composition, and Weight of each? 331. Allowance for Deviation in Weight. 333. Trade Dollar. 334. Alloy of a Coin. 335. Weight of Coin. 336. Value of Gold and Silver. 337. Canada Money. 338. Coins of Canada Money. 339. English, or Sterling Money. 340. Monetary Unit of Great Britain. Table of English Money. 341. Of what does the money of Great Britain consist? 342. French Money and Table. Of what does French Money consist? Where is the Franc used? 343. German Money. Unit of German Money. 344. Table. 345. Time. 346. Unit of Measure of Time. 347. A Year. 348. A Day. 349. Solar Day. 350. Mean Solar Day. 351. Civil, or Legal Day. 352. Astronomical Day. 353. Solar Year. 354. Common, or Civil Year. 355. How to determine what years are Leap Years. Table of Time Measure. 356. Extension. 357. Unit of Measure of Extension. 358. A Yard. 359. A Line. 360. A Surface. 361. A Solid. 362. Line, or Linear Measure and Table. 363. Chain Measure and Table. 364. Mariners' Measure and Table. 365. A Minute of the Earth's Circumference. Length of a Degree. 366. Shoemakers' Measure. 367. Unit of Shoemakers' Measure. Table. 368. Cloth Measure. Table. Flemish Ell; English Ell; French Ell. 369. Square, or Surface Measure. 370. A Surface. 371. Area of a Surface. 372. A Square. Table of Square Measure. 373. The Square of Architects and Carpenters. 374. A Square Foot, Yard, or Mile. Difference between *feet square* and *square feet*. 375. Surveyors' Square Measure and Table. 376. Solid, or Cubic Measure. 377. A Solid or Body. 378. A Cube. 379. Contents, or Volume of

a Body. Table of Cubic Measure. 380. A Square of Earth. 381. The Unit in Civil Engineering. 382. The Unit in Freight Charges. 383. Liquid Measure. 384. Unit of Liquid Measure. Table. 386. Imperial Gallon. 387. Apothecaries' Fluid Measure and Table. 388. O., Cong. · 389. A common teaspoonful; a common teacup; a common tablespoon; a pint of water. 390. ℞., ä., aa., j,, ij., ss., gr., P., P. æq., q. p. 391. Dry Measure. 392. Unit of Dry Measure. 393. Dry Gallon. 394. Imperial Bushel. Imperial Quarter. Table of Dry Measure. 395. Troy, or Mint Weight. 396. Standard Unit. Table of Troy Weight. 397. Avoirdupois, or Commercial Weight, and Table. The *cwt.* in England. 398. The Long Ton. 399. Apothecaries' Weight and Table. 400. Diamond Weight and Table. 401. Assayers' Weight and Table. A Gold Carat. 402. Circular, or Angular Measure. 403. Unit for Measuring Angles. 404. A Degree. 405. A Circle. 406. Circumference. 407. Radius. 408. Diameter. 409. An Arc. 410. A Chord. 411. A Segment. 412. A Sector. 413. An Angle. 414. A Semi-Circumference. 415. A Quadrant. 416. A Sextant. 417. A Sign. Table of Circular Measure. 419. Table of Old French Linear Measure. 420. Old Spanish Linear Measure. 421. Old French Square and Cubic Measure. 422. Table of Comparative Weights, Measures, and Values. 423. An Acre; an Arpent (Paris); an Arpent (Woodland). 424. Miscellaneous Tables. 425. Weight of Grain and Produce per bushel. 426. Table for Copying Legal Papers, Recording Deeds, etc.

Reduction of Denominate Numbers.

427. Reduction is the operation of changing an expression in one or more denominations to an equivalent expression in some other denomination or denominations of the same kind of measurement; as 3 yards may be changed to its equivalent 108 inches, and 7 lbs. 6 oz. (Troy) may be changed to its equivalent $7\frac{1}{2}$ lbs., or 90 ounces.

Reduction is of two kinds *Descending* and *Ascending*.

428. Reduction Descending is changing the forms of denominate quantities from a higher to a lower order of units, or denomination; as in changing or reducing dollars to cents, pounds to grains, etc.

429. Reduction Ascending is the converse of reduction descending, and hence it is the changing of the form of denominate quantities from a lower to a higher denomination; as cents to dollars, grains to pounds, etc.

(279,

REDUCTION DESCENDING.

430. *To Reduce a Simple Denominate Number to a Lower Denomination, (in the same System of Measure).*

1. Reduce 6 feet to inches.

OPERATION.

	6 feet.	
	12 inches.	
—	———	
	72 inches, Ans.	

or,

ft. in.
6. 0
  —
  12
  6
  —
72 inches, Ans.

Explanation and Reason.— By considering the conditions of the problem, we see that we are required to reduce 6 feet to inches, i. e. to find the equivalent of 6 feet in the unit inches. Before we can perform the operation we must know the units of the lower denominations from feet to inches. And by referring to the table of Linear Measure we find that 1 foot = 12 inches. This gives the premise for the solution and from it we reason as follows: Since 1 foot is = to 12 inches, 6 feet are = to 6 times as many, which is 72 inches, the answer.

2. Reduce 3 bushels to pints.

OPERATION.

bu. pks. qts. pts.
3 0 0 0
— — —
4 8 2
3 12 96
— — —
12 96 192 pts. Ans.

Explanation and Reason.— In this problem, we are required to reduce bushels to pints; but before we can perform the operation we must know either the different units of the lower denominations from bushels to pints, or the equivalent of *one* bushel in pints. By reference to the table of Dry Measure we see that 1 bushel = 4 pecks; 1 peck = 8 quarts; and 1 quart = 2 pints. These equivalents furnish our premises and from them we develop the solution.

We first write the problem as shown in the operation, filling all vacant denominations, from bushels to the denomination required, pints, with naughts; then below each denomina-

tion we draw a line and write thereunder the number of units of each order which make one of the next higher order. Having thus stated the problem we reason as follows: Since 1 bushel is = to 4 pecks, 3 bushels are = to 3 times as many, which is 12 pecks; then since 1 peck is = to 8 quarts, 12 pecks are = to 12 times as many, which is 96 quarts; then since 1 quart is = to 2 pints, 96 quarts are = to 96 times as many which is 192 pints, the answer.

If we work from the basis of the number of pints in a bushel we would reason thus: 1 bushel is = to 64 pints; Since 1 bushel is = to 64 pints, 3 bushels are = to 3 times as many, which is 192 pints.

   3.   Reduce $7 to mills.   7000 m.
   4.   Reduce 3 shillings to farthings.   144 far.
   5.   Reduce 4 pounds Troy to pennyweights.
                                            960 pwt.
   6.   Reduce 5 bushels to quarts.   160 qts.
   7.   Reduce 3 days to minutes   4320 m.
   8.   Reduce 2° to ''.   7200 ''.
   9.   Reduce 2 square feet to square inches.
                                     288 sq. in.

431. *To Reduce a Compound Denominate Number to a Lower Denomination (in the same System of Measure).*

   1.   Reduce 3 bu., 2 pks., and 1 pint, to pints.

OPERATION.

bu.	pks.	qts.	pts.	
3	2	0	1	
4	8	2		
3	14	112		
14	112	225 pts., Ans.		

Explanation and Reason.— Here we are required to determine the number of pints in the whole expression. We first state the problem as shown in the operation filling the vacant units or denominations in the scale with a naught and writing under each term the number of units of that order which make one of the next higher order. Having thus stated the

3 6

problem we reason as follows: Since 1 bushel = 4 pecks. 3 bushels are = to 3 times as many which is 12 pecks + the ? pecks = 14 pecks; then since 1 peck = 8 quarts, 14 pecks are = to 14 times as many, which is 112 quarts; then since 1 quart = 2 pints, 112 quarts are = to 112 times as many, which is 224 pints + 1 pint = 225 pints, answer.

In all problems of like character to the three preceding, the form of operation and the process of reasoning here given should be used.

2. Reduce 3 £. 2s. 8d. 3 far. to farthings.
<div align="right">3011 far.</div>

3. Reduce 6 lbs. 12 oz. 13 drs. to drams.
<div align="right">1741 drs.</div>

4. Reduce 5 lbs. 2 oz. 12 grs. to grains.
<div align="right">29772 grs.</div>

5. Reduce 3 rods 20 links 6 inches to inches.
<div align="right">758.4 or 758⅖ in.</div>

6. Reduce 3 f.oz. 4 f.drs. 20 m. to minims.
<div align="right">1700 m.</div>

7. Reduce 3 cable-lengths 5 fathoms to feet.
<div align="right">2190 ft.</div>

8. Reduce 3 rds. 12 ft. 6 in. to inches.   744 in.

432. *To Reduce a Fractional Denominate Number to a Lower Denomination (in the same System of Measure).*

1. Reduce ⅔ of a gallon to gills.

OPERATION

8	3 gallons.
	4 quarts.
	2 pints.
	4 gills.
—	————————
	12 gills, Ans.

Explanation and Reason.—In this problem, we are required to find the equivalent of ⅔ of a gallon in the unit of gills. By referring to the table of Wine Measure, we see that 4 gills = 1 pint; 2 pints = 1 quart; 4 quarts = 1 gallon. Then with this data as our premises we write the ⅔ of a gallon on the statement line and reason as

follows: Since 1 gallon = 4 quarts, there are 4 times as many **quarts** as gallons; then since 1 quart = 2 pints, there are two **times** as many pints as quarts; then since 1 pint = 4 gills, **there are** 4 times as many gills as pints   This completes the **reasoning,** and by working out the statement we obtain 12 **gills** as the equivalent of $\frac{3}{8}$ of a gallon.

2. Reduce $\frac{3}{5}$ of a dollar to cents.   Ans. 60 cts.

3. Reduce $\frac{5}{6}$ of a pound (£) to farthings.
                                        800 far.

4. Reduce $\frac{3}{10}$ of a Troy pound to grains.
                                        1728 grs.

5. Reduce $\frac{4}{11}$ of a yard to inches. $13\frac{1}{11}$ in.

6. Reduce $\frac{3}{7}$ of a bushel to pints. $27\frac{3}{7}$ pts.

7. Reduce $\frac{2}{3}$ of a week to seconds.
                                        403200 sec.

8. Reduce $\frac{2}{7}$ of a franc to millimes. $285\frac{5}{7}$ m.

Reduction of Denominate Fractions.

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433. A Denominate Fraction is a fraction whose integral unit is a denominate number.

Thus, ¾ of a day, .9 of a mile, are ·denominate fractions.

434. *To Reduce Denominate Fractions, or Fractional Denominate Numbers to Lower Denominations or to Compound Denominate Numbers.*

1. Reduce ⅖ of a bushel to lower denominations.

OPERATION.

bu.	pks.	qts.	pts.
⅖	0	0	0

	4		8		2
5	2	5	3	5	4
	8		24		8
	1⅗		4⅘		1⅗

1 pk.   4 qts.   1⅗ pts.

Explanation and Reason. According to the conditions of this problem, we are to find not the equivalent of ⅖ of a bushel in pecks, quarts, or pints, but we are to determine what compound denominate number composed of the denominations of *pecks, quarts, and pints,* will be equivalent to ⅖ of a bushel. In the operation, we first write the ⅖ of a bushel and fill each lower denomination with a naught, below which we draw a line and underneath write the number of units of each order which make one of the next higher

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order. We then draw a vertical line to the left of the 4 pecks and reason as follows: Since 1 bushel = 4 pecks ⅓ of a bushel = the ⅓ part, and ⅔ bushels = 2 times as many, which is, as shown by the operation, 1⅔ pecks. The 1 peck is now written below a long horizontal line which we call the answer line, and is the first denomination in the number sought.

Then we draw a vertical line to the left of the 8 quarts and reason thus: Since 1 peck = 8 quarts, ⅓ of a peck = the ⅓ part, and ⅔ = 3 times as many, which work gives 4⅓ quarts.

The 4 quarts are written below the answer line and is the second denomination of the compound number required by the terms of the problem. Lastly, drawing a vertical line to the left of the 2 pints we reason thus: Since 1 quart = 2 pints, ⅓ of a quart = ⅓ part and ⅔ = 4 times as many, which is 1⅔ pints. This being the last denomination in the Dry Measure table of measurement, we write it in full below the answer line and thus complete the solution—having obtained 1 pk., 4 qts., and 1⅔ pts., as the equivalent compound denominate value of ⅔ of a bushel.

2. Reduce ⅓ £ to a compound denominate number.   Ans. 6s. 8d.

3. Reduce ⅖ yard to a compound denominate number.   Ans. 1 ft. 2⅖ in.

4. Reduce ⅔ lbs. Troy to a compound denominate number.   Ans. 5 oz. 2 pwt. 20½ grs.

5. Reduce ⅚ mile to a compound denominate number.   Ans. 6 fur. 26 rds. 3 yds. 2 ft.

6. Reduce ⅞ degree to a compound denominate number.   Ans. 52', 30''.

7. Reduce ⅘ Cong. to a compound denominate number.   Ans. 4 O. 12 fl. oz. 6 fl. dr. 24 m.

435. *To Reduce Denominate Decimal Fractions, or Decimal Denominate Numbers, to Lower Denominations (in the same System of Measure).*

1. Reduce .875 of a bushel to pints.

OPERATION.

bu.	pks.	qts.	pts.
.875	0	0	0
	—	—	—
	4	8	2
	.875	3.5	28
	—	—	—
	3.500	28.0	56 pts. Ans.

or,

```
    .875
       4
   ------
   3.500
       8
   ------
    28.0
       2
   ------
```

56 pints, Ans.

Explanation and Reason— This problem is very similar to the first and second problems in denominate numbers, page 280. We first write the problem as shown in the operation, filling all vacant denominations from bushels to pints with naughts; then, below each lower denomination we draw a line and write thereunder the number of units of each order which make one of the next higher order. Then, from these premises we reason as follows: Since 1 bushel = 4 pecks, .875 of a bushel = .875 times as many, which is 3.5 pecks; then, since 1 peck = 8 quarts, 3.5 pecks = 3.5 times as many, which is 28 quarts; then, since 1 quart = 2 pints, 28 quarts = 28 times as many, which is 56 pints, answer.

2. Reduce 0.755 of a gallon to gills.
   Ans. 24.16, or $24\frac{4}{25}$ gills.

3. Reduce 0.375 of a rod to inches.
   74.25, or $74\frac{1}{4}$ in.

4. Reduce 0.25 of an acre to sq. ft.   10890 sq. ft.

5. Reduce 0.42 of a ton to drams.   215040 drs.

6. Reduce 0.3 of a yd. to nails. $4\frac{4}{5}$ nails.

7. Reduce 0.16 of a chain to inches.   126.72 in.

436. *To Reduce a Denominate Decimal Fraction, or a Decimal Denominate Number, to Lower Compound Denominate Numbers.*

1. Reduce .374 lbs. apothecaries' weight, to a compound denominate number.

OPERATION.

lbs.	oz.	dr.	sc.	gr.
.374	0	0	0	0
	12	8	3	20
	.374	.488	.904	.712
	4.488	3.904	2.712	14.240
	4 oz.	3 dr.	2 sc.	14.24 grs., Ans.

Explanation and Reason.—This problem is very similar to the elucidated problem on page 284. In the operation, we first write the .374 of a pound and fill the place of each lower denomination with a naught; then drawing a line below each naught, we write the number of units of each order of Apothecaries' weight, which it takes to make one of the next higher order. Having thus stated the problem, we reason as follows: Since 1 pound = 12 ounces, .374 of a pound = .374 times as many, which is 4.488 ounces. We now draw the answer line and write the 4 ounces below it. Then, since 1 ounce = 8 drams, .488 of an ounce = .488 times as many, which is 3.904 drams. The 3 drams we write below the answer line, and continue thus: Since one dram = 3 scruples, .904 of a dram = .904 times as many, which is 2.712 scruples. The 2 scruples we write below the answer line, and continue thus: Since 1 scruple = 20 grains, .712 of a scruple = .712 times as many, which is 14.24 grains.

This being the lowest denomination, it is written below the answer line and completes the solution.

2. Reduce 0.27 of a bu. to a compound denominate number. Ans. 1 pk. 0 qts. 1.28 pts.

3. Reduce 0.7 of a lb. Troy to a compound denominate number. Ans. 8 oz. 8 pwt.

4. Reduce 0.35 of a mi. to a compound denominate number. Ans. 2 fur. 32 rds.

5. Reduce 0.32 of a day to a compound denominate number. Ans. 7 hrs. 40 min. 48 sec.

6. Reduce 0.3 of a cu. yd. to a compound denominate number. Ans. 8 cu. ft. 172.8 in.

7. Reduce 0.65 of a gal. to a compound denominate number. Ans. 2 qts. 1 pt. 0.8 gi.
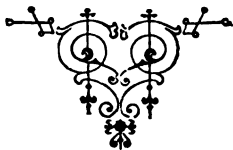
REDUCTION ASCENDING.

437. *To Reduce a Simple Denominate Number to a Compound Denominate Number of Higher Denominations.*

1. Reduce 942 gills wine measure to a compound denominate number.

OPERATION.

4	942 gills
2	235 pints + 2 gi. remainder.
4	117 quarts + 1 pt. remainder.
	29 gallons + 1 qt. remainder.

29 gal. 1 qt. 1 pt. 2 gi. Ans.

Explanation and Reason.—In this problem, we are to determine the equivalent of 942 gills in higher denominations. According to the table of Wine Measure, 4 gills = 1 pint; 2 pints = 1 quart, and 4 quarts = 1 gallon. Being in possession of these equivalent denominations, we first reduce the gills to the next higher denomination, pints; then we reduce the pints to the next higher denomination, quarts; and thus continue to reduce each lower denomination to the next higher, until the highest denomination in the table of Wine Measure is reached, or until the last quotient is less than the next higher denomination. Our reasoning is as follows: Since 4 gills = 1 pint, there are ¼ as many pints as gills, which is 235 pints and 2 gills remainder, as shown in the operation. Then, since 2 pints = 1 quart there are ½ as many quarts as pints, which is 117 quarts + 1 pint remainder. Then, since 4 quarts = 1 gallon there are ¼ as many gallons as quarts, which is 29 gallons and 1 quart remainder. This completes the reasoning and gives 29 gal., 1 qt., 1 pt., and 2 gills as the equivalent of 942 gills.

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2. Reduce 3721 pints to a compound denominate number, (dry measure).

Ans. 58 bu. 0 pk. 4 qt. 1 pt.

3. Reduce 1391 inches to a compound denominate number, (linear measure).

Ans. 6 rds. 5 yds. 1 ft. 11 in.

4. Reduce 2756 grains, Troy, to a compound denominate number.   Ans. 5 oz. 14 pwt. 20 gr.

5. Reduce 3457 drams, Avoirdupois, to a compound denominate number.

Ans. 13 lbs. 8 oz. 1 dr.

6. Reduce 17353 farthings to a compound denominate number.   Ans. £18 1s. 6d. 1 far.

7. Reduce 34567 inches, chain measure, to a compound denominate number.

Ans. 43 ch. 64 links 4.12 inches.

NOTE.—In chain measure, the chain consists of 100 links.

438. *To Reduce a Simple or a Compound Denominate*
Number to a Denominate Fraction of a
Higher Unit.

1. Reduce 3 pints to the fraction of a bushel.

OPERATION.

| 3 pints. |
| 2 |
| 8 |
| 4 |
| — |
| $\frac{3}{64}$ bushel, Ans. |

Explanation and Reason.— Since we are to find the equivalent of 3 pints in the fraction of a bushel, we must first know either the scale of units from pints to bushels, or the number of pints in a bushel. By referring to the table of Dry Measure, we find that 2 pints = 1 quart; 8 quarts = 1 peck; and 4 pecks = 1 bushel.

With this knowledge, we place the 3 pints on the right of

the statement line and reason as follows: Since 2 pints = 1 quart, there are $\frac{1}{2}$ as many quarts as pints, which we indicate by writing the 2 on the division side of the statement line. This gives us the value of 3 pints in the fraction of quarts. Then, since 8 quarts = 1 peck, there will be $\frac{1}{8}$ as many pecks as quarts, which we indicate by writing the 8 on the division side of the statement line. This gives us the value of 3 pints in the fraction of pecks. Then, since 4 pecks = 1 bushel, there will be $\frac{1}{4}$ as many bushels as pecks, which we indicate by writing the 4 on the division side of the statement line. This gives us what the condition of the problem required, the value of 3 pints in the fraction of a bushel.

2. Reduce 8s. 4d. to the fraction of a pound sterling.

OPERATION INDICATED.

8s. 4d. = 100d.   ·£$\frac{100}{240}$=£$\frac{5}{12}$ Ans.
                       or,

$\begin{array}{c|c} & 100d. \\ 12 & \\ 20 & \overline{} \\ \underline{} & £\frac{5}{12} \text{ Ans.} \end{array}$

3. Reduce 5 gills to the fraction of a gallon.
        Ans. $\frac{5}{32}$ of a gal.

4. Reduce 72 drams to the fraction of a ton.
        Ans. $\frac{9}{64000}$ of a ton.

5. Reduce 16″ to the fraction of a degree.
        Ans. $\frac{1}{225}$ of a degree.

6. Reduce 2 hours and 20 minutes to the fraction of a day.   Ans. $\frac{7}{72}$ of a day.

7. Reduce 3 minims to the fraction of a pint.
        Ans. $\frac{1}{2560}$ of a pint.

8. Reduce 25 inches to the fraction of a yard.
        ·Ans. $\frac{25}{36}$ of a yard.

9. Reduce 3 oz. 4 pwt. 16 grs. to the fraction of a pound Troy.

OPERATION INDICATED.

3 oz. 4 pwt. 16 grs. = 1552 grs.

$$\begin{array}{r|l} 24 & 1552 \\ 20 & \\ 12 & \\ - & \underline{} \\ & \tfrac{97}{360} \text{ of a pound, Ans.} \end{array}$$

or thus,

$\tfrac{1552}{5760}$ grs.$= \tfrac{97}{360}$ of a pound, Ans.

10. Reduce 1 cu. ft. 200 cu. in. to the fraction of a cu. yd. Ans. $\tfrac{241}{6932}$ of a cu. yd.

11. Reduce 3′ 5″ to the fraction of a degree. Ans. $\tfrac{37}{720}$ of a deg.

12. Reduce 5 cable-lengths 3 fathoms to the fraction of a mile. Ans. $\tfrac{403}{880}$ of a mi.

13. Reduce 6 rds. 4 yds. 2 ft. 9 in. to the fraction of a furlong. Ans. $\tfrac{91}{528}$ of a fur.

14. Reduce 50 min. 30 sec. to the fraction of an hour. Ans. $\tfrac{101}{120}$ of an hr.

439. *To Reduce a Simple or Compound Denominate Number to a Denominate Decimal of a Higher Unit.*

1. Reduce 10 inches to the decimal of a yard.

OPERATION.

$$\begin{array}{r|l} 12 & 10 \text{ inches.} \\ & \overline{} \\ 3 & .83\tfrac{1}{3} = \text{decimal of a foot.} \\ & \overline{} \\ & .27\tfrac{7}{9} \text{ decimal of a yard.} \end{array}$$

or thus:

$$\begin{array}{r|l} 12 & 10 \text{ inches.} \\ 3 & \\ - & \overline{} \\ & .27\tfrac{7}{9} \text{ of a yd., Ans.} \end{array}$$

or thus:

$\tfrac{10}{36}$ of a yd. $= .27\tfrac{7}{9}$ of a yd.

Explanation and Reason. This problem is closely related to the one under Art. 438, and the operation and reasoning are somewhat similar. We first write the 10 inches on the statement line, and considering that 12 inches = 1 foot, and 3 feet = 1 yard, we reason thus: Since 12 inches = 1 foot, there are $\tfrac{1}{12}$ as many feet as inches, which is, as shown by the operation, .83¼ of a foot. Then,

since 3 feet = 1 yard, there are ⅓ as many yards as feet, which . is .27⅓ of a yard, answer.

2. **Reduce 3 quarts, 0 pints, and 1 gill to the decimal of a gallon.**

OPERATION.

4	1. gill.
2	0. 25 pints.
4	3.125 quarts.
	.78125 of a gallon.

or thus:

3 qts. 0 pts. 1 gill = 25 gills.

	25 gills.
4	
2	
4	
—	
	.78125 of a gal., Ans.

or thus:

²⁵⁄₃₂ of a gal. = .78125 of a gal. Ans.

Explanation and Reason.— For convenience, we here write the different denominations in column on the statement line, and remembering that 4 gills = 1 pint, 2 pints = 1 quart, and 4 quarts = 1 gallon, we reason as in the above problem, and divide the gills by 4, which gives a decimal of .25 of a pint. This decimal is written to the right of the 0 pints. Then we divide the .25 of a pint by 2 and produce .125 of a quart, which is written to the right of the 3 quarts. The 3.125 quarts are then divided by 4, and the required decimal of .78125 of a gallon is obtained.

3. Reduce 3 gills to the decimal of a gallon.
   Ans. .09375 of a gal.

4. Reduce 2 ounces and 5 grains, Troy, to the decimal of a pound. Ans. .1675²⁵⁄₇₂ of a lb.

5. Reduce 3″ to the decimal of a degree.
   Ans. .0008⅓ of a deg.

6. Reduce 7 seconds to the decimal of an hour.
   Ans. .0019⁴⁄₉ of an hr.

7. Reduce 108 square inches to the decimal of a sq. yd. Ans. .08⅓ of a sq. yd.

8. Reduce 12 drams to the decimal of a ton.
   Ans. .0000234375 of a ton.

9. Reduce 3 dimes, 4 cents, 2 mills, to the decimal of a dollar. Ans. \$.342.

10. Reduce 3 oz. 2 drams to the decimal of a pound. Ans. .1953125 of a lb.

11. Reduce 3 qts. 1 pt. 2 gills, to the decimal of a gallon. Ans. .9375 of a gal.

12. Reduce 3 sq. ft. 72 sq. in. to the decimal of a sq. yd. Ans. .38⅔ of a sq. yd.

13. Reduce 3 quarters, 2 nails, to the decimal of a yd. Ans. .875 of a yd.

14. Reduce 6 minutes, 40 seconds, to the decimal of an hour. Ans. .11⅑ of an hr.

440. *To Reduce a Denominate Fraction to a Fraction of a Higher Denomination.*

1. Reduce ⅘ of an ounce avoirdupois to the fraction of a ton.

OPERATION.

$$
\begin{array}{c|c}
5 & 4 \\
16 & \\
25 & \\
4 & \\
20 & \\
\hline
 & \frac{1}{40000} \text{ of a ton, Ans.}
\end{array}
$$

or thus:

$$
\begin{array}{c|c}
5 & 4 \\
16 & \\
2000 & \\
\hline
 & \frac{1}{40000} \text{ of a ton, Ans.}
\end{array}
$$

Explanation and Reason.— This problem is also very similar to the one under Article 438, and requires about the same process of reasoning.

We first write the ⅘ of an ounce on the statement line, and then reduce it to the next higher denomination, pounds, by dividing it by 16; and thus by successively dividing by that number of the lower order which make one of the next higher, we reduce the ⅘ of an ounce to the fraction of a ton, the denomination required.

The reasoning for the work is as follows: Since 16 ounces = 1 pound, there are $\frac{1}{16}$ as many pounds as ounces; then, since

25 pounds = 1 quarter there are $\frac{1}{25}$ as many quarters as pounds; then, since 4 quarters = a hundredweight, there are $\frac{1}{4}$ as many hundredweight as quarters; then, since 20 hundredweight = 1 ton, there are $\frac{1}{20}$ as many tons as hundredweight.

If it is desired, after dividing by 16, the reasoning may be given as follows: Since 2000 pounds = 1 ton, there are $\frac{1}{2000}$ as many tons as pounds.

2.   Reduce $\frac{3}{4}$ of a gill to the fraction of a quart.
Ans. $\frac{3}{40}$ of a quart.

3.   Reduce $\frac{1}{8}$ of a millime to the fraction of a franc.   Ans. $\frac{1}{8000}$ of a franc.

4.   Reduce $\frac{5}{6}$ of a fluid drachm to the fraction of a gallon.   Ans. $\frac{5}{6144}$ of a gal.

5.   Reduce $\frac{2}{3}$ of a pennyweight to the fraction of a pound Troy.   Ans. $\frac{1}{360}$ of a pound.

6.   Reduce $\frac{3}{7}$ of a square rod to the fraction of a square mile.   Ans. $\frac{3}{716800}$ of a sq. mi.

7.   Reduce $\frac{2}{3}$ of an inch to the fraction of a quarter, (cloth measure).   Ans. $\frac{2}{45}$ of a qr.

441.   *To Reduce a Denominate Fraction to a Denominate Decimal of a Higher Denomination.*

1.   Reduce $\frac{3}{4}$ of a penny to the decimal of a pound sterling.

OPERATION.

4	3
12	
20	
—	

$\frac{1}{320} = .003125$ of a £.

Explanation and Reason.— This problem is very similar to the one under Article 439. Remembering the table for English Money, we write the $\frac{3}{4}$d. on the statement line and reason thus: Since 12 pence = 1 shilling, there are $\frac{1}{12}$ as many shillings as pence, which we indicate by writing the 12 on the division side of the statement line. Then, since 20 shillings = 1£, there are $\frac{1}{20}$ as many pounds as shillings,

which is indicated by writing the 20 on the division side of the statement line. The result of the work thus far gives the fractional equivalent of $\frac{1}{8}$d. in the unit of pounds, which is $\frac{1}{320}$£. This we reduce to a decimal in the usual way and obtain 003125 of a £, answer.

2.   Reduce $\frac{1}{8}$ of a pint to the decimal of a bushel.
Ans. .0130208$\frac{1}{3}$ of a bu.

3.   Reduce $\frac{7}{10}$ of an inch to the decimal of a rod.
Ans. .0035$\frac{35}{99}$ of a rod.

4.   Reduce $\frac{2}{7}$ of a grain to the decimal of a pound Troy.
Ans. .0000744$\frac{1}{21}$ of a lb.

5   Reduce $\frac{9}{11}$ of a square inch to the decimal of a sq. yd.
Ans. .0006$\frac{31}{33}$ of a sq. yd.

6.   Reduce $\frac{15}{16}$ of a second to the decimal of an hour.
Ans. .00026$\frac{1}{24}$ of an hour.

7.   Reduce $\frac{2}{5}$ of a '' to the decimal of a degree.
Ans. .0001$\frac{1}{9}$ of a degree.

442.   *To Reduce a Decimal Denominate Number to a Decimal of a Higher Denomination.*

1.   Reduce .35 of a pint to the decimal of a gallon.

OPERATION.

2 | .35 of a pint.
———
4 | .175 of a quart.
———
| .04375 of a gallon.
  or thus:
| .35
2 |
4 |
— |
———
| 04375 of a gallon.

Explanation and Reason.—
Here again we have a problem very much like the one under Art. 439.   We are to find the equivalent of .35 of a pint in the decimal of a gallon.   We first write the .35 of a pint on the statement line, and reason as follows: Since 2 pints = 1 quart, there are $\frac{1}{2}$ as many quarts as pints, which is, as shown by the operation, .175 of a quart. Then, since 4 quarts = 1

gallon, there are ¼ as many gallons as quarts, which is .04375 of a gallon.

In the second statement the reasoning is the same as in the first, but the operation of division is not performed in detail, with each separate divisor.

2. Reduce .75 of a grain to the decimal of a pound Troy.   Ans. .0001302$\frac{1}{12}$ of a lb.

3. Reduce .96 of a minim to the decimal of a pint.   Ans. .000125 of a pint.

4. Reduce .16 of a rod to the decimal of a mile.   Ans. .0005 of a mi.

5. Reduce .36 of a foot to the decimal of a cable-length.   Ans. .0005 of a c.-l.

6. Reduce .012 of a rod to the decimal of a league.   Ans. .0000125 of a L.

7. Reduce .072 of a farthing to the decimal of a £.   Ans. .000075 of a £.

443. *To Reduce a Decimal Denominate Number to a Fraction of a Higher Denomination.*

1. Reduce .35 of a second to the fraction of an hour.

OPERATION.

Explanation and Reason.—

100	35 of a second.
60	
60	
—	

$\frac{7}{72000}$ of an hour, Ans.

In this problem we wish to determine the equivalent, in the fraction of an hour, of .35 of a second expressed as a *common fraction*. We therefore write the .35 of a second on the statement line as a common fraction, $\frac{35}{100}$, and then reason as follows: Since 60 seconds = 1 minute, there are $\frac{1}{60}$ as many minutes as seconds; then, since 60 minutes = 1 hour, there are $\frac{1}{60}$ as many hours as minutes. This statement, when worked, gives $\frac{7}{72000}$ of an hour, answer.

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2. Reduce .36 of a dram to the fraction of a pound.　　　　　　Ans. $\frac{9}{6400}$ of a lb.

3. Reduce .75 of a pint to the fraction of a barrel.　　　　　　Ans. $\frac{1}{336}$ of a bbl.

4. Reduce .124 of a farthing to the fraction of a £.　　　　　　Ans. $\frac{31}{240000}$ of a £.

5. Reduce .27 of a square inch to the fraction of a rod.　　　　　　Ans. $\frac{1}{145200}$ of a rod.

6. Reduce .0156 of an inch to the fraction of a mile.　　　　　　Ans. $\frac{13}{52800000}$ of a mi.

7. Reduce .324 of a grain to the fraction of a Troy ounce.　　　　　　Ans. $\frac{27}{40000}$ of an oz.
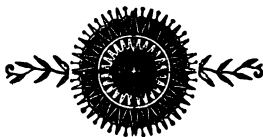
Addition of Compound Denominate Numbers.

444. Addition of Compound Denominate Numbers is the process of uniting two or more compound denominate numbers into one equivalent number.

The process of adding is the same as adding simple numbers, except that the scale of increase and decrease, by passing from one denomination to another, varies with every system of measurement and with almost every denomination of each system.

We shall therefore not discuss the subject at length, but shall proceed to illustrate the process by which results in compound addition are determined.

1. Add 5 bu. 3 pks. 7 qts. 1 pt., 8 bu. 3 qts. 1 pt., and 3 bu. 3 pks. 1 pint.

OPERATION.

bu.	pks.	qts.	pt.
5	3	7	1
8	0	3	1
3	3	0	1
17	3	3	1 Ans.

Explanation.—In all problems of this kind, we first write the numbers so that units of the same denomination stand in the same column, and begin with the lowest denomination to add.

3 pts., ÷ 2, No. of pts. in a qt., = 1 qt. and 1 pt.

11 qts. ÷ 8, No. of qts. in a pk., = 1 pk. and 3 qts.

7 pks. ÷ 4, No. of pks. in a bu., = 1 bu. and 3 pks.

Accordingly, we here first add the pints and find the sum to be 3 pts., which we divide by 2, (since 2 pts. = 1 qt.) and obtain 1 qt. and 1 pt. remainder. The 1 pt. we write under the column of pts., and carry or add the 1 qt. to the column of

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qts., which added gives 11 qts. and which we divide by 8, (since 8 qts. = 1 pk.) and btain 1 pk. and 3 qts. remainder. The 3 qts. we write under the column of qts., and add the 1 pk. to the column of pks., which added gives a sum of 7 pks. This we divide by 4 (since 4 pks. = 1 bu.) and btain 1 bu. and 3 pks. The 3 pks. we write under the column of pks. and add the 1 bu. to the column of bushels, which gives a sum of 17 bu., which we write under the column of bushels.

This completes the operation and produces 17 bu., 3 pks., 3 qts., 1 pt. as the complete result.

2. Add 15£. 10s. 9d.,   8£. 9s. 7d., 1£. 12s. 10d., and 1£. 18s. 6d.   Ans. 27£. 11s. 8d.

3. Add 12 yds. 3 qrs. 2 na. 2 in., 5 yds. 2 qrs. 1 na. 1¾ in.,   2 qrs. 1 na. 1½ in., 8 yds. 2 in.
                    Ans. 27 yds. 0 qrs. 3 na. ½ in.

NOTE.—In working the above problem, the student must remember, when dividing the inches, that whenever the dividend is *fourths,* the remainder is also *fourths.* The remainder is always like the dividend.

4. Add 10 lbs. 1 ℥. 6 ʒ. 0 Ɔ. 10 grs.,   10 ℥. 2 ʒ. 2 Ɔ. 16 grs., and 13 lbs. 2 ℥. 7 ʒ. 3 Ɔ. 12 grs.
                    Ans. 24 lbs. 3 ℥. 0 ʒ. 2 Ɔ. 18 grs.

5. Add 3 yds. 2 ft., 9 in., 4 rds. 2 yds. 1 ft. 11 in., and 5 rds. 4 yds 2 ft. 8 in.
                    Ans. 11 rds. 0 yd. 1 ft. 4 in.

6. Add 21 gals. 3 qts. 1 pt. 3 gi., 32 gals. 1 qt. 0 pt. 2 gi., and 47 gals. 2 qts. 1 pt. 1 gi.
                    Ans. 101 gals. 3 qts. 1 pt. 2 gi.

7. Add 18 cu. yds. 13 cu. ft. 1431 cu. in., 16 cu. yds. 12 cu. ft. 931 cu. in., and 30 cu. yds. 20 cu. ft. 1246 cu. in.

Ans. 65 cu. yds. 20 cu. ft. 152 cu. in.

8. Add 9 mi. 67 ch. 3 rds. 17 l., 17 mi. 61 ch. 1 rd. 12 l., and 16 mi. 42 ch. 2 rd. 14 l.

Ans. 44 mi. 11 ch. 3 rd. 18 l.

9. Add $\frac{3}{4}$ wk. $\frac{2}{5}$ da. $\frac{5}{8}$ hr. and $\frac{2}{3}$ min.

Ans. 5 da. 16 hrs. 14 min. 10 sec.

OPERATION INDICATED.

		da.	hr.	min.	sec.
$\frac{3}{4}$ wk.	=	5	6	0	0
$\frac{2}{5}$ da.	=		9	36	0
$\frac{5}{8}$ hr.	=			37	30
$\frac{2}{3}$ min.	=				40
		5,	16,	14,	10, Ans.
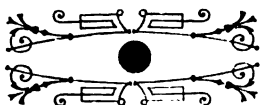
Subtraction of Compound Denominate Numbers.

445. Subtraction of Compound Denominate Numbers is the process of decreasing one compound denominate number by another of the same system of measurement.

As in addition, the scale of increase and decrease varies, otherwise the work is the same as in subtraction of simple numbers.

1. From 18£. 4s. 7d. 3f. subtract 11£. 9s. 11d. 2 f.

OPERATION.

£.	s.	d.	f.
18.	4.	7.	3.
11.	9.	11.	2.
6.	14.	8.	1., Ans.

Explanation.— In all problems of this kind, we first write the numbers so that units of the same denomination stand in the same column, and begin with the lowest denomination to subtract. Accordingly, we here commence with the farthings and say 2 far. from 3 far. leaves 1 far. which we write under the column of farthings. We now come to the column of pence and observe that 11d. cannot be taken from 7d., because the 11d. is the greater number; we, therefore, according to the law *that the difference between two numbers is the same as the difference between the two numbers when equally increased,* as demonstrated in Art. 94, add 12d. to the 7d., making 19d. From this we subtract

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the 11d. and have 8d. remainder, which we write under the column of pence. Then, as we added 12d. to the minuend, we, now, to compensate therefor, according to the above law, add 1s., the equivalent of 12d., to the column of shillings in the subtrahend and thus we have 10s. to subtract from 4s. which we cannot do. Hence, for reasons above given, we add 20s. to the 4s. which make 24s., from which we take 10s. and have 14s. remainder, which we write under the column of shillings. We now add 1£ the equivalent of 20s. to the 11£ making 12£ , which we take from 18£ and have a remainder of 6£ which we write under the column of pounds. This completes the operation.

It will be observed that when we added the 12d. to the column of pence, and the 20s. to the column of shillings, that, in each case, we added such a number of that order as made *one* of the next higher order. This must always be done in simple numbers or in any of the systems of compound numbers, when the subtrahend figure or denomination exceeds the minuend figure of like denomination.

2.   From 3 mi. 5 fur. 30 rds. take 1 mi. 7 fur. 32 rods.   Ans. 1 mi. 5 fur. 38 rds.

3.   From 7 lbs. 3 oz. 12 pwt. 20 grs. take 3 lbs. 5 oz. 10 pwt. 15 grs.
                    Ans. 3 lbs. 10 oz. 2 pwt. 5 grs.

4.   From 30° 25′ 32″ take 25° 34′ 35″.
                    Ans. 4° 50′ 57″.

5.   From 4 lbs. 12 oz. 13 drs. take 2 lbs. 9 oz. 15 drs.   Ans. 2 lbs. 2 oz. 14 drs.

6.   From 13 hrs. 24 min. 7 sec. take 10 hrs. 30 min. 12 sec.   Ans. 2 hrs. 53 min. 55 sec.

7.   From 3 sq. rds. 5 sq. ft. 95 sq. in. take 1 sq. rd. 10 sq. ft. 100 sq. in.

<div align="center">Ans. 1 sq. rd. 267 sq. ft. 31 sq. in.</div>

SOLUTION.

sq. rd.	sq. ft.	sq. in.
3	5	95
1	10	100
1	$266\frac{1}{4}$	139
	$\frac{1}{4} =$	36
1	267	31

Explanation.—By proceeding as in the explanation above given, we obtain 1 sq. rd. $266\frac{1}{4}$ sq. ft. 139 sq. in. as a result; but it is not proper to have a fractional expression in any but the lowest denomination of a compound denominate number, so we proceed to reduce $\frac{1}{4}$ of a sq. ft. to sq. in ; $\frac{1}{4}$ of 144 sq. in. = 36 sq. in.; 139 sq. in. + 36 sq. in. =175 sq. in., or 1 sq. ft. and 31 sq. in. So we write 31 sq. in. and add 1 sq. ft. to 266 sq. ft. and find the answer to be 1 sq. rd. 267 sq ft. 31 sq. in.

8.   From $\frac{3}{5}$ of a hogshead, subtract $\frac{2}{3}$ of a gallon.

<div align="center">Ans. 37 gal. 0 qt. 1 pt. $0\frac{4}{15}$ gi.</div>

OPERATION INDICATED.

$\frac{3}{5}$ of a hhd.	=	37 gal.	3 qt.	0 pt.	$1\frac{3}{5}$ gi.
$\frac{2}{3}$ of a gal.	=		2 qt.	1 pt.	$1\frac{1}{3}$ gi.
		37 gal.	0 qt.	1 pt.	$0\frac{4}{15}$ gi.

9.   From $2\frac{3}{5}$ bu. subtract $\frac{7}{9}$ of a peck.

<div align="center">Ans. 2 bu. 1 pk. 4 qt. $1\frac{13}{45}$ pt.</div>
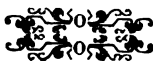
446. Multiplication of Compound Denominate Numbers is the process of determining the product of two numbers, when the number to be multiplied is a compound denominate number.

Compound multiplication differs from multiplication of simple numbers, in that when the product in any denomination equals or exceeds one of the next higher denomination in the same system of measurement, said product must by a process of division be reduced to that next higher denomination, before the number in it can be multiplied.

1. Multiply 4 gals. 3 qts. 1 pt. 2 gi. by 9.

OPERATION.

gal.	qts.	pt.	gi.
4.	3.	1.	2
			9
44.	1	1	2

18 gi. ÷ 4 = 4 pts. 2 gi.
13 pts. ÷ 2 = 6 qts. 1 pt.
33 qts. ÷ 4 = 8 gals. 1 qt.

Explanation.—In all problems of this kind, we write the multiplier under the lowest denomination of the multiplicand and multiply. We first say 9 times 2 gi. are 18 gi. which, as shown in the operation, equals 4 pts. and 2 gi. The 2 gi. we write under the gills and reserve the 4 pts. to add to the product of pints. We then say 9 times 1 pt. are 9 pts. and 4 pts. added make 13 pts. which reduced to the next higher denomination equals 6 qts. and 1 pt. The 1 pint we write under the unit or denomination pints, and reserve the 6 qts. to add to the product of quarts. Then we say 9 times 3 qts. are 27 qts. and 6 qts added make 33 qts. equal to 8 gals. and 1 qt. The 1 qt. we write under the quarts, and reserve the 8 gals. to add to the product of gallons. Lastly we say 9 times 4 gals. are 36 gals. plus the 8 gals. reserved, are 44 gals. which we write under the denomination of gallons. This gives 44 gals. 1 qt. 1 pt. 2 gi. for the entire product.

NOTE.—The multiplier must always be considered an abstract number, Art. 108.

2.   Multiply 7 Cong. 3 O. 15 fl. ℥ by 6.

Ans. 44 Cong. 7 O. 10 fl. ℥ .

3.   Multiply 27 cords, 34 cu. ft. by 12.

Ans. 327 cords, 24 cu. ft.

4.   Multiply 3 mi. 7 fur. 20 rds. by 7.

Ans. 27 mi. 4 fur. 20 rds.

5.   Multiply 7 hrs. 30 mi. 24 sec. by 5.

Ans. 1 da. 13 hrs. 32 min.

6.   Multiply 3 francs, 7 dec. 3 cen. 4 mil. by 5.

Ans. 18 fr. 6 dec. 7 cen.

7.   Multiply 4£. 3s. 4d. by 12.

Ans. £50.

8.   In 5 bbls. of pecans, each containing 2 bu. 3 pks. 5 qts. 1 pt., how many bushels?

Ans. 14 bu. 2 pks. 3 qts. 1 pt.

Division of Compound Denominate Numbers.

447. Division of Compound Denominate Numbers is the process of determining any required part of a compound denominate number.

It is the reverse of compound multiplication, and we first divide the highest denomination in the number and if any fraction occurs in the result, it must be reduced to the next lower denomination before the number in that denomination is divided.

1. Divide 32 lbs. 12 oz. 12 drs. by 5.

<table>
<tr><td colspan="4">OPERATION.</td></tr>
<tr><td></td><td>lbs.</td><td>oz.</td><td>dr.</td></tr>
<tr><td>5)</td><td>32</td><td>12</td><td>12</td></tr>
<tr><td></td><td>6</td><td>8</td><td>15⅕ Ans.</td></tr>
</table>

Explanation.— In all problems of this kind, we write the quantity to be divided in the order of its denomination and place the divisor on the left, as in division of simple numbers.

32 lbs. ÷ 5 = 6 lbs. and 2 lbs. remainder.

2 lbs. × 16 = 32 oz. + 12 oz. = 44 oz.
44 oz. ÷ 5 = 8 oz. and 4 oz. remainder.

4 oz. × 16 = 64 drs. + 12 drs. = 76 drs.
76 drs. ÷ 5 = 15 drs. and 1 dr. remainder.

1 dr. ÷ 5 = ⅕ dr.

Having thus stated the problem, we first divide the 32 lbs. by 5 and obtain a quotient of 6 lbs. with 2 lbs. remainder. We write the 6 lbs. in the quotient line under the pounds, and reduce the 2 lbs. remainder to ounces = 32 oz. + 12 oz. = 44 oz. which we divide by 5 and obtain 8 oz. with a remainder of 4 oz. The 8 oz. we write in the quotient line, under the ounces, and reduce the 4 oz. to drams, = 64 drs. + 12 drs. = 76 drs. which ÷ 5 = 15 drs. with 1 dr. remainder. or 15⅕ drs.; this is written in the quotient line under the drams, and completes the operation.

2. A box contains 8 bu. 3 pks. 5 qts. How many smaller boxes, each holding 1 pk. 1 qt. 1 pt., can be filled from the larger box?

Ans. 30 boxes.

OPERATION INDICATED.

8 bu. 3 pks. 5 qts. = 570 pts.

1 pk. 1 qt. 1 pt. = 19 pts.

570 pts. ÷ 19 = 30 boxes, Ans.

Explanation.—In all problems of this kind, we first reduce both dividend and divisor to the same denomination and then divide as in simple numbers.

3. Divide 37 mi. 3 fur. 4 rds. by 4.

Ans. 9 mi. 2 fur. 31 rds.

4. Divide 6° 24′ 32″ by 7.   Ans. 54′ 56″.

5. Divide 24 lbs. 3 oz. 12 pwt. 12 grs. by 12.

Ans. 2 lbs. 0 oz. 6 pwt. 1 gr.

6. Divide 25 yds. 3 qrs. 2 nails by 13.

Ans. 1 yd. 3 qrs. 3 nails, $1\frac{4}{5}\frac{1}{2}$ in.

7. Divide 3 cu. yds. 19 cu. ft. 996 cu. in. by 12.

Ans. 8 cu. ft. 659 cu. in.

8. Divide 3 mi. 75 ch. 21 l. by 7.

Ans. 45 ch. 3 l.

9. What is $\frac{3}{8}$ of 2 m. 3 fur. 1 yd. 2 ft.

Ans. 7 fur. 5 rds. 1 ft. $10\frac{1}{2}$ in.

ᚠISCELLANEOUS PROBLEMS,

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448. 1. Paid $2 for sawing 3 cds. 26 cu. ft. of wood. How much could be sawed for $1, at the same rate ? Ans. 1 cd. 77 cu. ft.

OPERATION INDICATED.

$$\begin{array}{c|cc} & \text{cd.} & \text{cu. ft.} \\ 2 & 3 & 26 \\ \hline & 1 & 77 \text{ Ans.} \end{array}$$

2. Henry traded 4 rubber balls for 3 pks. 4 qts. 1 pt. of pecans. How many did he get for each ball? Ans. 7 qts. $\frac{1}{4}$ pt.

3. An English weaver sold $7\frac{1}{2}$ webs of cloth of an equal number of yards in each web at 30£. 8s. 3d. 2 far. a web. How much did he receive for all.

OPERATION INDICATED.

£.	s.	d.	far.		£.	s.	d.	far.
30	8	3	2		30	8	3	2
			$7\frac{1}{2}$	or,	$7\frac{1}{2}$			
228	2	$1\frac{1}{2}$	3	$7\frac{1}{2}$ times 30£ =225	0	0	0
			2	$7\frac{1}{2}$ times 8d. =		60	0	0
				$7\frac{1}{2}$ times 3d. =			22	2
228	2	2	1 Ans.	$7\frac{1}{2}$ times 2 far. =				15

Ans. 228 2 2 1

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4. 38 lbs. 12 oz. 15 drs. of butter cost $7¼. How much can be bought for $1.

Ans. 5 lbs. 4 oz. 10$\frac{17}{22}$ drs.

OPERATION INDICATED.

℔.	oz.	dr.	
38	12	15 ÷ 7⅓ or $\frac{22}{3}$.	
		3	
22)116	6	13	
5	4	10$\frac{17}{22}$ Ans.	

Explanation and Reason.— In this problem, we have the quantity that $7¼=$\frac{22}{3}$ will buy, and we reason as follows: Since $\frac{22}{3}$ will buy 38 lbs. 12 oz. 15 drs., ⅓ of a dollar will buy the 22d part and ⅓, or $1 will buy 3 times as much. In the operation of all problems of this kind, it is best to first multiply the dividend by the denominator of the divisor, and then divide the product by the numerator of the divisor, as shown in the above partially worked operation.

5. A farmer sowed 3 bu. 3 pks. 6 qts. of oats on each of 3.5 acres. How much oats did he sow on all? Ans. 13 bu. 3 pks. 1 qt.

6. Divide 26 tons, 13 cwt. 3 qrs. 12 lbs. of coal equally among 12 families, and find what each will have to pay at $12 per ton. Ans. $26.6935.

7. How many days at $1.25 per day will it take a man to earn 200 lbs. 12 oz. of beef at 6 cents a pound? Ans. 9.6 + days.

8. A merchant exchanged 3 yds. 2 qrs. of broad cloth worth $4 per yard, for 26 gals. 3 qts. 1 pt. of molasses. What was the molasses worth per gallon? Ans. 52$\frac{4}{43}$ cents.

9. At a certain distance 20 lbs. of steam expanded the mercury in a Fahrenheit thermometer 3.5 degrees. How many lbs. at the same distance would be required to expand it from zero to the freezing point? Ans. 182$\frac{4}{7}$ lbs.

10. From 17 lbs. 5 oz. take 2 lbs. 9 oz.

Ans. 14 lbs. 12 oz.

11. A grocer has 8 jars of butter, each weighing 14 lbs. 7 oz. How many pounds in all, and what is it worth at $32\frac{1}{2}$¢ per pound?   Ans. 37.53\frac{3}{4}$.

12. What will 4 bu. 3 pks. 1 qt. 1 pt. cost at 10¢ per pint?   Ans. $30.70.

13. What cost 4 bu. 3 pks. 1 qt. 1 pt. at $6 per bushel.   Ans. 28.78\frac{1}{8}$.

14. What cost 4 bu. 3 pks. 1 qt. 1 pt. at 25¢ per quart?   Ans. 38.37\frac{1}{2}$.

15. What cost 4 bu. 3 pks. 1 qt. 1 pt. at $1.75 per peck?   Ans. 33.57\frac{13}{16}$.

16. What cost 1521 pounds of corn at 84¢ per bushel, and how many bushels are there?
                    Ans. 22.81\frac{1}{2}$; 27 bu. 9 lbs.

17. What cost 2842 bu. 16 lbs. of wheat at $1.22 per bushel?   Ans. 3467.56\frac{8}{15}$.

18. What cost 342506 lbs. of wheat at $9.80 per imperial quarter?   Ans. 6992.83\frac{1}{12}$.

19. Corn is 60¢ per bu. What is it worth per cental?   Ans. 1.07\frac{1}{7}$.

20. Wheat is $2.90 per cental. What is it worth per bushel?   Ans. $1.74.

21. Cloth is $1.80 per yard. What is it worth per metre?   Ans. $1.968+.

22. Cloth is $3.937 per metre. What is it worth per yard?   Ans. $3.60.

23. Cloth cost in ¡Mexico $2 per vara. What is it worth per yard?   Ans. $2.16.

Note.—A vara is 33.3864, practically 33.38 inches

24. Tea is worth $.75 per pound. How much will you sell for 20¢ ?   Ans. $4\frac{4}{15}$ oz.

25. Butter is worth 35¢ per pound. How much can you buy for 10¢.   Ans. $4\frac{4}{7}$ ounces.

26. Sell $4\frac{1}{2}$ inches of silk at $2.75 per yard, and state the amount.   ·   Ans. $.34\frac{3}{8}$.

27. A lady wishes to buy 40¢ worth of silk which is $3.00 per yard. How much will you sell her?   Ans. $4\frac{4}{5}$ in.

28. Maple syrup is worth $1.92 a gallon. How much can be bought for 25¢ ?
      Ans. $4\frac{1}{6}$ gills, or 1 pt. $\frac{1}{6}$ gi.

29. A clerk commenced work on the 17th of January, and discontinued April 1st. He received $65 per month. How much was due him counting January 17th, but not April 1st ?
      Ans. 160.33\frac{1}{3}$.

NOTE.—In computing salaries and rents, all months are considered as containing 30 days.

30. What cost 4 tons 1420 pounds of hay, at $16.25 per ton?   Ans. 76.53\frac{3}{4}$.

31. What cost 376 pounds of hay at $15 per ton ?
      Ans. $2.82.

32. What cost 1265 pounds of bran at 80¢ per cwt.?   Ans. $10.12.

To Find the Interval of Time Between Two Dates.

449. 1. How many years, months, days, hours. and minutes from 4:20 o'clock P. M. June 10, 1881, to 9:15 o'clock A. M. August 14, 1886, not allowing for leap years and counting 30 days to the month?

OPERATION.

yr.	mo.	d.	hr.	min.
1886	8	14	9	15
1881	6	10	16	20

5 yrs. 2 mos. 3 ds. 16 hrs. 55 min. Ans.

Explanation.—In all problems of this kind, we write the later date first, since it expresses the greater period of time, and the earlier date beneath. Then subtract as in compound denominate numbers.

NOTE.—The months are numbered from January, and the hours are counted from 12 o'clock at night.

SECOND OPERATION.

The time from June 10, 1881 to June 10, 1886	=	5 yrs.
" " " June 10, 1886 to Aug. 10, 1886	=	2 mos.
" " " 4:20 P. M. Aug. 10, 1886 to 4:20 P. M. Aug. 13, 1886	=	3 ds.
" " " 4:20 P. M. Aug. 13, 1886 to 8:20 A. M. Aug. 14, 1886	=	16 hrs.
" " " 8:20 A. M. Aug. 14, 1886 to 9:15 A. M. Aug. 14, 1886	=	55 min.

NOTE.—While both of the above operations are correct according to the conditions of the problem, and conform to the usual method of finding the time between dates, neither operation gives an accurate result, for the reason that some years and some months contain more days than other years and other months. The only accurate way is when the time is less than one year, to count the exact number of days in each month of intervening time, or refer to a time table. When the time is more than one year, find the days for the months, as above, and allow 365 days for common, and 366 for leap years.

2. The Declaration of Independence was ratified July 4, 1776; the battle of New Orleans was fought Jan'y 8, 1815. What is the time between these two dates, by the usual method? Ans. 38 yrs. 6 mos. 4 ds.

3. Jamestown, Va., was first settled May 23, 1607, and the Pilgrims landed at Plymouth December 22, 1620. What time intervened, by the usual method?   Ans. 13 yrs. 6 mos. 29 ds.

4. A note was dated January 10, 1884, and was made payable 2 years after date. When did it become due; how many days did it run, by the usual method; and how many days counting actual time; no allowance to be made for days of grace?

Ans. January 10, 1886, it matured;
720 ds. by the usual method;
731 ds. actual time.

5. A note is drawn Oct. 15, 1885, and made payable 3 months after date. When does it mature, allowing 3 days of grace according to business custom, and how many days does it run, actual time?   Ans. Matures Jan'y 18, 1886;
Runs 95 days.

6. A note is drawn Oct. 15, 1885, and made payable 90 days after date. When does it mature, allowing the customary 3 days of grace, and how many days does it run, actual time?

Ans. Matures Jan'y 16, 1886.
Runs 93 days.

Note.—It is the custom, when maturing commercial instruments, to be governed strictly by the terms expressed therein. When they are made payable in years or months, they are matured in years or months, counting them as they run from the date of the instrument. When they are made payable in days, they are matured in days, the actual number of days in each month of the intervening time being counted.

The day that a note or other instrument is dated is not counted as one of the days which it has to run. The day it matures is however counted.

When discounting notes, bills of exchange, etc., the actual number of days which the instrument has to run, is counted, to compute the interest, whether it is drawn in years, months, or days.

For an elaborate elucidation of all kinds of discount operations, see Soulé's Philosophic Practical Mathematics.

450. ‡IME TABLE

Showing the Time in Days from January 1 to any Day in the Year, inclusive of the First and Last Day.

In leap years when the time embraces February, 1 day must be added to the result of the table.

Day	Jan'ry	Feb'ry	March	April	May	June	July	August	Sep'ber	Octob'r	Nov'er	Dec'ber
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	841
8	8	39	67	98	128	159	189	220	251	281	812	842
9	9	40	68	99	129	160	190	221	252	282	313	843
10	10	41	69	100	130	161	191	222	253	283	314	844
11	11	42	70	101	131	162	192	223	254	284	315	845
12	12	43	71	102	132	163	193	224	255	285	316	846
13	13	44	72	103	133	164	194	225	256	286	317	847
14	14	45	73	104	134	165	195	226	257	287	318	848
15	15	46	74	105	135	166	196	227	258	288	319	849
16	16	47	75	106	136	167	197	228	259	289	320	850
17	17	48	76	107	137	168	198	229	260	290	321	851
18	18	49	77	108	138	169	199	230	261	291	322	852
19	19	50	78	109	139	170	200	231	262	292	328	853
20	20	51	79	110	140	171	201	232	263	293	824	854
21	21	52	80	111	141	172	202	233	264	294	325	855
22	22	53	81	112	142	173	203	234	265	295	826	856
23	23	54	82	113	143	174	204	235	266	296	327	857
24	24	55	83	114	144	175	205	236	267	297	828	858
25	25	56	84	115	145	176	206	237	268	298	329	859
26	26	57	85	116	146	177	207	238	269	299	330	860
27	27	58	86	117	147	178	208	239	270	300	831	861
28	28	59	87	118	148	179	209	240	271	301	832	862
29	29		88	119	149	180	210	241	272	302	833	868
30	30		89	120	150	181	211	242	273	303	834	864
31	31		90		151		212	243		304		865

TO DETERMINE THE DAY OF THE WEEK ON WHICH AN EVENT DID OR MAY OCCUR.

451. A convenient method of determining immediately what day of the week any date transpired, or will transpire, from the commencement of the Christian era, for the term of three thousand years.

452. The following table shows the ratio to be added for each month:

TABLE OF MONTHS.

January, ratio is	3	July, ratio is	2
February, "	6	August, "	5
March, "	6	September, "	1
April, "	2	October, "	3
May, "	4	November, "	6
June, "	0	December, "	1

NOTE.—In leap years, the ratio of January is 2 and the ratio of February is 5. There is no change in the ratios of the other months.

453. The following table shows the ratios to be added for each century of the Christian era:

TABLE OF CENTURIES.

200, 900, 1800, 2200, 3000, the ratio is	0
300, 1000, the ratio is	6
400, 1100, 1900, 2300, 2700, the ratio is	5
500, 1200, 1600, 2000, 2400, 2800, the ratio is	4
600, 1300, the ratio is	3
700, 1400, 1700, 2100, 2500, 2900, the ratio is	2
100, 800, 1500, the ratio is	1

454. DIRECTIONS FOR THE OPERATION.

1. *Add to the given year, (omitting the century figures) one-fourth part of itself, rejecting the fractions, if any.*

2. *To this sum add,* 1°, *the day of the given month;* 2°, *add the ratio of the month, as per table of months;* 3°, *add the ratio of the century as per table of centuries.*

3. *Divide this sum by* 7. *The remainder is the day of the week counting Sunday as the first day, Monday as the second, etc.*

PROBLEMS.

1. The battle of Shiloh was commenced on the 6th of April, 1862. What was the day of the week?

<div align="right">Ans. Sunday.</div>

OPERATION.

The given year, omitting the centuries, is............	62
¼ of same, rejecting fractions, is	15
The day of the month was the	6
The ratio of April is................................	2
The ratio of 1800 is.................................	0

This sum, divided by 7, gives — 7) 85

12 and 1 remainder .... 12+1
The 1 remainder signifies the first day of the week—SUNDAY.

2. The Declaration of Independence was signed July 4, 1776. What was the day of the week?

<div align="right">Ans. Thursday.</div>

OPERATION.

Given year is.......	76
¼ of same is........	19
Day of month is....	4
Ratio of July is....	2
Ratio of 1700 is	2

Divide by 7) 103

14+5 remainder—THURSDAY.

3. Gen. R. E. Lee was born June 19, 1807. What was the day of the week?   Ans. Friday.

4. Martin Luther was born Nov. 10, 1483. What was the day of the week?   Ans. Monday.

5. What day of the week will January 1 occur in 2000?   Ans. Saturday.

6. On what day of the week were you born, and if you live to be 150 years old, as we wish you may, on what day of the week will you die?

DIFFERENCE OF LATITUDE.

455. Latitude is the distance in degrees, minutes, and seconds, of any place on the globe, North or South of the equator.

Latitude is reckoned from the equator to each pole of the earth, and like arcs of all circles, is measured in degrees, minutes, and seconds, and can never be greater than a quadrant, or 90 degrees. The earth not being a perfect sphere, but oblate, or flattened at the poles, the degrees vary slightly in length toward the poles.

The difference of latitude between two places is found by subtraction or addition, as in compound denominate numbers.

1. The latitude of Washington City is 38° 53′ 39″ North, and that of New Orleans is 29° 56′ 59″ North. Required the difference of latitude.

<div align="center">

OPERATION.

Lat. of Washington = 38° 53′ 39″ N.
Lat. of New Orleans =   29° 56′ 59″ N.
_____
Dif. of latitude   =   8° 56′ 40″ Ans.

</div>

Explanation.— Since both places are on the same side of the equator, that is, in North latitude, we subtract the lesser latitude from the greater. When the latitude of one place is North, and that of the other South of the equator, we add the two latitudes together, and their sum will be their difference of latitude.

2. The latitude of Mobile is 30° 41′ 26″ North, and that of Quebec is 46° 48′ 17″ North. What is their difference?   Ans. 16° 6′ 51″.

3. Philadelphia is in latitude 39° 56′ 53″ North, and Rio de Janeiro 22° 54′ 24″ South. What is their difference?   Ans. 62° 51′ 17″.

<div align="center">(318)</div>

DIFFERENCE OF LONGITUDE.

456. The Longitude of a place is its distance in degrees, minutes, and seconds, East or West, from a given meridian, measured on the equator.

A degree of longitude on the equator is 69.1638 statute miles, but since all meridians are drawn through the poles and meet in a point, they gradually converge as we advance from the equator, and vary in length with each degree of latitude, until they meet in the poles, and the longitude becomes nothing.

457. A Meridian is an imaginary circular line on the surface of the earth, passing through the poles and any given place.

The meridian from which longitude is reckoned is called the *first meridian*, or **standard meridian**, and is marked 0°. All places east of the first or standard meridian, within 180°, are in east longitude; and all places west of the first or standard meridian, within 180°, are in west longitude.

The English reckon longitude from the meridian of Greenwich; the French from that of Paris. The government of the United States usually reckon longitude from the English standard meridian, Greenwich. In American maps, the meridian of Greenwich is printed at the top and the meridian of Washington, the capital of the U. S., is printed at the bottom.

The difference of longitude between two places is found like the difference of latitude, by subtraction or addition, as in compound denominate numbers.

1   The longitude of Galveston is 94° 47′ 26″ West; of Liverpool 3° 4′ 16″ West.   What is their difference?   Ans. 91° 43′ 10″.

OPERATION.

Long. of Galveston   =   94° 47′ 26″ W.
Long. of Liverpool   =   3° 4′ 16″ W.
───────────────
Dif. of Longitude   =   91° 43′ 10″ Ans.

Explanation.—Since both places are in West longitude, we subtract the lesser from the greater. When one place is in West and the other in East longitude, we add the longitudes of the two places together, and if their sum is more than 180 degrees we subtract it from 360 degrees, because the shortest distance between the places would then be the other way around the world.

2.   Cape Flattery, W. T., is in longitude 124° 44′ 30″ West.   Hong Kong, Ch., is in longitude 114° 9′ 32″ East.   What is their difference?
                                   Ans. 121° 5′ 58″.

OPERATION.

Long. of Cape Flattery = 124° 44′ 30″ W. | 360° 0′ 0″
Long. of Hong Kong   = 114° 9′ 32″ E. | 238° 54′ 2″
─────────────────────────────
                          238° 54′ 2″ | 121° 5′ 58″

3.   The longitude of Calcutta is 88° 20′ 11″ East, of Trieste, 13° 46′ East.   What is their difference?
                                   Ans. 74° 34′ 11″.

4.   The longitude of Havana is 82° 21′ 17″ West, and of Antwerp 4° 24′ 44″ East.   What is their difference?   Ans. 86° 46′ 1″.

5.   St. Petersburg is in longitude 30° 19′ 22″ East.   Cedar Keys is in longitude 83° 1′ 57″ West.   Find their difference,   Ans. 113° 21′ 19″.

LONGITUDE AND TIME.

458. The circumference of the earth, being a great circle, is divided into 360 equal parts called *Degrees of Longitude*.

The earth revolves on its axis, from *west* to *east*, once in 24 hours—which gives the sun the appearance of passing around the earth from *east* to *west*.

Now, since the earth revolves once in 24 hours, all parts of its surface or circumference—the 360° of longitude—pass under the sun during that space of time. Hence, since 360° are passed under the sun in 24 hours, $\frac{1}{24}$ part of 360°, or 15° of longitude are passed in 1 hour. Since 15° are passed in 1 hour, or 60 minutes, $\frac{1}{60}$ part of 15°, or 15′ of longitude are passed in 1 minute. Since 15′ of longitude are passed in 1 minute, or 60 seconds, $\frac{1}{60}$ part of 15′, or 15″ of longitude are passed in 1 second.

Then since 15° of long.=1 hour of time,
      1°   " $=\frac{1}{15}$ of 1 hr. (60 min.)=4 min. of time.
Then since 15′ of long.=1 minute of time,
      1′   " $=\frac{1}{15}$ of 1 min. (60 sec.)=4 sec. of time.
Then since 15″ of long.=1 second of time,
      1″   " $=\frac{1}{15}$ of a second of time;
Or, since   1′   "   =4 seconds of time,
      1″   " $=\frac{1}{60}$ of 4 sec.$=\frac{4}{60}$, or $\frac{1}{15}$ of a second of time.

459.   From the foregoing, we deduce the follow-ing:

COMPARATIVE TABLE OF LONGITUDE AND TIME

360° long. make a difference of 24 hours of time.
15° "   "   "   "   1   "   "
15′ "   "   "   "   · 1 minute   "
15″ "   "   "   "   1 second   "
1°   "   "   "   "   4 minutes   "
1′   "   "   "   "   4 seconds   "
1″   "   "   "   " $\frac{1}{15}$ second   "

460.   Longitude and Time give rise to two classes of problems, as follows:

1.   To reduce time to longitude, or to find the difference of longitude between two places, when the difference of time is known. ·

2.   To reduce longitude to time, or to find the difference of time between two places, when the difference of longitude is known.

PROBLEMS UNDER THE FIRST CLASS.

461.   1.   Reduce 11 hrs. 20 min. 40 sec of time to longitude.

OPERATION.

11 hrs.   20 min.   40 sec.
                          15
—————————————————————
170°   10′   0″ Ans.

Explanation. — Since according to the foregoing elucidations, 1 hr. = 15° of longitude, 1 min. = 15′ of longitude, and 1 second = 15″ of longitude there are 15 times as many °, ′, and ″ of longitude as there are hrs., min., and sec. of time. Hence in all problems of this kind, we multiply the different units of time by 15, as in multiplication of compound denominate numbers, and thus convert them into units of longitude.

2. The difference of time between two places is 3 hrs. 15 min. 24 sec. What is their difference of longitude? Ans. 48° 51′ 0″.

3. The difference of time between New York and Chicago is 54 min. 19 sec. What is the difference of longitude? Ans. 13° 34′ 45″.

4. When it is 11 o'clock 16 minutes $1\frac{4}{15}$ seconds A. M. at Boston, it is 10 o'clock A. M. at New Orleans. Find the difference of longitude.

OPERATION INDICATED.

Time at Boston	11 hrs.	16 min.	$1\frac{4}{15}$ sec.
Time at New Orleans	10 hrs.	0 min.	0 sec.

Difference of time	1 hr.	16 min.	$1\frac{4}{15}$ sec.
			15

19°	0′	19″.

5. The longitude of Rome, Italy, is 12° 27′ E.; the difference of time between Rome and Mobile, Ala., is 6 hrs. 41 min. $57\frac{1}{15}$ sec. What is the longitude of Mobile, West? Ans. 88° 2′ 28″ W.

OPERATION INDICATED.

6 hrs. 41 min. $57\frac{1}{15}$ sec.=dif. of time.
15

100°	29′	28″ = dif. of longitude.
12°	27′	0″ = longitude of Rome, E.
88°	2′	28″ = longitude of Mobile, W.

NOTE.—When the Dif. of Long. given is that of two places which are in opposite longitudes, and the Long. of one of them is given to find that of the other, we subtract the longitude of the given place from the Dif. of Long. and thereby find the longitude of the other.

This is done because the Dif. of Long. between the two places is equal to their sum (one being E. and the other W.) and consequently, when the Long. of one is given, that of the other is found by subtraction. Should the Dif. have been that of two places in the same kind of Long., we would then have added to find the longitude of the other.

6. When it is 12 M. in New York, it is 18 min. 2⅘ sec. past 11 o'clock at Cincinnati. What is their difference of longitude? Ans. 10° 29′ 21″.

OPERATION INDICATED.

12 hrs.   0 min.   0 sec.
11 hrs. 18 min. 2⅘ sec.
_____

41 min. 57⅖ sec. × 15 = 10° 29′ 21″.

PROBLEMS OF THE SECOND CLASS.

462. 1. Reduce 30° 42′ 50″ of longitude to time.

OPERATION.

15) 30°   42′   50″

2 hrs. 2 min. 51⅓ sec. Ans.

Explanation—As shown by the foregoing elucidated table of longitude and time equivalents, 15° of longitude = 1 hr. of time, 15′ of longitude = 1 min. of time, and 15″ of longitude = 1 sec. of time; therefore, there will be ₁⁄₁₅ as many *hours, minutes,* and *seconds* of *time* as there are *degrees, minutes,* and *seconds* of *longitude,* and hence we divide the °, ′, and ″ by 15, as in division of compound denominate numbers, and thus reduce longitude to time.

2. The longitude of Paris, France, is 2° 20′ 15″ E., and of Berlin, Germany, 13° 23′ 44″ E. What is the difference of time? Ans. 44 min. 13¹⁴⁄₁₅ sec.

OPERATION.

13°   23′   44″ = longitude of Berlin, E.
2°   20′   15″ =   "   " Paris, E.
_____

15) 11°   3′   29″ = difference of longitude.

44 min. 13¹⁴⁄₁₅ sec. = difference of time.

3. The longitude of Bombay, India, is 72° 54′ East; of St. Louis, Mo., 90° 15′ 15″ West. What is the difference of time, and when it is 10 A. M. in St. Louis, what is the time in Bombay?

OPERATION.

72°	54′	0″	= longitude of Bombay, E.
90°	15′	15″	= " St. Louis, W.

15) 163°   9′   15″ = difference of longitude.

10 hrs. 52 min. 37 sec. = difference of time, or the length of time it
10 hrs. = St. Louis time. is 10 o'clock in Bombay, before it is 10
20 hrs. 52 min. 37 sec., A. M. in St. Louis.
or   8 hrs. 52 min. 37 sec. P. M. in Bombay.

4. The difference of longitude between St. Paul and Cincinnati is 10° 35′ 24″. What is the difference of time?   Ans. 42 min. 21⅗ sec.

5. The longitude of New Orleans is 90° 4′ 9″. The longitude of San Francisco is 122° 26′ 45″. What is the difference in time, and when it is 12 M. in New Orleans, what is the time in San Francisco?

OPERATION INDICATED.

122°	26′	45″	= longitude of San Francisco
90°	4′	9″	= " New Orleans.

15) 32°   22′   36″ = difference of longitude.

2 hrs. 9 min. 30⅖ sec. = dif. of time.—*1st Ans.*

12 hrs. 0 min. 0 sec. = New Orleans time.
2 hrs. 9 min. 30⅖ sec. = dif. of time, or time before 12 M. in San Francisco.

9 hrs. 50 min. 29⅗ sec. = 50 minutes 29⅗ seconds past 9 A. M., San Francisco time.—*2d Ans.*

6. The longitude of Boston is 71° 3′ 30″, and the longitude of Chicago is 87° 37′ 45″. What is the time in Boston when it is 10 o'clock A. M. in Chicago?

Ans. 11 o'clock 6 min. and 17 sec. A. M.

OPERATION.

87°	37′	45″	= longitude of Chicago.
71°	3′	30″	= " Boston.

15) 16°   34′   15″ = difference of longitude.

1 hr.   6 min. 17 sec. = difference of time, or the length of time it is 10 A. M. in Boston before 10 hrs. = Chicago time) it is 10 A. M. in Chicago. added.

11 hrs. 6 min. 17 sec. A. M. Ans.

7. In traveling from Washington, longitude 77° 0′ 15″ W., to New Orleans, longitude 90° 4′ 9″ W., how much time will an exactly running watch appear to gain?   Ans. 52 min. 15⅔ sec.

8. The longitude of Greenwich is 0, of Astoria, Oregon, 123° 49′ 42″. How much earlier does the sun rise in Greenwich than in Astoria, Or.?

Ans. 8 hrs. 15 min. 18⅘ sec.

SYNOPSIS FOR REVIEW.

Define the following words and phrases:

427. Reduction of Denominate Numbers. 428, 430, and 431. Reduction Descending. 432, 433, and 434. Reduction of Denominate Fractions. 435 and 436. Reduction of Denominate Decimals. 429 and 437. Reduction Ascending. 438. Reduction of Denominate Numbers to Denominate Fractions of a Higher Unit. 439. Reduction of Denominate Numbers to Denominate Decimals of a Higher Unit. 440. Reduction of Denominate Fractions to Fractions of Higher Denominations. 441. Reduction of Denominate Fractions to Denominate Decimals of Higher Denominations. 442. Reduction of Decimal Denominate Numbers to Decimals of Higher Denominations. 443. Reduction of Decimal Denominate Numbers to Fractions of Higher Denominations. 444. Addition of Compound Denominate Numbers. 445. Subtraction of Compound Denominate Numbers. 446. Multiplication of Compound Denominate Numbers. 447. Division of Compound Denominate Numbers. 448. Miscellaneous Problems in Compound Denominate Numbers. 449. Time between Dates. 454. Directions for Finding the Day of the Week on which an Event Did or May Occur. 455. Latitude. 456. Longitude. 457. Meridian. 458. Longitude and Time. 459. Table of Longitude and Time. 461. Reduction of Time to Longitude. 462. Reduction of Longitude to Time.

RATIO AND PROPORTION.

463. Ratio is derived from a Latin word signifying relation or connection. It originates in the comparison and numerical measurement of numbers.

464. We would define **Ratio** to be the *measure* of the relation of two similar quantities. Thus, the ratio of 6 to 2, is $6 \div 2$, and is equal to 3. If we ask what is the *relation* of 6 to 2, the correct answer would be 6 is 3 times 2. We thus see that the ratio *three* is the number which *measures* the relation of 6 compared with 2, and therefore, that ratio is not merely the *relation* of two similar numbers, but the *measure* of this relation.

465. The Terms of a **Ratio** are the numbers compared. The first term of a ratio is the ANTECEDENT, which means *going before;* the second term is the CONSEQUENT, which means *following.*

466. The **Sign** of **Ratio** is the colon (:), which is the sign of division, with the horizontal line omitted. Thus the ratio of 6 to 2, is written, 6 : 2. Ratio is also indicated by writing the consequent under the antecedent in the form of a fraction. Thus, the ratio of 6 : 2 is often written $\frac{6}{2}$.

467. An **Inverse Ratio** is the quotient of the consequent divided by the antecedent. Thus, the inverse ratio of 8 : 4, is $\frac{4}{8}$.

468. The **Value** of a ratio is the quotient of the antecedent divided by the consequent, and is always an abstract number.

469. A **Simple Ratio** is the ratio of two numbers, as 8 : 4.

470. A **Compound Ratio** is the ratio of the products of the corresponding terms of two or more simple ratios, as follows:

$$\left.\begin{matrix} 6:2 \\ 8:4 \end{matrix}\right\} = 6 \times 8 : 2 \times 4; \text{ or } \tfrac{6}{2} \times \tfrac{8}{4} \text{ is a compound}$$

ratio, = 6.

471. The **Reciprocal** of a ratio is the quotient of 1 divided by the ratio, or it is the quotient of the consequent divided by the antecedent. Thus, the ratio of 6 to 2 is 6 : 2 or $\tfrac{6}{2}$, and its reciprocal is, $1 \div \tfrac{6}{2} = \tfrac{2}{6}$, or $\tfrac{1}{3}$.

472. The **Ratio of two fractions** is obtained by reducing them to a common denominator and then comparing their numerators. Thus, the ratio of $\tfrac{5}{7} : \tfrac{2}{7}$ is the same as 5 : 2.

473. The **Ratio of Compound Denominate** numbers is found by reducing them to the same denomination and then making the comparison.

From the foregoing definitions and elucidations, the following *formulas* and *general principles* are deduced :

FORMULAS.

474. 1. *The Ratio = the Antecedent ÷ Consequent.*

2. *The Consequent = Antecedent ÷ Ratio.*

3. *The Antecedent = Consequent × Ratio.*

42

14. ⌐ A grocer has coffee at 12½¢ per pound, and a coal dealer has coal at 40¢ per barrel. If, in exchanging, the grocer puts his coffee at 14¢, what should the coal dealer charge for his coal?

Ans. 44⅘¢ per barrel.

CLASSIFICATION. OPERATION.

$$\begin{array}{cc}
¢ & ¢ \\
12\frac{1}{2} & 14 \\
40 & \\
\end{array}$$

$$25 \,\Big|\, \begin{array}{l} 14 \\ 2 \\ 40 \\ \hline \end{array}$$

— │ 44⅘¢ Ans.

or thus:

CLASSIFICATION. OPERATION.

¢ gain.

14¢ — 12½¢ = 1½¢ gain.
40¢

$$\begin{array}{c|c} 2 & 3 \\ 25 & 2 \\ & 40 \\ \hline \end{array}$$

NOTE.—The student should write the reasoning for the operations.

4⅘¢ gain.
40¢ value of coal.
___
44⅘¢ price of coal.

15. If, with $8.10, you can buy 9 yards of cloth, how many yards can you buy for $5.40?
Ans. 6 yds.

16. If 6 yds. cost $7.20, how much will 12 yds. cost? Ans. $14.40.

17. If, for 12 cents, you can buy ⅔ of a yard, how many yards can you buy for 36 cents?
Ans. 2 yds.

18. If ¾ of a ton of hay cost $15, how much will 3000 pounds cost? Ans. $30.

ℙROPORTION.

478. Proportion arises from the comparison of ratios. It is a comparison of the results of two previous comparisons. Every proportion involves three comparisons; the first two were those which produced the ratios; and the third that which compares or equates the ratios.

Proportion is the expression of the equality of equal ratios; or it is the comparison of two equal ratios. Thus, $6 : 2 :: 15 : 5$ is a proportion, and is read 6 is to 2 as 15 is to 5.

Thus, the ratio of $6 : 2$ as $15 : 5$ is a proportion, i. e. *four* quantities are in *proportion*, when the *first* is the *same multiple* or *part* of the *second*, that the *third* is of the *fourth*.

479. The **Sign of Proportion** is a double colon $(::)$, or the sign of equality $(=)$. Thus, the above proportion is expressed $6 : 2 :: 15 : 5$, or $6 : 2 = 15 : 5$. The *first* is read, 6 is to 2 as 15 is to 5. The *second* is read, the ratio of 6 to 2 equals the ratio of 15 to 5.

480. The **Terms** of a proportion are the numbers compared.

481. The **Antecedents** of a proportion are the *first* and *third* terms.

482. The **Consequents** are the *second* and *fourth* terms.

483. The **Extremes** are the *first* and *fourth* terms.

484. The **Means** are the *second* and *third* terms.

(331)

To aid the calculator in this contracted work, we present the following

515.   T A B L E

$1\tfrac{1}{4}\% = \tfrac{1\tfrac{1}{4}}{100} = \tfrac{1}{80}$; and conversely $\tfrac{1}{80} = 1\tfrac{1}{4}\%$

$1\tfrac{1}{3}\% = \tfrac{1\tfrac{1}{3}}{100} = \tfrac{1}{75}$; "   " $\tfrac{1}{75} = 1\tfrac{1}{3}\%$

$1\tfrac{2}{3}\% = \tfrac{1\tfrac{2}{3}}{100} = \tfrac{1}{60}$; "   " $\tfrac{1}{60} = 1\tfrac{2}{3}\%$

$2\tfrac{1}{2}\% = \tfrac{2\tfrac{1}{2}}{100} = \tfrac{1}{40}$; "   " $\tfrac{1}{40} = 2\tfrac{1}{2}\%$

$3\tfrac{1}{3}\% = \tfrac{3\tfrac{1}{3}}{100} = \tfrac{1}{30}$; "   " $\tfrac{1}{30} = 3\tfrac{1}{3}\%$

$6\tfrac{1}{4}\% = \tfrac{6\tfrac{1}{4}}{100} = \tfrac{1}{16}$; "   " $\tfrac{1}{16} = 6\tfrac{1}{4}\%$

$8\tfrac{1}{3}\% = \tfrac{8\tfrac{1}{3}}{100} = \tfrac{1}{12}$; "   " $\tfrac{1}{12} = 8\tfrac{1}{3}\%$

$10\ \% = \tfrac{10}{100} = \tfrac{1}{10}$; "   " $\tfrac{1}{10} = 10\ \%$

$12\tfrac{1}{2}\% = \tfrac{12\tfrac{1}{2}}{100} = \tfrac{1}{8}$; "   " $\tfrac{1}{8} = 12\tfrac{1}{2}\%$

$16\tfrac{2}{3}\% = \tfrac{16\tfrac{2}{3}}{100} = \tfrac{1}{6}$; "   " $\tfrac{1}{6} = 16\tfrac{2}{3}\%$

$18\tfrac{3}{4}\% = \tfrac{18\tfrac{3}{4}}{100} = \tfrac{3}{16}$; "   " $\tfrac{3}{16} = 18\tfrac{3}{4}\%$

$20\ \% = \tfrac{20}{100} = \tfrac{1}{5}$; "   " $\tfrac{1}{5} = 20\ \%$

$25\ \% = \tfrac{25}{100} = \tfrac{1}{4}$; "   " $\tfrac{1}{4} = 25\ \%$

$33\tfrac{1}{3}\% = \tfrac{33\tfrac{1}{3}}{100} = \tfrac{1}{3}$; "   " $\tfrac{1}{3} = 33\tfrac{1}{3}\%$

$37\tfrac{1}{2}\% = \tfrac{37\tfrac{1}{2}}{100} = \tfrac{3}{8}$; "   " $\tfrac{3}{8} = 37\tfrac{1}{2}\%$

$40\ \% = \tfrac{40}{100} = \tfrac{2}{5}$; "   " $\tfrac{2}{5} = 40\ \%$

$50\ \% = \tfrac{50}{100} = \tfrac{1}{2}$; "   " $\tfrac{1}{2} = 50\ \%$

$62\tfrac{1}{2}\% = \tfrac{62\tfrac{1}{2}}{100} = \tfrac{5}{8}$; "   " $\tfrac{5}{8} = 62\tfrac{1}{2}\%$

$66\tfrac{2}{3}\% = \tfrac{66\tfrac{2}{3}}{100} = \tfrac{2}{3}$; "   " $\tfrac{2}{3} = 66\tfrac{2}{3}\%$

$75\ \% = \tfrac{75}{100} = \tfrac{3}{4}$; "   " $\tfrac{3}{4} = 75\ \%$

$83\tfrac{1}{3}\% = \tfrac{83\tfrac{1}{3}}{100} = \tfrac{5}{6}$; "   " $\tfrac{5}{6} = 83\tfrac{1}{3}\%$

$87\tfrac{1}{2}\% = \tfrac{87\tfrac{1}{2}}{100} = \tfrac{7}{8}$; "   " $\tfrac{7}{8} = 87\tfrac{1}{2}\%$

$100\ \% = \tfrac{100}{100} =$ the whole number, or the whole of a thing.

$150\ \% = \tfrac{150}{100} =$ one and one-half times the whole number.

$200\ \% = \tfrac{200}{100} =$ two times the whole number.

PROBLEMS.

1. What is 20% of $4264.20?   Ans. $852.84.

FIRST OPERATION.   SECOND OPERATION.
    $42.64.20
          20   5 | $4264.20
    _____   _____
    $852.8400 Ans. $852.84 Ans.

Explanation—Since 20% is ⅕ of the amount, we therefore, in the second operation, divide by 5 and thus obtain the correct result.

2. What is 2½% of $500?   Ans. $12.50.
3. What is 10% of 450 pounds?
                              Ans. 45 pounds.
4. What is 12½% of 1600 chickens?
                              Ans. 200 chickens.
5. What is 20% of 444 apples?
                              Ans. 88⅘ apples.

CLASSIFIED PROBLEMS.

516. *To find the Percentage and the Amount or Difference, when the Base and the Rate Per Cent are given. Or in Commercial Problems, to find the selling price, when the cost and the gain or the loss per cent are given.*

1. What is 20% of 550?   Ans. 110.
OPERATION.
    5.50   *Explanation.* — In all
       20   or 5 | 550   problems of this kind,
    _____   _____   for reasons above given
    110.00 Ans. 110, Ans. we first divide by 100,
                            which is done by point-
                            ing off two places and

then multiply by the given rate. Or, when the rate is an aliquot part of 100, we take such a part of the number as the rate per cent is part of 100.

2. What is 25% increase of 500 pounds, and what is the amount?

Ans. 125 percentage increase, and 625 amount.

OPERATION.

5.00		
25	or	4	500

125.00   125 = percentage.

Explanation.—In this problem we find the percentage as above, and then add the same to the base.

125 + 500 = 625 pounds, amount.

517. From the foregoing elucidations we derive the following general directions for finding the Percentage and the Amount or Difference:

1. *To find the Percentage, divide the base by 100, and then multiply by the rate. Or, take such a part of the base as the rate is part of 100.*

2. *To find the Amount, add the percentage to the base.*

3. *To find the Difference, subtract the percentage from the base.*

NOTE.—When the base is a compound number reduce the whole number to the lowest named denomination; or, reduce the lower denominations to a decimal of the highest.

3. What is $16\frac{2}{3}$% of 1272?   Ans. 212.

4. What is 17% of 3850 hats?

Ans. 654.50 hats.

5. What is 40% of $1611.15?   Ans. $644.46.

6. Bought goods at 24¢, and sold them at a profit of 25%. What was the selling price?

Ans. 30¢.

7. Goods cost $2.40 a yard, and sold at 20% loss. What was the selling price?

Ans. $1.92.

8. What is 52% of $514.63? Ans. $267.60+.

9. What is 62½% of 3240 peaches?

Ans. 2025 peaches.

10. What is 16⅔% of 600 books?

Ans. 100 books.

11. A steamer running at a speed of 12 miles an hour increased her speed 12½%. What was her speed after the increase?

Ans. 13½ miles per hour.

12. Bought 2462 bushels of corn and sold 33⅓% of it. How many bushels are left?

Ans. 1641⅓ bushels.

13. A capitalist had $85000 and invested 25% of it. How many dollars did he invest?

Ans. $21250.

14. The product of a factory in 1884 was 94400000 pounds, and in 1885, it was 12½% less. How many pounds was the deficiency?

Ans. 11800000 pounds.

15. Which is greater, and what is the difference between 8% on 1200, and 7% on 1300?

Ans. 1st, 8% on 1200; 2nd, 5.

16. A bankrupt merchant who owed $43000, paid 21%. How much did he pay? Ans. $9030.

17. A man had $1400 and spent 10% of it, and then 20% of the remainder. How much did he spend? Ans. $392.

18. Paid $8.50 per barrel for flour and sold it at a gain of 10%. What was the selling price?

Ans. $9.35.

19. Paid $15 for a coat and sold it at 20% gain. What was the profit? Ans. $3.

20. Goods cost $22.50 per dozen, and sold at 33⅓% gain. What was the gain per dozen?

Ans. $7.50.

21. A grocer bought 160 dozen eggs and sold 8⅓% of them. How many did he sell, and how many dozen are left?   Ans. 13⅓ doz. sold,

146⅔ doz. left.

22. A. resides 1½ miles from school; he walked 37½% of the distance and rode the remainder. How many feet did he ride?   Ans. 4950 feet.

518. *To find the Rate Per Cent, when the Base and the Amount or Difference are given, or when the Base and the Percentage are given. Or in Commercial problems, to find the Rate Per Cent when the cost and the selling price, or the Cost and the Gain or Loss are given.*

1. The base is 40 and the amount is 50. What was the rate per cent?   Ans. 25%.

OPERATION.

40 = base.

50 = amount, or the base } + the Percentage. }

———

10 = Percentage, gain, increase.

40 | 100

———

25 = gain or increase on 100.

Explanation.— In all problems of this kind, we first find the percentage, increase, or decrease—the gain or loss, as shown in the operation. Then by inspection and reason, we see that it was the base 40 which produced the 10 percentage or gain. We then reason as follows: Since 40 gained 10, 1 will gain the 40th part, and 100 will gain 100 times as many, which is 25.

It may and should be here asked, where we get the 100 and how do we know that the result represents per cent? We answer: the 100 comes from the problem itself. The question

of the problem, "*What was the rate per cent,*" if expressed as
it is understood, would read: *What was the gain or increase on*
100? By this we see where the 100 comes from, and as the
reasoning showed that the result was a gain on the 100, it is
therefore clear that it is gain per cent.

2. The population of a city, in 1885, was 68000;
in 1886 it was 72760. What was the rate per cent
increase? Ans. 7%.

OPERATION.

68000 = base.
72760 = amount, base + P.
or increase.

4760 = Percentage = increase or gain.

68000 | 100

7 = increase or gain on
every 100, and hence is 7%.

Explanation.— The reasoning for this problem is the same as in the preceding one, and hence is omitted.

3. A hat cost $3.50 and sold for $4.20. What
was the rate per cent gain? Ans. 20%.

OPERATION.

$3.50 = cost = base.
4.20 = selling price = amount.

70¢ = gain = percentage.

350 | 100

20 = gain on every 100¢ and
hence is rate % gain.

Explanation.— As in the second problem above, we first find the gain or percentage. We then reason thus: Since $3.50 gain 70¢, 1¢ will gain the 350th part and 100¢ will gain 100 times as much, which is 20.

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4. Goods cost $3.50 and sold for $2.80. What was the rate per cent loss? Ans. 20%.

OPERATION.

$3.50 = cost = base.
2.80 = selling price = difference.
—————
70¢ = loss = percentage.

350 | 100
——
20 = loss on every 100¢ and hence is rate % loss.

Explanation.— In this problem we first find the loss or percentage, and then reason as in the above problem.

5. Goods cost $22 and sold at a profit of $5.50. What was the per cent gain? Ans. 25%.

OPERATION.

$
5.50 = gain = percentage.

22.00 | 100
——
25 = gain = gain %.

Explanation — In this problem, we have the gain or percentage given, and hence have but to make the proportional statement, the reasoning for which is the same as above.

6. Yesterday the thermometer registered 76 degrees; to-day it stands 11 degrees lower. What is the per cent decrease in temperature?

Ans. $14\frac{2}{19}$%.

OPERATION.

11 = percentage = decrease.

76 | 100
——
$14\frac{2}{19}$ = % decrease in temperature.

Explanation.— The reasoning for the proportional statement of this problem is the same in the main as in the preceding one and is therefore omitted.

519. From the foregoing elucidations, we derive the following directions for finding the *rate per cent :*

1. *To find the Rate % when the Base and Amount, or the base and the difference, or the cost and the selling price, are given, first find the percentage—the increase or decrease, the gain or loss—and then make the proportional statement shown in the operations ; or thus : the base or cost : the increase or decrease, the gain or the loss : : 100 : the required rate per cent.*

2. *When the base and the percentage of increase or decrease, or the base and the gain or loss, are given, then make the above proportional statement at once.*

Or, if it is desired to ignore all processes of reasoning, thus : multiply the increase or decrease, the gain or loss, by 100, and divide by the base or cost.

PROBLEMS.

7. The base is 1750; the percentage is 43.75. What is the rate %?   Ans. $2\frac{1}{2}\%$.

8. A yard of cloth cost 16¢ and sold for 18¢. What was the gain %?   Ans. $12\frac{1}{2}\%$.

9. Paid $120 for a horse and sold it at a profit of $22. What was the gain %?   Ans. $18\frac{1}{3}\%$.

10. In a population of 240000, there were 6214 deaths in 12 months. What was the rate per cent?
Ans. $2.58\frac{11}{12}\%$.

11. What per cent of 54 is 6?   Ans. $11\frac{1}{9}\%$.

12. What per cent of $540 is $67.50?
Ans. $12\frac{1}{2}\%$.

13. $2.50 is what % of $60.40?   Ans. $4\frac{21}{151}\%$.

14. 9 is what % of 216?   Ans. $4\frac{1}{6}$%.

15.   An invoice of goods cost in New York \$3840; the freight was \$57.60.   What was the rate %?
          Ans. $1\frac{1}{2}$%.

16.   On a bill of \$421.80, a discount of \$21.09 was allowed.   What was the % discount?
          Ans. 5%.

17.   What % of a number is 5% of 15% of it?
          Ans. $\frac{3}{4}$%, or .75%.

18.   What % of a number is 10% of 20% of 25% of it?   Ans. $\frac{1}{2}$%, or .5%.

19.   15% of 60 is what % of 75?   Ans. 12%.

20.   40 is what per cent of 8?   Ans. 500%.

21.   6 is what % of 24?   Ans. 25%.

22.   A passenger train runs 25 miles an hour; an express train runs 35 miles an hour.   What % slower does the passenger train run, and what % faster does the express train run, when compared with each other?
          Ans. Pass. train runs $28\frac{4}{7}$% slower.
          Ex.   "   "   40 % faster.

23.   Single tickets cost 25¢; in packages of 10 they cost 20¢ each.   What is the % gain by buying by the package, and what is the % loss by buying by the single ticket?
Ans. 25% gain by buying by the package.
     20% loss by   "   "   single ticket.

24.   What % of a number is $\frac{5}{8}$ of it?
          Ans. $62\frac{1}{2}$%.

25.   What % of a number is $\frac{1}{2}\frac{7}{5}$ of it?
          Ans. 68%.

26.   What is 2% of $12\frac{1}{2}$% of $66\frac{2}{3}$% of 25% of a number?   Ans. $\frac{1}{24}$%.

27. What is 48% of $16\frac{3}{5}$% of $37\frac{1}{2}$% of $62\frac{1}{2}$% of a number?   Ans. $1\frac{1}{8}$%.

28. The cotton crop of the Southern States ending the fiscal year, Sept. 1, 1884, was 5713200 bales; and for the fiscal year ending Sept. 1, 1885, was 5655900 bales. 1. What % more was produced in 1884 than in 1885? 2. What % less in 1885 than in 1884? 3. What % is 5713200 bales of 5655900 bales? 4. What % is 5655900 bales of 5713200?

    Ans. 1.0131+% more was produced in 1884.
         1.00296+% less   "   "   1885.
         101.0131+%, 5713200 is of 5655900.
         98.99705+%, 5655900 is of 5713200.

29. A man has due him $45, and compounds on receipt of $36. What % did he lose?   Ans. 20%.

30. A broker bought bonds at 90¢ on the dollar and sold them at 95¢ on the dollar. What % did he gain?   Ans. $5\frac{5}{9}$%.

31. In a year of 365 days, 67 days are rainy. What % of the days are not rainy?   Ans. $81\frac{44}{73}$%.

32. According to the Carlisle mortality tables, 43 persons of every 5879 of 25 years of age die annually. What is the rate % of deaths?   Ans. .731+%.

520. *To find the* BASE *when the Amount or Difference and the Rate per cent Increase or Decrease are given. Or, in Commercial Problems, to find the* COST *when the Selling price and the Gain or Loss per cent are given.*

1. The manufactured value of goods is $2100,

which is 20% more than the value of the raw mate-
rial. What was the value of the raw material?

Ans. $1750.

FIRST OPERATION.

$100=assumed base or value of
raw material.
20=20% increased value.

$120=manufactured value.

$
```
     | 100=assumed base.
 120 | 2100
 ——  | ————
     | $1750 cost of raw material.
```

Explanation.—By considering the problem, we see that the $2100 is the amount of the value of the raw material and the 20% cost to manufacture the goods. We also see that the 20% was calculated on the value of the raw material, and not on the $2100, the value of the manufactured goods. Hence there are no figures in the problem upon whose face we can calculate the 20% cost to manufacture. We therefore, as shown in the operation, assume 100, as the base or value of the raw material, and on this we calculate and add thereto the 20% cost to manufacture. This gives an amount of $120, as the manufactured value of goods from a base or raw material value of $100.

Now with these values which contain the same ratio of base and amount, or of raw material and of manufactured goods, that exists between the $2100 of manufactured goods and the required value of the raw material from which it was produced, we make the proportional statement, shown by the operation and obtain the required base or raw material value.

In making the solution statement, we place the $100 assumed base or value of raw material on the line and reason thus: Since $120 amount or manufactured value at a gain of 20% require $100 base or value of raw material, $1 will require the 120th part, and $2100 amount or manufactured value will require 2100 times as much.

In assuming a number to represent cost, it is immaterial so far as correct results are concerned, what number we assume; but we always select 100 for the reason that per cent being on the hundred, our operation is facilitated to a greater extent by 100 than by any other number.

The foregoing reasoning and solution are applicable to all problems of like character, regardless of the rate of gain or

loss per cent. But when the gain or loss per cent are an aliquot part of 100, the operation may be very much shortened by the following process of work and reasoning:

SECOND OPERATION.

6) $2100=

350 = ⅕ of base or value
  5   of r. m.

$1750 = base or value of r. m.

Explanation—In this solution, since the rate % is an aliquot of 100, we reason thus: Since the amount, $2100, contains a gain of 20%, and since 20% is equal to ⅕ of a thing, the $2100 is therefore the base or value of the raw material, plus ⅕ of the same, which makes ⅚, and since $2100, is ⅚ of the base, ⅕ of the base is the ⅙ part, and ⅘ or the whole base is 5 times as much.

2. Sold goods for $40 and gained 25%. What was the cost of the goods? Ans. $32.

FIRST OPERATION.

$100 = cost assumed.
  25 = 25% gain added.

$125 = selling price.

$
125 | 100
    | 40
    ————
    | $32 cost, Ans.

Explanation. — The reason for each step of the operation of this problem is essentially the same as in the preceding problem and hence is omitted.

SECOND OPERATION.

5) $40 = selling price.
    8 = ¼ of cost.
    4

$32 cost, Ans.

Explanation. — For this operation we reason thus: Since the $40 selling price contains a gain of 25%, and since 25% is ¼ of the cost, the $40 is hence ⁵⁄₄ of the cost; and since $40 is ⁵⁄₄ of the cost, ¼ of the cost is ⅕ part, which is $8; and ¼ or the whole cost is 4 times as much, which is $32.

3. Sold goods for $30 and lost 25%. What did the goods cost? Ans. $40.

FIRST OPERATION.

$100 = cost assumed.
25 = 25% loss deducted.
___

$75 = selling price.

$
75 | 100
    | 30
  — | ——
    | $40 cost, Ans.

Explanation—The reasoning for the proportional statement is as follows: Since $75 selling price at a loss of 25% required $100 cost, $1 selling price will require the 75th part, and $30 selling price will require 30 times as much.

NOTE.—In practice the line statement is all that needs be made. The assumed base or cost and the amount or selling price resulting therefrom, should be mentally performed.

SECOND OPERATION.

3) $30 = selling price.

10 = ⅓ of cost.
4
___

$40 cost, Ans.

cost is 4 times as much which is $40.

Explanation—In this solution we reason as follows: Since the $30 selling price is the cost less 25%, it is therefore ¾ of the cost; and since ¾ of the cost is $30, ¼ is the ⅓ part which is $10, and ⅓ or the whole

521. From the foregoing elucidations, we derive the following general directions for finding the base or cost when the amount, or difference, or selling price, and the rate per cent are given :

1. *To find the Base or Cost, first assume 100 to represent base or cost ; on it calculate the given rate % to find the percentage ; then either add it to or subtract it from the base or cost according as the rate % is a gain or a loss, an increase or a decrease, and thus produce an amount or difference, which has the same ratio of value to the 100 assumed base or cost as the given Amount or Difference has to the required base or cost. Now, with this ratio of values and the given*

amount or difference, make the proportional statement shown in the operation; or thus : *The produced amount* **or difference :** *the assumed base or cost : : the given* **amount or difference :** *the required base or cost.*

2. *If it is desired to ignore all processes of reasoning in the solution, the following brief and arbitrary directions will solve this class of problems :*

Multiply the given amount or difference or selling price by 100, *and divide by* 100 *plus the gain* % *or minus the loss* %.

PROBLEMS.

4. Sold goods for $20.62½ and gained 25%. What did the goods cost ?   Ans. $16.50.

5. Sold a watch for $70 and lost 20%. What did it cost ?   Ans. $87.50.

6. The rent of a house is $67.50 per month, which is 12½% advance on last year's rent. What was the rent last year?   Ans. $60.

7. The amount is 266⅔, the rate % is 33⅓. What is the base?   Ans. 200.

8. The difference is 400, the rate % is 20. What is the base?   Ans. 500.

9. What number increased by 10% of itself is 1815 ?   Ans. 1650.

10. $18.17 is 15% more than what sum ?
         Ans. $15.80.

11. $4.50 is 50% less than what sum ? Ans. $9.

12. A produce merchant sold corn at 60½¢ per bushel, which is 10% more than he paid for it. What did he pay for it ?   Ans. 55¢.

13. A carpenter after using 80% of his lumber had 500 feet on hand. How many feet had he at first ?   Ans. 2500 feet.

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14. A shoe factory manufactured 2000 pairs of shoes, which weighed 5500 pounds, not considering the thread and nails. There was a waste of $8\frac{1}{3}\%$ in manufacturing. How many pounds of leather were used in making the shoes ?

Ans. 6000 pounds.

15. Sold sugar at $8\frac{1}{2}\mathcal{g}$ per pound and gained $6\frac{1}{4}$ per cent. What did it cost ? Ans. $8\mathcal{g}$.

16. My merchandise account is debited with $78500 and credited with $72225. The gain $\%$ on sales has been $12\frac{1}{2}$. What was the cost of the sales, and how much merchandise have I on hand ?

Ans. $64200, cost of sales.
$14300, mdse. on hand.

17. A planter lost by a storm 20% of his grain and has 29120 bushels left. How many bushels had he before the storm, and how many bushels did he lose ? Ans. 36400 bu. before the storm.
7280 bu. lost.

18. Sold goods for $100 and gained 40%. What did they cost ? Ans. $71.42\frac{6}{7}$.

19. Sold goods for $175 and gained 150%. What did they cost ? Ans. $70.

20. A cotton factory received from a merchant 27530 pounds of cotton, to manufacture into shirtings. He is to receive $3\frac{1}{2}\mathcal{g}$ per yard for manufacturing. The factory manufactured and delivered to the merchant 34000 yds., which weighs 8 ounces per yard. It is now agreed to settle, the factory to retain the cotton on hand at $11\mathcal{g}$ per pound. Allowing a waste of 15% in manufacturing, how much does the factory owe the merchant or the merchant owe the factory ?

Ans. $361.70 the merchant owes the factory.

522. *To find the Base when the Rate per cent and percentage are given. Or, in Commercial Problems, to find the cost when the Rate per cent and the Gain or Loss are given.*

1. The rate per cent is 8 and the percentage is 36. What is the base?   Ans. 450.

OPERATION.

100 = assumed base.

8% calculated on assumed base.

8.00 = percentage on assumed base.

$$8 \mid \frac{100 = \text{assumed base.}}{36}$$
$$\overline{450 \text{ base, Ans.}}$$

Explanation.—In all problems of this kind, we first assume 100 base and find the percentage thereon at the given rate %. By this work we produce a *base* and a *percentage* containing the same ratio that exists between the given percentage and the required base. Having these ratio numbers, we then make a proportional statement as shown in the operation.

In this problem we find the percentage to be 8. Then placing the assumed base on the statement line, we reason thus: Since 100 base at 8% produced 8 percentage, conversely 8 percentage required 100 base; and since 8 percentage required 100 base, 1 percentage will require the 8th part, and 36 percentage will require 36 times as much, which is 450.

NOTE.—In practice only that part of the operation shown by the statement line should be made; the other work should be mentally performed.

2. 40 is 5% of what number?   Ans. 800.

OPERATION INDICATED.

$$5 \mid \frac{100}{40}$$

3. On a sale of a lot of go   s, a loss of $17 was sustained. The % loss was 20. What was the cost?   Ans. $85.

OPERATION.

100=assumed cost.

20%
——————
$20.00=loss on assumed cost.

$
     | 100
  20 | 17
  — | ——
     | $85 cost, Ans.

Explanation—In this problem the reasoning is the same as in the first problem, and hence is omitted.

523.   From the foregoing elucidations, we derive the following general directions for finding the base or cost when the rate per cent and the percentage or gain or loss are given :

1. *To find the Base or Cost first assume* 100 *to represent base or cost, and then find the percentage or gain or loss thereon at the given Rate %. Then, ꞌor reasons given in the explanation of the first problem, make the proportional statement as shown in the operation.*

2. *If it is desired to ignore all reasoning in the solution, then multiply the percentage, gain or loss, by* 100, *and divide by the given rate %.*

PROBLEMS.

4. The rate per cent is 10 and the percentage is 108. What is the base?   Ans. 1080.

5. 3021 is 12½% of what number?

Ans. 24168.

6. Goods were sold at a profit of 25% and a gain of $5.60 was realized. What was the cost?

Ans. $22.40.

7. Sold a watch for $26.50 more than it cost and gained 50%. What did it cost, and what was the selling price? Ans. $53 cost.

$79.50 selling price.

8. City taxes are 2%, and a man paid $886.42. What was the assessed value of his property?

Ans. $44321.

9. $\frac{21}{5000}$ is $\frac{7}{8}$% of what number? Ans. $1\frac{2}{8}$.

10. $8\frac{1}{2}$ is 1% of what number? Ans. 850.

11. State taxes are .6%, and a man paid $67.50. What is the assessed value of his property?

Ans. $11250.

12. 5% of $180 is $12\frac{1}{2}$% of what sum?

Ans. $72.

13. A man lost $35.88, which was 8% of what he had at first. How much did he have left?

Ans. $412.62.

14. A planter sold 2430 barrels of molasses and had 25% of his yearly product left. What was his yearly product? Ans. 3240 barrels.

MARKING GOODS.—PROFIT AND LOSS.

524. *To mark Goods at a given Gain or Loss %.*

1. Bought butter at 25¢ per pound. At what price must it be sold to gain 25%?

Ans. $31\frac{1}{4}$¢.

OPERATION.—See Article 516, page 365.

$$25\% = \tfrac{25}{100} = \tfrac{1}{4}.$$

$$\begin{array}{|l} 25 \\ 6\frac{1}{4} = 25\% \text{ gain.} \\ \hline 31\frac{1}{4}\text{¢, Ans.} \end{array}$$

2. Bought cheese at 15¢ per pound. At **what** price must it be sold to lose 10%?   Ans. 13½¢.

OPERATION.—See Article 516, page 365·

$$10\% = \tfrac{10}{100} = \tfrac{1}{10}. \quad - \left|\begin{array}{l} 15 \\ 1.5 = 10\% \text{ loss.} \\ \hline 13.5\text{¢, or }13\tfrac{1}{2}\text{¢, Ans.} \end{array}\right.$$

NOTE.—When the % gain or loss is an aliquot part of 100, all we have to do, when marking goods, is to add to or subtract from the cost, such a part of itself as the rate % is part of 100.

Find the selling price of the following:

3. Of goods which cost 18¢ and sold at 33⅓% gain.   Ans. 24¢.

4. Of goods which cost $240 and sold at 40% gain. ·   Ans. $336.

5. Of goods which cost $4.25 and sold at 16⅔% gain.   Ans. $4.95⅚.

6. Of goods which cost $14.50 and sold at 30% loss.   Ans. $10.15.

7. A merchant paid $108 per dozen for coats. At what price per coat must he sell them to gain 50%?   Ans. $13.50.

8. Bought silk for $1.60 per yard. At what price must it be sold to gain 140%?
                               Ans. $3.84.

9. A merchant wishes to mark his goods in such a manner that when he sells at wholesale he may discount 20% from his retail price and still gain 10% on cost. Accordingly, what must be the retail price of goods that cost $1.60 per yard?
                               Ans. $2.20.

OPERATION.

$1.60=cost.		100=assumed retail price.
16=10% gain. 80		1.76

$1.76=wholesale | $2.20=retail price, Ans.
price.

or thus: 80 | 110   or thus: 80 | 100
                  | 1.60   100 | 110
                                              | 1.60

10. Goods cost $10. What must be the asking price so that 15% may be deducted and a gain of 25% realized on first cost?   Ans. 14.70\frac{1}{2}$$\frac{1}{9}$.

11. Paid 18¢ per dozen for eggs. Allowing 5% for breakage and 10% for uncollectable sales, how much per dozen must they be sold for to gain 14% on first cost?   Ans. 24¢.

OPERATION to find selling price to gain 14%.	OPERATION to allow 10% for uncollectable sales.	OPERATION to allow for 5% breakage.	Or thus :
18¢.			
14%	90 | 100	95 | 100	90 | 114
——	| .2052	| .2280	100 | 100
＇.0252	— |———	— |———	95 | 18
⁄ .18	| .2280¢.	| $.2400	— |———
——		Ans.	| 24 ¢
.2052			| Ans.

12. Goods cost $12. What must be asked for them so that 20% may be deducted from the asking price, 6$\frac{1}{4}$% be allowed for waste, 10% allowed for "bad debts," and 25% gain made on first cost?
Ans. 22\frac{2}{9}$.

13.   A nest of five tubs cost $3.00.   What should be the selling price of each tub, to gain $33\frac{1}{3}\%$ on cost?

OPERATION TO OBTAIN ONE OF MANY ANSWERS.

$3.00 cost $+ 33\frac{1}{3}\%$ $=$ $4.00, selling price for the nest of 5 tubs. $2\cancel{c} + 3\cancel{c} + 5\cancel{c} + 8\cancel{c} + 12\cancel{c} = 30\cancel{c}$, which is the sum of the assumed ratio of values. Then the following proportional statements give the respective value of each tub:

First or smallest tub.		Second tub.		Third tub.		Fourth tub.		Fifth tub.	
30	$2\cancel{c}$ 4.00	30	$3\cancel{c}$ 4.00	30	$5\cancel{c}$ 4.00	30	$8\cancel{c}$ 4.00	30	$12\cancel{c}$ 4.00
	$26\frac{2}{3}\cancel{c}$, Ans.		$40\cancel{c}$, Ans.		$66\frac{2}{3}\cancel{c}$, Ans.		$1.06\frac{2}{3}$, Ans.		1.60, Ans.

Practically, where the nickle is the smallest coin used in trade, the prices would be as follows:

1st tub, $25\cancel{c}$;   2d tub, $40\cancel{c}$;   3d tub, $65\cancel{c}$; 4th tub, $1.10; 5th tub, $1.60.

NOTE.—Other ratios of value may be assumed according to the judgment of the calculator. Or, if it is deemed equitable, the prices may be made proportional to the capacity or volume of the respective tubs.

14.   What must be the selling price per box of a nest of 7 boxes which cost $15, the gain % being 25?   Ans.

525.   *To find the Gain or Loss per cent, when the Cost and the Selling price are given.*

1.   Goods cost $3 and sold for $4.   What was the gain % ?   Ans. $33\frac{1}{3}\%$.

NOTE.—For the operation and reasoning for this class of problems, see Articles 518 and 519, pages 368 and 371.

2.   Bought a horse for $180 and sold it for $160. What per cent was lost ?   Ans. $11\frac{1}{3}\%$.

3. Sold flour which cost $7.50 at $8 per barrel. What was the gain per cent? Ans. 6⅔%.

4. Bought shirts at $16.50 per dozen and sold them at $1.75 a piece. What % did I gain?
Ans. 27 3/11 %.

5. Bought 3 apples for 10¢ and sold them at 5¢ each. What % did I gain? Ans. 50%.

6. Bought tea at $1.25 per lb. and sold it at 7½¢ per oz. What % did I lose? Ans. 4%.

526. *To find the Cost when the Gain per cent and the Selling price or the Gain or Loss are given.*

1. Sold goods for $15 and gained 25%. What did they cost? Ans. $12.

NOTE.—For the operation and reason for the solution of this class of problems, see Article 520.

2. Sold a hat for $1 more than it cost and gained 33⅓%. What did it cost? Ans. $3.

3. If 3¢ per pound is lost by selling coffee at a loss of 20%, what was the cost? Ans. 15¢ per lb.

4. Sold goods for $19.80 per dozen and lost 10%. What was the cost per single article?
Ans. $1.83⅓.

5. A loss of 1¢ per pound was sustained by selling at 7 9/13 % loss. What was the cost?
Ans. 13¢.

6. Sold corn at 56¼¢ per bushel and gained 12½%. What did it cost? Ans. 50¢.

7. Sold gloves at a profit of 25¢ per pair, and gained 30%. What was the cost per dozen?
Ans. $10.

8. Sold butter at 35¢ a pound and lost 6⅔%. What was the cost? Ans. 37½¢.

9. I gained $15 by selling a horse at 16⅔% gain? What did the horse cost me? Ans. $90.

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10.   A clothier sold a coat for $18 and thereby lost 11¼%.   What did the coat cost?

Ans. $20.25.

11.   A dealer asked $25 for a suit of clothes, but took off 10% to effect a sale, and thereby gained 12½%.   What did the suit cost him?   Ans. $20.

12.   14⅖% was lost by selling silk after deducting 25% from $4 per yard asking price.   What was the cost per yard?   Ans. $3.50.

527.   DISCOUNT, REBATE, AND INCREASE.

1.   A merchant who owed $1200 was offered 15% discount for an immediate settlement by cash, which he accepted.   How much did he pay?

Ans. $1020.

2.   A business man insured property to the amount of $65000 at 1¾%.   He was allowed a rebate of 20% for cash.   What was the net premium paid?   Ans. $910.

3.   A faithful accountant, who was receiving a compensation of $720 per year, had his salary increased 33⅓%.   How much does he now receive per month?   Ans. $80.

4.   A firm paid $300 per month rent and was raised 25%.   What is their current rent?

Ans. $375.

5.   A clerk who received $125 per month had his salary reduced 20%.   What did he then receive?

Ans. $100.

6.   A man who was receiving $2.50 per day demanded an increase of 40%.   What would then be his daily pay?   Ans. $3.50.

7. A merchant sold goods amounting to $137.40, and allowed a discount of 10%. What was the net amount of the bill? Ans. $123.66.

8. On a bill of $97.16 a discount of 5% was allowed. What was the balance due? Ans. $92.302.

9. Allow a discount of 10% and then 5% on a bill of $3214.80. How much money will you receive, and what will be the discount?

Ans. 1st, $2748.654.
2d, $466.146.

10. A merchant owed $5716, and paid $3800 on account, with the understanding that he is to be allowed 5% discount on the amount of the bill that the $3800 cash will pay. How much does he still owe? Ans. $1716.

528.   MISCELLANEOUS PROBLEMS.

1. Sold goods for 16¢ per yard and lost 20%. What should they have been sold for to gain 25%?

Ans. 25¢.

OPERATION.

80	100=cost. 16	or thus : 80	125 16
	20¢ cost of goods. 5=25% gain.		25¢, Ans.

25¢ selling price, Ans.

2. Sold goods for $18 and gained 12½%. What should they have sold for to gain 40%?

Ans. $22.40.

3. Whiskey which cost 90¢ per gallon is compounded with water in the proportion of 1 gallon of water to 2 gallons of whiskey, and the mixture is sold for 85¢ per gallon. What % is gained?

Ans. 41⅔%.

4. A merchant compounded, in equal quantities, sugar that cost 6¢ per pound and sugar that cost 9¢ per pound. He then sold the mixture at 9¢ per pound. What % did he gain? Ans. 20%.

OPERATION.

1 pound at 6¢ = 6¢.	9¢ sales. 2	3 gain. Ans.
1 " " 9¢ = 9¢.	7½¢ cost. 15	2
		100
2)15¢.	1½¢ gain. —	———
1 lb. of the com-		20% gain,
pound cost 7½¢.		Ans.

5. A merchant bought 5 bbls. flour for $40 and sold the same at a gain of 25%. He then bought 5 bbls. more for $40, and sold the same at a loss of 25%. Did he gain or lose and if so, how much? Ans. He neither gained nor lost.

OPERATION.

$40 cost @ 25% gain gives $50 selling price=
$10 gain.
$40 sold @ 25% loss gives $30 selling price=
$10 loss.

6. A merchant sold 5 bbls. of inferior flour for $40 and gained 25%. He then sold 5 bbls. of superior flour for $40 and lost 25%. Did he gain or lose in the two transactions, and if so how much? Ans. He lost $5⅓.

OPERATION.

$40 selling price @ 25% gain gives $32 cost
=$ 8 gain.
$40 selling price @ 25% loss gives $53⅓ cost
=$13⅓ loss.

Net loss $ 5⅓

7. Sold a barrel of oranges for $6 and gained 20%. I then invested the $6 in merchandise which I sold at a loss of 20%. What was the gain or loss by the two transactions?

<div align="right">Ans. 20¢ loss.</div>

8. A merchant marked his goods at 40% gain, but supplies his wholesale buyers at a discount of 20%. What % does he make?   Ans. 12% gain.

<div align="center">OPERATION.</div>

$100 = assumed cost.
  40 = 40% gain.
——
$140 = retail selling price.
  28 = 20% discount.
——
$112 = wholesale selling price.
 100 = cost.
——
$12 gain = 12 %.

9. If a merchant marks his goods at 50% profit and then effects sales at 40% discount on retail price, what per cent does he gain or lose?

<div align="right">Ans. Loses 10%.</div>

10. A grocer bought 10 barrels of molasses each containing 40 gallons, at 50¢ per gallon. He then put the molasses in kegs holding 8 gallons each, and sold the same as 10 gallon kegs, at 55¢ per gallon. What was his profit, and what % did he gain?

<div align="right">Ans. $75 profit; 37½% gain.</div>

11. A fruit dealer sold 4 peaches for 5¢ and gained 56¼%. What % would he have gained by selling 5 for 6 cents?   Ans. 50%.

12. A. and B. are two merchants; they desire to barter rice and sugar. A. has rice, market value 6¢ per pound; and B. has sugar, market value 8¢ per pound. At the time of the exchange or barter, A. suggests to B. that in order to influence the market reports, he will place his rice at 7¢ per pound and that B. shall advance his sugar accordingly. B. accepts the proposition. What should be the exchange price of B.'s sugar?

Ans. 9⅓¢ per pound.

FIRST OPERATION.

7¢ = exchange value of A.'s rice.
6¢ = market value of same.
———
1¢ = the increase on same.

$6\left\vert\begin{array}{l}1 = \text{gain.} \\ 100 \\ \overline{} \\ 16\tfrac{2}{3}\% \text{ gain.}\end{array}\right.$

8¢ = market value of B's sugar.
1⅓¢ = 16⅔% gain.
———
9⅓¢ = exchange value.

SECOND OPERATION.

6¢ + 1¢ = 7¢ = exchange value of A.'s rice.

$6\left\vert\begin{array}{l}1 = \text{gain on } 6¢. \\ 8 \\ \overline{} \\ 1\tfrac{1}{3}¢ \text{ gain on } 8¢.\end{array}\right.$

8¢ + 1⅓¢ = 9⅓¢ = exchange value of B.'s sugar.

13. Supposing, in the above problem, that B. had proposed to reduce his sugar 1¢ per pound, and that A. should reduce his rice accordingly. What would be the exchange price of A.'s rice?

Ans. 5¼¢.

14. Supposing in the above problem that A. and B. had each raised 1¢ on the market value of their rice and sugar, how much % would B. have lost, and what % would A. have gained?

<div align="center">

Ans. B. would have lost 3⅘%.

A.   "   "   gained 3¹⁹⁄₂₇%.

Operation to find B.'s % of loss.

</div>

9⅛¢=correct exchange value, as above.

9¢ =incorrect exchange value, as supposed.

⅛¢=loss by incorrect exchange value.

$$\begin{array}{c|l} & ¢ \\ 3 & 1 = \text{loss.} \\ 28 & 3 \\ & 100 \\ \hline & \overline{} \\ & 3\tfrac{4}{7}\% \text{ loss,} \\ & \quad\text{Ans.} \end{array}$$

<div align="center">

Operation to find A.'s % gain.

</div>

$$\begin{array}{c|l} & 9¢=\text{B's selling price.} \\ 8 & 6 \\ \hline & 6\tfrac{3}{4}¢=\text{price A. should have sold for when} \\ & \qquad \text{B. sold for } 9¢. \end{array}$$

7¢ =A.'s incorrect s. price.

6¾¢=A.'s correct s. price.

¼¢=A.'s gain by selling at 7¢.

$$\begin{array}{c|l} & ¢ \\ 4 & 1 = \text{gain.} \\ 27 & 4 \\ & 100 \\ \hline & \overline{} \\ & 3\tfrac{19}{27}\% \text{ gain, Ans.} \end{array}$$

<div align="center">

EXPLANATION FOR SECOND OPERATION.

</div>

8¢ B.'s market value + 1¢ = 9¢, B.'s selling price; then since 8¢ sell for 9¢, 1¢ will sell for the ⅛ part, and 6¢, A.'s market value, will sell for 6 times as many, which is 6¾¢; then since A. really sold for 7¢, when he should have sold for 6¾¢, he gained 7¢ — 6¾¢, = ¼¢; and if 6¾¢ gain ¼¢, 1¢ will gain the 6¾ or ²⁷⁄₄ part, and 100¢ will gain 100 times as much, which is 3¹⁹⁄₂₇¢ or %.

15. How many apples must I buy so that after allowing 25% of them to be eaten and 20% of the remainder to be given away; I may sell just 1 dozen? Ans. 20 Apples.

16. A fruit dealer has pears worth 5¢ a piece, but will sell 6 for 25¢. What % would be gained by buying 6 for 25¢, and what % would be lost by buying them separately at 5¢ each?

Ans. 1°, 20% gain; 2°, 16⅔% loss.

SYNOPSIS FOR REVIEW.

Define the following words and phrases:

503. Per Cent. 504. The Sign of Per Cent. 505. Percentage. How Applied. 506. Elements Considered in Percentage. 507. The Base. 508. The Rate. 509. The Percentage. 510. The Amount. 511. The Difference. 512. How may Per Cent be Expressed? 515. Table of Aliquots. 517. General Directions to find the Percentage and the Amount or the Difference. 519. General Directions to find the Rate % when Base and Amount or Difference, or when Base and Percentage are given. 521. General Directions to find Base or Cost when Amount or Difference and Rate % are given. 523. General Directions for finding Base or Cost when Rate % and Percentage are given. 524. Marking Goods. 527. Discount, Rebate, and Increase.

INTEREST.

529. **Interest** is money charged or paid for the use of money.

530. The **Principal** is the sum of money on which interest is charged or paid.

531. The **Rate of Interest** is the per cent paid on the principal, for its use, for a specified time.

532. The **Amount** is the sum of the principal and interest.

533. **Simple Interest** is the interest on the principal unincreased by interest, however long overdue.

534. **Legal Interest** is the rate per cent fixed by the law of each State, to apply when no agreement is made. In Louisiana it is 5%.

535. **Conventional Interest** is the rate per cent agreed upon by the parties concerned. The law of many of the States places a limit to this interest. In Louisiana the limit is 8%.

536. **Usury** is a higher rate per cent than the law allows. The law of different States prescribes different penalties for usury. In La. the penalty is the forfeiture of all interest above legal.

537. **Time** is the period for which the principal is loaned or bears interest.

538. In all interest computations, the element of time is combined with the applications of percentage.

539. The Principal, the Interest, the Rate, the Time, and the Amount constitute the five quantities involved in interest questions; and when any three of these are given, the others may be found. Hence there are five classes of interest questions.

INTEREST AND INTEREST DIVISOR.

540. Interest is a specified per cent or number of hundredths, of the principal, paid for its use, for one year of 360 days. It is therefore a compound of per cent, *on a* 100, and per annum *for a year*, 360 days. Hence to obtain 1% interest for 1 year, on any principal, we simply divide by 100. And to obtain 1% interest for 1 day, on any principal, we first divide by 100 to get 1% interest for 1 year and then divide that quotient by 360 to get the interest for one day. Or we may divide the principal at once by 36000 which is the product of 100, the basis of %, and 360 days, the basis of a year.

The quotient arising by this division, being interest, we therefore name the 36000 the **Interest Divisor** for 1 per cent.

Having thus produced the interest at 1% for *one* year or *one* day, to find the interest at any desired rate per cent and for any desired number of years or days, we have but to multiply this interest by the desired rate per cent and the desired number of years or days.

The foregoing is the basis of all interest computations, and by working in accordance therewith we avoid all the arbitrary rules which confuse and confound the millions.

But to perform the operations of interest in detail, as above indicated, would require considerable time and labor. Hence, with a view to economise both, and still work from the foundation principles of interest, we combine reason and cancellation with the foregoing principles and evolve a brief, a simple, and a universal formula, applicable to all interest computations.

A UNIVERSAL FORMULA FOR COMPUTING INTEREST.

541. The solution of the following problem shows the application of the foundation principles of interest, and the evolution in the operations by which the brief, simple, and universal fomula is obtained :

What is the interest of $72000 at 8% for 11 days?
Ans. $176.

First Operation, in detail.

36000)$72000($2 = interest at 1% for 1 day.
　　　　　8 = 8%.
　　　　———
　　$16 = interest at 8% for 1 day.
　　11 = 11 days.
　　———
　$176 = interest at 8% for 11 days.

Second Operation in interest evolution.		Third Operation in interest evolution.		Fourth Operation in interest evolution.
$	$	$		$
72000 or,	72000	72000		720.00
190 | 8　　36000 | 8		4500 | 11	45 | 11
350 | 11	11			
—— |　　　　——		$176,	$176.00
$176, Ans.	$176, Ans.	Ans.		Ans.

Explanation.—In the FIRST operation, we divided by 36000,

the 1% *interest divisor*, and obtained $2, the interest at 1% for 1 day; this we multiplied by the rate, 8%, and obtained $16, the interest at 8% for 1 day; this we multiplied by the days, 11, and obtained $176, the interest at 8% for 11 days.

In the SECOND operation we indicated, on the statement line, the work of the first operation, and then cancelled.

In the THIRD operation, we mentally divided the 36000 by the rate per cent, 8, and produced 4500, the 8% *interest divisor.* By this mental cancellation, we very much shortened the operation.

In the FOURTH operation, we first produce the 4500, the 8% *interest divisor*, then cancel the two 0's and in compensation therefor point off two places in the principal. By this mental cancellation, we shorten the operation to the greatest practical limit, and present a universal formula for interest computations, far superior to the arbitrary rules given in most of the arithmetics.

542. To aid in understanding the interest divisor and the use of the same, we present the following table which gives the interest divisors at 1, 2, 3, 4, 4½, 5, 6, 8, 9, 10, 12, 15, 18, 20, and 24 per cent:

TABLE OF INTEREST DIVISORS.

%	Interest Divisors.		%	Interest Divisors.
$36000 \div 1$	$= 36000$		$36000 \div 9$	$= 4000$
$36000 \div 2$	$= 18000$		$36000 \div 10$	$= 3600$
$36000 \div 3$	$= 12000$		$36000 \div 12$	$= 3000$
$36000 \div 4$	$= 9000$		$36000 \div 15$	$= 2400$
$36000 \div 4½$	$= 8000$		$36000 \div 18$	$= 2000$
$36000 \div 5$	$= 7200$		$36000 \div 20$	$= 1800$
$36000 \div 6$	$= 6000$		$36000 \div 24$	$= 1500$
$36000 \div 8$	$= 4500$			

When the rate of interest will not cancel the 36000 without a remainder, then we proceed as shown in the statement for the second operation.

In case 365 days to the year were used in interest computations, as is the custom in some communities, then we would have $100 \times 365 = 36500$ as the interest divisor at 1 per cent; and hence to find the interest at any other per cent, we would proceed as explained above.

In many computations, Equation of Accounts, Accounts Current and Interest Accounts by the product or equation method, Interest on Daily Cash Balances, Cash Notes, True Discount, etc., the Interest Divisor is of great service, and should be well understood by all accountants.

A PHILOSOPHIC METHOD OF USING THE FACTORS OF THE INTEREST DIVISOR.

543. In the practical computation of interest, we prefer to use the factors of the Interest Divisor (100 and 360, or 365) in a slightly different manner from that shown by the four operations above, yet strictly in accordance with reason and logic.

In order to fully elucidate the work, we will re-state and re-work the above problem.

What is the interest of $72000 at 8% for 11 days?
                                                    Ans. $176.

FIRST OPERATION.   SECOND OPERATION.

$$
360 \,\bigg|\,
\begin{array}{l}
720.00 \\
8 \\
11 \\
\hline
\$176.00 \text{ Ans.}
\end{array}
\qquad
45 \,\bigg|\,
\begin{array}{l}
720.00 \\
11 \\
\hline
\$176.00 \text{ Ans.}
\end{array}
$$

Explanation—In the first operation, our statement conforms to the statement made in the second operation preceding, except

that we first divide the principal, $72000, by 100, by pointing off two places, and then by 360, instead of using the 36000 as a single divisor.

The reasoning for the work based upon the foregoing elucidations, is as follows: 1% interest on any principal for 1 year is the $\frac{1}{100}$ part of it; which we produce by pointing off two places. Then, at 8% it is 8 times as much, which is indicated by writing the 8 on the increasing side of the statement line. Then; since the interest, as indicated by the statement is for 1 year, for one day it is the 360th part, which is indicated by writing the 360 on the decreasing side of the statement line; and for 11 days, it is 11 times as much as for 1 day, which is indicated by writing the 11 on the increasing side of the statement line.

In the second operation, our statement is the same as that in the fourth operation preceding and constitutes the most valuable method known, of computing interest.

The reasoning is the same as in the first operation, except, instead of multiplying by 8 and dividing by 360, we mentally divide the 360 by 8, and then use the quotient, 45, as a *contracted interest divisor.* In this manner we contract the operation to the greatest practical limit and use reason and logic throughout the solution.

———

CONTRACTIONS IN INTEREST OPERATIONS.

544. There are a great many methods of contracting interest calculations, but the greater number of them are applicable only to special combinations of numbers, and others require the memorizing of arbitrary rules and rule exceptions, instead of the exercise of the reasoning faculties. Those that we here present are general in their application, and are based upon one beautiful system of work by which we solve all interest problems, and give a reason for every figure of the work without the aid of rules, no matter what may be the principal, the rate per cent, or the time.

CONTRACTED INTEREST DIVISORS.

545. The following table shows the *Contracted Interest Divisors,* for the most usual rates *per cent:*

TABLE OF CONTRACTED INTEREST DIVISORS.

Ds	%	Interest Divisors.	Ds.	%	Interest Divisors.
360 ÷	1	= 360	360 ÷	9	= 40
360 ÷	2	= 180	360 ÷	10	= 36
360 ÷	3	= 120	360 ÷	12	= 30
360 ÷	4	= 90	360 ÷	15	= 24
360 ÷	4½	= 80	360 ÷	18	= 20
360 ÷	5	= 72	360 ÷	20	= 18
360 ÷	6	= 60	360 ÷	24	= 15
360 ÷	8	= 45	360 ÷	30	= 12

NOTE.—When the rate per cent is not a factor of 360, such as 7%, 11%, etc., then 360 will be the Interest Divisor, and the rate % will be used as a multiplier as shown in the first operation under the Philosophic method.

PROBLEMS IN INTEREST,

WORKED BY THE PHILOSOPHIC SYSTEM.

546. *The Principal, Rate Per Cent, and Time given to find the Interest, the Amount, or the Proceeds.*

1. What is the interest on $560 at 8% for 3 years?   Ans. $134.40.

OPERATION.

$5.60 = 1% of principal.
8 = 8%.

$44.80 = int. for 1 year.
3 = years.

$134.40 = int. for 3 yrs., Ans.

Explanation.— Considering that interest involves per cent and per annum, as elucidated in the foregoing work, and in consonance with the foregoing logical method of operation, we here reason as follows: The interest on $560 at 1% for 1 year is the

hundredth part, $5.60, which we express by pointing off two places; and at 8% it is 8 times as much, which is $44.80; and for 3 years it is 3 times as much as for one year, which is $134.40.

2. What sum must be paid for the interest on $820 at 8% for 1 year and 9 months?

<div style="text-align:right">Ans. $114.80.</div>

OPERATION.

$$12 \left|
\begin{array}{l}
8.20 = 1\% \text{ of principal.} \\
8 \ = 8\%. \\
21 \ = \text{months.} \\
\hline
\$114.80 = \text{int. for 1 yr.} \\
\qquad\qquad 9 \text{ mos., Ans.}
\end{array}
\right.$$

(1 year and 9 months reduced to much as it is for 1 month.

Explanation—In this and in all problems where there are months in the time, we reason as follows: The interest on $820 at 1% for 1 year, or 12 mos., is the *hundredth part*, $8.20; and at 8% it is 8 times as much; and for 1 month instead of 12, it is the 12th part; and for 21 months) it is 21 times as

3. What is the interest on $1230.40 at 9% for 2 years, 5 months, and 24 days?   Ans. $274.9944.

FIRST OPERATION.

$$360 \left|
\begin{array}{l}
12.3040 = 1\% \text{ of principal} \\
9 \qquad = 9\%. \\
894 \qquad = \text{days.} \\
\hline
\$274.9944. = \text{int. for 2 yrs.} \\
\qquad\qquad 5 \text{ mos. and 24 ds.}
\end{array}
\right.$$

SECOND OPERATION.

$$40 \left|
\begin{array}{l}
12.3040 \\
894 \\
\hline
\$274.9944.
\end{array}
\right.$$

Explanation—In this and in all problems where there are days in the time, in accordance with the foregoing elucidated principles, we reason as follows in the first operation: The interest on $1230.40 at 1% for 1 year is the *hundredth part*, $12.3040; and at 9% it is 9 times as much; and for 1 day, instead of 1 year, it is the 360th part; and for 894 days it is 894 times as much.

In the second operation, the same reasoning governed the statement, but instead of writing the 9% and the 360, we used the contracted Interest Divisor, as elucidated in the foregoing work in Article 543.

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GENERAL DIRECTIONS FOR CALCULATING INTEREST.

547. From the foregoing elucidations, we derive the following general directions for calculating interest:

1. *For years, first find* 1% *of the principal, by dividing by* 100 *(pointing off two places); then multiply by the rate* %, *and this product by the number of years.* *See problem* 1, *page* 399.

2. *For months, write the principal on the statement line and find* 1%, *by dividing by* 100, *or pointing off two places; then indicate on the statement line the division by* 12, *and the multiplication by the rate* % *and the number of months, and work out the statement.* *See problem* 2, *page* 400.

3. *For days, write the principal on the statement line and find* 1% *by pointing off two places, or by indicating the division by* 100; *then indicate on the statement line, the division by* 360, *(or* 365 *when that is used) and the multiplication by the rate* % *and the number of days, and work out the statement, cancelling as much as possible.* *See problem* 3, *page* 400.

NOTE —Instead of indicating the division by 360 and the multiplication by the rate %, we may simply divide by the quotient, (the contracted Interest Divisor,) of 360 divided by the rate %, as explained on page 398.

4. *To find the amount, add the interest to the principal.*

5. *To find the proceeds, subtract the interest from the principal.*

NOTE.—There are always as many decimals in the answer as there are on the statement line, and no more.

51

PROBLEMS.

4. What is the interest on $1350 at 8% for 64 days?   Ans. $19.20.

5. What is the interest on $550 at 7% for 72 days?   Ans. $7.70.

6. What is the interest on $727.20 at 5% for 11 days?   Ans. $1.111.

7. What is the interest on $7200 at 10% for 121 days?   Ans. $242.

8. What is the interest on $3155.16 at 6% for 5 years?   Ans. $946.548.

9. What is the interest on $2344.80 at 8% for 3 months?   Ans. $46.896.

10. What is the interest and the amount of $5000 at 8% for 2 years, 5 months, and 15 days?
Ans. $983.33⅓ interest; $5983.33⅓ amt.

Note.—To obtain the amount, add the interest to the principal.

11. What is the interest and the proceeds of $45000 at 4% for 8 months and 20 days.
Ans. $1300 interest; $43700 proceeds.

Note.—To obtain the proceeds, subtract the interest from the principal.

12. What is the interest on $1440 at 7% for 123 days?   Ans. $34.44.

13. What is the interest on $1711.90 for 34 days at 15%?   Ans. $24.25+.

14. What is the interest on $3240 for 94 days at 9%?   Ans. $76.14.

15. What is the interest on $21636.72 for 63 days at 10%?   Ans. $378.64+.

16. What is the interest on $12\frac{1}{2}$¢ at $4\frac{1}{2}$% for $10\frac{1}{2}$ days ?   Ans. $1\frac{21}{1280}$¢.

OPERATION INDICATED.

```
   ¢                        ¢                   ¢
2 | 25             2 | 25              2 | 25
100|        or   36000|         or   8000|
2 | 9              2 | 9               2 | 21
360|               2 | 21
2 | 21
```

17. What is the interest on $16\frac{2}{3}$¢ at $5\frac{1}{2}$% for 8 days, 6 hours, and 24 minutes ?   Ans. $1\frac{341}{16200}$¢.

18. What is the interest on $1500000 for 3 days at $\frac{1}{4}$% per day ?   Ans. $11250.00.

OPERATION INDICATED.   or thus :

```
    $
    | 15000.00                        $15000.00
    | 90=¼% per day=90% per yr.           ¼%
360 | 3                                _____
 ── |                                  $ 3750.00=int. for 1 day.
    | $11250.00, Ans.                       3 =days.
                                       _____
                                       $11250.00=int. for 3 ds.
```

19. What is the interest on $30000 at 4% per month for 28 days ?   Ans. $1120.00.

OPERATION INDICATED.

```
    $                                    $
    | 300.00                             | 300.00
    | 48=4% per mo.=48% per yr.          | 4
360 | 28                       or,  30   | 28
 ── |                                ──  |
    | $1120.00, Ans.                     | $1120.00, Ans.
```

20. What is the interest on \$2500 at 6% for 146 days, counting 365 days as the interest year?

<div align="right">Ans. \$60.00.</div>

OPERATION INDICATED.

$$
365 \underline{\Big|} \begin{array}{l} \text{\$} \\ 25.00 \\ 6 \\ 146 \\ \hline \text{\$60.00, Ans.} \end{array}
$$

NOTE.—The law of the State of New York requires 365 days to be used as a divisor in calculating interest. In all the other States, 360 is used.

21. What is the interest of \$1111.11 at 11% for 11 times 11 days, counting 365 to the year?

<div align="right">Ans. \$40.51+.</div>

MERCHANTS' AND BANKERS' DISCOUNT.

548. For full information in regard to the Special Laws and Business Customs pertaining to Interest and Discount Calculations, see Soulé's Philosophic Work on Practical Mathematics.

549. Bankers' and Merchants' Discount is simple interest on the Principal, at the Rate per cent for the Time that Notes or other obligations have to run.

550. A Promissory Note is a written promise by one party to pay to another party, or his order, a specified sum at a future time, unconditionally.

The following is the usual form of a negotiable promissory note:

\$1480.   NEW ORLEANS, Sept. 20, 1886.

Ninety days after date, for value received, I promise to pay to the order of C. REYNOLDS, One Thousand Four Hundred Eighty Dollars.   S. C. HEPLER.

Due Dec. 19/22/86.

551. The **Parties** to a promissory note are the *maker* and *payee.* In the above note, S. C. Hepler is the *maker* or *promisor*, and C. Reynolds is the *payee* or *promisee.* The *holder* of a note is the party who owns it.

If, in the above note, the words "the order of," before the name of the payee, had been omitted, it would have been unnegotiable.

552. Negotiable Paper is that which may be transferred from one owner to another by assignment or indorsement.

There are several kinds of negotiable paper, namely: Promissory Notes, Bills of Exchange, Due Bills, Bank Notes, Checks on Banks or Bankers, Coupon Bonds, Certificates of Deposit, and Letters of Credit.

553. An **Indorser** of a note is the party who writes his name on the back of the note, and thereby becomes security for its payment. The first indorser of a note is he to whom the note is made payable. A note is not negotiable without the indorsement of the payee.

554. The **Face** of a note, draft, etc., is the sum specified or named therein and promised to be paid.

555. The **Maturity** of a note is the day that the note becomes due.

556. Days of Grace are days allowed for the payment of a note, draft, etc., after the expiration of the time specified in the instrument. By custom in the United States, 3 days of grace are allowed on all notes, drafts, etc., that are not drawn without grace, at sight, or on demand.

557. Dishonoring a note is the failure to pay it when due.

-**558. Discount Day** is the day that a note, draft, etc., is discounted. Many bankers and busi-

ness men when discounting notes, etc., charge interest for this day.

559. The **Proceeds** or **Cash Value** of notes, etc., is what remains after the interest or discount is deducted.

560.   PROBLEMS.

1. March 8, 1886, a note is drawn for $2000 and made payable one year after date, with interest at 8 per cent. What is the interest and the amount due the holder at the maturity of the note?

Ans. $160, interest. $2160, amount.

OPERATION.

$20.00 face of note.
    8%
—————
$160.00 interest for 1 year.

Explanation—Notes of this character that bear interest, are entitled only to interest for 360 days to the year.

$2160 principal and int. added.

2. March 8, 1886, a note was drawn for $2000 and made payable one year after date, without interest. When does this note mature? If discounted at 8% on the day that it was drawn, what was the discount and the proceeds?

Ans. March 8/11, 1887, it matures.
        $164 discount. $1836 proceeds.

OPERATION.
        $
      | 20.00
45 | 369
— | —————
      | $164.00 = discount.

Explanation.—This note being drawn in years, we therefore, in conformity with custom, mature it in years, and when we discount it, in conformity with a different business custom, we count the actual days of unexpired time, including 3 days of grace and discount day.

NOTE.—In maturing notes, drafts, etc., that are drawn in years or months, it is the custom to count from the **day of**

the month that the instrument is dated to the same day of the month in which it matures. Thus, a note dated July 15, 1886, and made payable 3 months after date, matures October 15/18, 1886. But when discounting notes or drafts drawn in years, months, or days, it is the custom to count the actual days of unexpired time, including 3 days of grace and discount day.

561. *To Mature and Discount Notes, when Drawn in Months and when Drawn in Days.*

NOTE.—Observe carefully the difference in the maturity and discount of the two following notes.

$2540.80　　　New Orleans, December 18, 1886.

3. Two months after date, for value received, I promise to pay to the order of Frank Draxler, Two Thousand Five Hundred Forty and $\frac{80}{100}$ Dollars.

A. D. HOFELINE.

When does this note mature? What are the proceeds, if discounted the day it was drawn at 9 per cent? Ans. It matures February 18/21, 1887.

The net proceeds are $2498.88.

OPERATION.

$
40 | 25.40.80
    | 66
—— | ——————
    | $41.9232 discount.
    $2498.88 proceeds.

Explanation. — In maturing this note in accordance with law we count the months, but in discounting in accordance with business custom we count the actual number of days in the two months and add thereto 3 days of grace and discount day.

GENERAL DIRECTIONS FOR BANKERS' AND MERCHANTS' DISCOUNT.

562. From the foregoing elucidations, we derive the following general directions for Bankers' and Merchants' Discount:

1. *Calculate the interest on the note at the given rate for the actual number of days that the note has to run, plus three days of grace and discount day.*

2. *Subtract the interest thus found from the face of the note; the remainder will be the proceeds.*

NOTE 1.—When notes bear interest, find the amount or value of the same at maturity, and calculate the discount on such maturity value.

NOTE 2.—In many Cities and States, interest is not charged for discount day.

$2540.80.   New Orleans, December 18, 1886.

4. Sixty days after date, for value received, I promise to pay to the order of C. Reynolds, Two Thousand Five Hundred Forty and $\frac{80}{100}$ Dollars.

                                   J. B. ANDERSON.

When does this note mature? What are the proceeds, if discounted the day it was drawn, at 9 per cent?   Ans. February 16/19, 1887, it matures.
                        $2500.15, proceeds.

OPERATION.

$
| 25.40.80
40 | 64
— |————————
| $40.6528 discount.
$2500.15 proceeds.

Explanation.—In this problem, according to law, we mature in days, and according to custom we discount in days, counting grace and discount day.

$6231.50.   New Orleans, November 1, 1886.

5. Ninety days after date, for value received, I promise to pay to the order of T. C. W. Ellis, Six Thousand Two Hundred Thirty-one and $\frac{50}{100}$ Dollars, payable at the Germania National Bank, New Orleans.   T. L. MACON, JR.

When does this note become due? What are the proceeds, if discounted December 23, 1886, at 6%?
            Ans. January 30/2, 1887, it is due.
                  $6187.88 proceeds.

NOTE.—Time that has elapsed is not counted when discounting notes.

6. Find the proceeds of a, note for $1428 at 60 days at 8%. Ans. $1407.69.

7. Find the proceeds of a note for $6200 at 90 days at 7%. Ans. $6086.68.

8. Find the proceeds of a note for $91543 at 30 days at 4½%. Ans. $91153.94.

9. What is the maturity, interest, and proceeds of a note for $23875 at 4 months, dated June 15, 1886, and discounted June 26, 1886, at 5% ?

Ans. Matures Oct. 15/18,/86.
Interest, $381.34.
Proceeds, $23493.66.

10. A merchant borrows $50000 for five years at 10% and agrees to pay the principal and interest in 5 equal annual installments. What are the yearly payments ? Ans. $13189.87.

NOTE.—For a solution of this difficult problem see the miscellaneous problems in the back of this book.

TO DISCOUNT NOTES THAT BEAR INTEREST.

$4500.   New Orleans, June 4, 1886.

1. Four months after date, for value received, I promise to pay to the order of O'Neil, Sullivan & Co., Four Thousand Five Hundred Dollars, with six per cent interest.   W. B. McCRACKEN.

When does this note mature? If discounted the day it was drawn by a note broker at 8 per cent, what proceeds would the holder receive?

Ans. It matures October 4/7, 1886.
The proceeds are $4461.48.

OPERATION

To find the amount or value of the note at maturity.

$
12 | 45.00
     | 6
     | 4

$90.00=int. for 4 mos. at 6%.
4500 00=face of note added.

$4590.00=amt. or value of note at maturity.

OPERATION

To discount the note and find the proceeds at maturity.

$45.90
45 | 126

$128.52=int., or discount.
4590.00=value of note at maturity.

$4461.48=proceeds of note the day it was disc't'd.

Explanation —In this problem, we first find the amount or value of the note at maturity. This we do by calculating and adding to the face of the note, the 6 per cent interest that it bears for even 4 months. This work gives us $4590 as the value of the note when it matures; hence it is clear that this is the amount to be discounted. We then discount the $4590 according to business custom for the actual unexpired time, including 3 days of grace and discount day, at the specified 8%.

2. A note bearing 8% interest is given August 15, 1886, payable one year after date, for $5000. What are the proceeds if discounted December 5, 1886, at 8%? Ans. $5091.60.

"CASH NOTES,"

Or Notes and Drafts which, when Discounted, will Produce a Specified Sum.

563. For what sum must a 60 day note be drawn, so that when discounted at 8 per cent, the proceeds will be $8872? Ans. $9000.

FIRST OPERATION.
$100 note assumed.

$
9 4⁵⁄ | 10̸0̸ 20
      | 64
      |————
      | 1280
      |————

$1.42⅔ interest.
100.00 note assu'ed.
————
$98.57⅓ proceeds.

$
887.20 | 100
       | 9
       | 8872.00
       |————
       | $9000, Ans.

Explanation.—As it is the custom of bankers to calculate their discount on the face of notes, drafts, etc., it is plain that if we were to add the simple interest of the $8872 to itself for the time and rate given, and draw the note for the amount thus produced it would not, when discounted, produce the required sum for the reason that the interest on this increased amount would be more than the interest on the first sum. The deficit would be the interest on the interest first obtained, plus the interest on each succeeding sum of interest, interminably. Consequently, to produce exact results, we cannot work on the face of the sum that we desire to obtain for the note when discount-ed. We are therefore constrained to assume *some number* to represent the face of the note to be drawn. In this solution we assume $100, which we discount for the time and rate, and thus produce $98.57⅓ proceeds. By the figures now before us and the exercise of our reason, we see that a $100 note for the time and rate given, is worth $98.57⅓ cash, and

by transposition that \$98.57⅝ cash proceeds are worth a **\$100** note. We now observe that as we discounted the \$100 for 64 days at 8 per cent, the same ratio exists between the \$98.57⅝ proceeds and the \$100 note, as exists between the \$8872 proceeds and the face of the note required to produce the same when discounted for 64 days at 8 per cent; and hence we have but to find the proportional result of these two ratios. To do this we place the \$100 assumed note on the statement line, and reason thus: if $^{885}_{8}{}^{20}\!\!\not{e}$ (\$98.57⅝ reduced) proceeds require \$100 note, $\frac{1}{8}\!\!\not{e}$ proceeds will require the 8872th part, and ⅝, or a whole cent, will require 9 times as much; and if 1$\not{e}$ require the result of the present statement, 887200$\not{e}$ proceeds will require 887200 times as much. This gives us \$9000 as the face of the note to be drawn.

SECOND OPERATION.

\$4500 note assumed.

\$64 int. for 64 ds. at 8%

———

\$4436 proceeds, cash.

$$
4436\ \Big|\ \begin{matrix} \$ \\ 4500 \\ 8872 \\ \hline \$9000,\ \text{Ans.} \end{matrix}
$$

Explanation.—In this solution, which we much prefer to the first, in order to save time and labor, we assume the 8% interest divisor to represent the face of the note. We assume the interest divisor for the reason that the interest thereon is always as many dollars as we have days. Hence to produce the required relationship numbers representing note and proceeds, we have but to subtract the days from the interest divisor. Having produced these numbers we reason thus: Since \$4436 cash require \$4500 note, \$1 cash will require the 4436th part and \$8872 will require 8872 times as much.

———

GENERAL DIRECTIONS.

564. From the foregoing elucidations, we derive the following general directions for finding the face of "Cash Notes:"

1. *Assume as the face of the note* \$100, *or the Interest Divisor for the rate % given, and find the proceeds of the same for the given time and rate %.*

2. *Then divide the assumed note multiplied by the* **required** *proceeds, by the proceeds of the assumed note.*

PROBLEMS.

2. What must be the face of a note, so that, when discounted for 94 days at 12 per cent, it will produce $10000 proceeds.   Ans. $10323.47.

Operation by the use of the Interest Divisor.	$
$3000 note assumed.	3000 note as-
94 int. for 94 ds. @ 12%. 2906	10000 sumed.
$2906 cash proceeds.	$10323.47, Ans.

3. A creditor owed me a balance of $3212.65 and settled the same with his "cash note," payable 4 months after date. The note was dated June 17, 1886. Allowing 8 per cent interest, what was the face of the note, and when does it mature?

Ans. $3305.19+ face of note.

Oct. 17/20, 1886, it matures.

4. What must be the face of a note to net or produce $1777.95, when discounted at 7% for 63 days?   Ans. $1800.

OPERATION.	$
$36000=note assumed	36000
441=int. for 63 ds. @ 7%. 35559	1777.95
$35559=proceeds.	$1800.00, Ans.

Explanation.—There being no 7% Interest Divisor, we assume as the face of the note the 1% Interest Divisor and then multiply the interest at 1%, which is $63 (as many dollars as there are days) by 7, the rate % and thus obtain $441 int. The statement is then made as in the preceding examples.

NOTE.—Whenever there is no interest divisor for the rate % given, the interest divisor for 1% should be assumed and the interest found thereon as above, or, if preferred, in the usual manner.

5. For what sum must a note be drawn for 45 days, without grace or discount day, to settle a cash balance of $10862.50, interest at 10%?

<div align="right">Ans. $11000.</div>

565.   "CASH NOTES"

With Interest, Commission, and Brokerage Combined.

1. A customer desires to obtain from a bank $8000 on his 90 day note. In conformity with bank custom, which prudence and safety demand, he is required to have one or more indorsers on the note, and as his correspondent I indorse and negotiate the note for him. The rate of bank discount is 8%; I charge 2½% commission for indorsing, and ¼% brokerage for negotiating. What must be the face of the note?   Ans. $8406.80.

OPERATION.

Face of note assumed, - - - -	$4500.
Int. on same for 94 ds. at 8% - $ 94.	
Com. " " @ 2½% - - 112.50	
Brok. " " @ ¼% - - 11.25 ——	217.75

Cash value, or pro. of the assumed note. $4282.25

	$	
	4500 = note assumed.	
4282.25	8000.00	
	——————	
	$8406.80, Ans.	

Explanation.—In this solution, for reasons given in the second solution of the first problem of "Cash Notes," we assume the 8% Interest Divisor as the face of the required note, and from it we deduct the interest, commission and brokerage, and thus produce the necessary relationship numbers, as explained in the first solution with which we make the proportional statement, the result of which gives the correct answer.

2. A merchant owes a cash balance of $4000, which he wishes to settle by note at 120 days for such a sum as, when discounted at 8 per cent, allowing 2½ per cent commission for indorsing, will net the exact cash balance. What must be the face of the note? Ans. $4221.88.

3. What must be the face of a note for 60 days to net $1720, discount at 5% and 2½% commission for indorsing? Ans. $1780.33+.

TRUE DISCOUNT

566. **True Discount** is such a deduction from the face of notes or debts, as is equal to the simple interest on the remainder for the same time and rate for which the deduction was made.

567. The **Present Worth** of notes or debts due at some future time without interest, is such a sum of money which, for a given time and rate %, will amount to the face of the note or debt at maturity.

1. What is the present worth and the true discount of a note of $8400 for 60 days at 8%?
Ans. $8282.20 present worth.
117.80 true discount.

OPERATION.

$4500=present worth assumed.
64=interest for 64 ds. @ 8%.
—————
$4564=amt. due 64 ds. hence

	$
	4500, p. w. assumed.
4564	8400
	—————
	$8282.20, pres. worth.
	$8400.00, face of note.
	—————
	$117.80, true disc't.

Explanation.—Since true discount is the interest on the present worth instead of the face of the note or debt, we cannot therefore operate on the $8400, the face of this note,

and hence we assume as the *present worth*, the Interest Divisor for the given %. Then, in order to obtain an amount of note and interest which bear the same relationship to the *present worth* assumed, that the given amount, $8400, bears to its present worth, we calculate the interest on the assumed note for the given time, and rate, and add the same to the note assumed. Having produced these relationship numbers we reason as follows: Since $4500 present worth give an amount of $4564, conversely $4564 amount required a $4500 present worth; and since $4564 amount require $4500 present worth, $1 amount will require the 4564th part and $8400 amount will require 8400 times as much.

NOTE.—We might have assumed any other sum to represent the present worth, and produced the same result. We assumed the 8% Interest Divisor because when that number is assumed, the interest is always as many dollars as there are days in the time.

———————

GENERAL DIRECTIONS.

568.   From the foregoing elucidations, we derive the following general directions for calculating True Discount :

1. *Assume as the* Present Worth, *the Interest Divisor, for the given rate % and find the* amount *of the same for the given time and rate %.*

2. *Then divide the assumed Present Worth multiplied by the given amount, by the amount of the assumed present worth.*

PROBLEMS.

2. What principal must be loaned for 183 days to amount to $3915.90, interest at 6% ?

Ans. $3800.

3. What is the present worth of $4391.68 due in 1 year and 4 months, money worth 5%, and no allowance for days of grace or discount day?
                                        Ans. $4117.20.

4. What is the true discount on $8765.25 at 9% for 495 days?   Ans. $965.25,

PROBLEMS INVOLVING INTEREST OPERATIONS.

569. *To find the Rate % when the Principal, Time, and Interest are given.*

1. The interest on $8400 for 63 days was $117.60. What was the rate %?   Ans. 8%.

OPERATION
to find the interest on $8400
for the given time at 1%
assumed.

	$
	84.00
360	63
—	—
	$14.70

OPERATION
to find the rate %.

	1% assumed.
14.70	117.60
—	—
	8%, Ans.

Explanation.—In all problems of this kind, we first assume 1%; then, in order to obtain a sum of interest that bears the same relationship to the 1% assumed, that the $117.60 bears to the required rate %, we calculate the interest on the principal for the given time at 1%. Accordingly we find, in this problem, $14.70 interest. We then reason as follows: Since 1% gives $14.70 interest, by transposition $14.70 required 1%; and since $14.70 interest required 1%, 1 cent interest will require the 1470th part and $117.60 interest will require 11760 times as much.

GENERAL DIRECTIONS.

570. From the foregoing elucidation, we derive the following general directions for finding the rate %:

53

1.   *Assume* 1% *and find the interest on the principal for the given time at the* 1%.

2.   *Then divide the given interest by the* 1% *interest on the principal for the given time.*

PROBLEMS.

2.   The interest on $1000 for 288 days was $48. What was the rate %?   Ans. 6%.

3.   A note for $524.86 was discounted for 47 days and $518.6336 proceeds received. At what rate % was it discounted?   Ans. 9%.

4.   A. received $3679.20 for his note of $4200 which had 186 days to run. At what rate % was it discounted?   Ans. 24%.

571.   *To find the Principal when the Rate %, Time, and Interest or Amount or Proceeds are given.*

1.   What principal loaned for 63 days at 8% will produce $117.60 interest?   Ans. $8400.

OPERATION to find the interest on the assumed principal.		OPERATION to find the principal.	
	$		$ [cipa.
	! 00		100 assumed prin-
45	63	1.40	117.60
—		—	
	$1.40		$8400, Ans.

SECOND OPERATION.

	$
	4500 assumed principal
63	117.60
—	
	$8400.00, Ans.

Explanation.—In all problems of this kind, we assume $100 or the Interest Divisor, as principal, and then calculate the

interest thereon for the given rate and time. This is done in order to produce an interest which bears the same relationship to the assumed principal that the given interest bears to the required principal that produced it.

In the first solution of this problem, we find the interest on the assumed principal to be $1.40. Having this interest, we make the relationship or proportional statement, reasoning as follows: Since $100 principal gives $1.40 interest, conversely $1.40 interest required $100 principal. And since $1.40 interest required $100 principal, 1 cent interest will require the 140th part, and $117.60 interest will require 11760 times as much, which is $8400 principal.

In the second operation we assumed the 8% Interest Divisor, $4500, as the principal. We much prefer the second operation, for the reason that the interest on the Interest Divisor, for the time and at the rate %, will always be the same as the number of days. And hence, this being known, we save in the operation making one interest calculation.

2.   What principal loaned for 121 days at 10% will amount to $7442 ?   Ans. $7200.

OPERATION.

$

Amount=3721 | $3600=$ assumed principal.
$7442=$ amount.

$7200, Ans.

Explanation—When the amount is given, instead of the interest, then add the interest on the assumed principal to the assumed principal, and then make the proportional statement.

3.   What principal discounted for 369 days at 8% will give $1836 proceeds ?   Ans. $2000.

OPERATION.

$

Proceeds=4131 | $4500=$ assumed principal.
$1836=$ proceeds.

$2000, Ans.

Explanation.— When the proceeds are given, instead of the interest, then subtract the interest on the assumed principal from the assumed principal and then make the proportional statement.

GENERAL DIRECTIONS.

572. From the foregoing elucidations, we derive the following general directions for finding the principal:

1. *Assume $100 or the Interest Divisor as principal, and calculate thereon the interest for the given time and rate %.*

2. *Then divide the assumed principal multiplied by the given interest, by the interest on the assumed principal.*

Note.—If the amount is given instead of the interest, then add the interest on the assumed principal to the assumed principal, and divide the sum into the product of the assumed principal and the given amount.

If the proceeds are given instead of the interest, then subtract the interest on the assumed principal from the assumed principal, and divide the difference into the product of the assumed principal and the given proceeds.

PROBLEMS

4. A banker loaned a sum of money for 369 days at 8% and received $164 interest. What was the sum loaned? Ans. $2000.

5. What principal loaned for 90 days at 6% will amount to $2030? Ans. $2000.

6. Discounted a note for 183 days at 5% and received $1637.30 proceeds. What was the face of the note? Ans. $1680.

7. What principal loaned for 64 days @ 8% will amount to $1369.20? Ans. $1350.

8. A merchant discounted a note for 64 days at 5% and received $178.40 proceeds. What was the face of the note? Ans. $180.

9. What principal loaned for 124 days at 7%
will produce $130.20 interest ? Ans. $5400.

OPERATION.	*Explanation.—*

868.00 | $36000=1\%$ int. Divisor as assumed principal. | Since there is no 7% Interest Divisor, we assume
| 130.20 | as principal 36000, the 1% Interest
| ————— | Divisor, and then
| **$5400, Ans.** | multiply the in-

terest at 1%, which is equal in dollars to the number of days,
by 7, the rate %, and thus produce $868.00 interest. The
solution statement is then made as in the preceding problems.

10. What principal loaned for 72 days at 11%
will produce $33 interest ? Ans. $1500.

573. *To find the Time, when the Principal, Rate
%, and Interest, or when the Principal and the
Amount or Proceeds and the Rate % are given.*

1. Loaned $8400 at 8% and received $117.60
interest for it. How long was it loaned ?
  Ans. 63 days.

OPERATION to find the interest on the $8400 @ 8% for 1 yr. assumed.		OPERATION to find the number of days.
$84.00		360 days, or 1 yr. assumed.
8	672.00	117.60
—————		—————
$672.00, interest.		63 days, Ans.

Explanation. —In all problems of this kind, we first assume
360 days, or 1 year. Then, in order to obtain a sum of inter-
est that bears the same relationship to the 360 days of
assumed time that the $117.60 given interest bears to the
required time, we calculate the interest on the principal
loaned for the assumed time at the given %. Accordingly,
we produce, in this problem, $672.00 interest. We then rea-
son as follows: Since 1 year's time, with the given principal
and rate, give $672.00 interest, by transposition $672.00 inter-

est required 1 year's or 360 days' time; and since $672.00 interest required 360 days, time, 1 cent interest will require the 67200dth part, and 11760 cents interest will require 11760 times as many, which is 63 days.

GENERAL DIRECTIONS.

574. From the foregoing elucidation, we derive the following general directions for finding the Time.

1. *Assume 360 days, 1 year, and find the interest on the given principal at the given rate % and for the assumed time.*

2. *Then divide the given interest multiplied by the assumed time, by the interest on the principal for the rate % and assumed time.*

NOTE.—When the amount is given, first subtract the interest from the same to find the principal.

When the proceeds are given, first add the interest to the proceeds to find the principal.

PROBLEMS.

2. A note for $6000 was discounted at 5% and the interest or discount was $78.33⅓. For how many days was it discounted ?

Ans. 94 days.

3. A merchant borrowed $2500 at 4½%, and paid $38.75 for its use. How long did he have the money ? Ans. 124 days.

4. Loaned a sum of money at 8% until it amounted to $461.37. The interest was $6.37. How long was it loaned ? Ans. 63 days.

5. A note was discounted at 10% and $800 proceeds received. The discount was $200. For what time was the note discounted?

Ans. 720 days., or 2 years.

SYNOPSIS FOR REVIEW.

Define the following words and phrases:

529. Interest. **530.** Principal. **531.** Rate of Interest. **532.** Amount. **533.** Simple Interest. **534.** Legal Interest. **535.** Conventional Interest. **536.** Usury. **537.** Time. **539.** Five *quantities* in Interest Questions. **540.** Interest Divisor. **541.** Elucidation of Interest Divisor. **542.** Table of Interest Divisors. **543.** Philosophic Method of using the Interest Divisor. **545.** Table of Contracted Interest Divisors. **547.** General Directions for Calculating Interest. **549.** Merchants' and Bankers' Discount. **550.** A Promissory Note. **551.** The Parties to a Promissory Note. How do the words "the order of" affect the negotiability of the note? **552.** Negotiable Paper. Kinds of Negotiable Paper. **553.** An Indorser. How does the indorsement of the Payee affect the negotiability of the Note. **554.** The Face of a Note. **555.** The Maturity of a Note. **556.** Days of Grace. **557.** Dishonoring a Note. **558.** Discount Day. **559.** Proceeds or Cash Value. **562.** General Directions for Bankers' and Merchants' Discount. **563.** Cash Notes. **564.** General Directions for Cash Notes. **565.** Cash Notes with Interest, Commission, and Brokerage Combined. **566.** True Discount. **567.** Present Worth. **568.** General Directions for True Discount. **570.** General Directions to find the Rate of Interest. **572.** General Directions to find the Principal. **574.** General Directions to find the Time.

MENSURATION.

575. Mensuration is the process of finding the length of lines, the area of surfaces, and the volume or solidity of solids. The principles that govern the process of work are derived from Geometry, a very important and interesting branch of mathematics, but which cannot be fully explained in a treatise of this character.

For the most extended and thoroughly elucidated work on Mensuration of Surfaces and Solids, ever presented in any other arithmetic or calculator, see Soulé's Philosophic Work on Practical Mathematics.

DEFINITIONS.

576. **A Point** is that which has position without measurable length, width, or thickness.

577. **A Line** is that which has length without measurable width or thickness.

578. **A Surface** is that which has length and width only.

579. **A Polygon** is a plane figure, or portion of a surface bounded by straight lines.

580. **An Ellipse** is a figure bounded by an oval curved line.

The *Transverse Diameter, or Axis* of an ellipse is a line passing through its center in the direction of its length.

The *Conjugate Diameter, or Axis,* is a line passing through the center of the ellipse in the direction of its width.
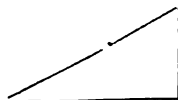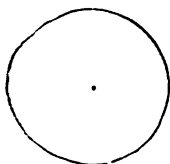
581. **A Square** is an equilateral rectangle.

A Square.

582. A Rectangle is a quadrilateral polygon which- has its opposite sides equal and parallel, and all its angles right angles.

A Rectangle.

583. A Right-Angled Triangle is a triangle that has one of its angles a right-angle.

A Right-Ang'ed Triangle.

584. A Circle is a plane figure bounded by a regular curved line, every point of which is equally distant from a point within called the center.

A Circle.

For the definition of *the Diameter, the Circumference, the Radius, a Chord, an Arc, a Sector, an Angle,* and a *Segment of a Circle,* see page 271.

585. The **Ratio** between the Diameter and the Circumference of a circle has been demonstrated in geometry to be as 1 to 3.1416; i. e., when the diameter of a circle is 1, the circumference is 3.1416.

586. The **Area** of a figure, or of a described object, is the measure of its surface in some unit, as the inch, the foot, the yard, the mile, etc.

587. The **Ratio** between the area of a circle and of a square one side of which is equal to the diameter of the circle has been demonstrated in geometry to be as 1 to .7854; i. e., when the area of a square is 1, the area of the circle is .7854.

54

588.   LINEAR MEASURE.

PROBLEMS.

1.   The diameter of a circle is 50 feet.   What is the circumference ?   Ans. 157.08 feet.

OPERATION.

$50 \times 3.1416 = 157.0800$ ft., Ans.

Explanation. — In all problems of this kind, we multiply the diameter by 3.1416, which is the ratio between the diameter and the circumference.

NOTE.—In all problems in Linear, Surface, or Solid measure, the student should draw on his paper the outline of the figure to be measured, before performing the operation.

2.   The circumference of a circle is 40 feet.   What is the diameter ?   Ans. 12.73+ft.

OPERATION.

$40.0000 \div 3.1416 = 12.73 +$ ft., Ans.

Explanation.—In all problems of this kind, we divide the circumference by 3.1416, which is the ratio between the diameter, and the circumference of a circle.

GENERAL DIRECTIONS.

589.   From the foregoing elucidations, we derive the following general directions for the mensuration of the diameter and the circumference of circles :

1°.   *To find the circumference when the diameter is given,* multiply *the diameter by* 3.1416.

2°.   *To find the diameter when the circumference is given, divide the circumference by* 3.1416.

3.   What is the circumference of a circular garden, the diameter being 25 yards and 2 feet ?
Ans. 241.9032 ft.

4.   What is the diameter of a circle whose circumference is 68 feet 9 inches ?   Ans. 262.605 in.

590.　　　MENSURATION OF SURFACES.

PROBLEMS.

1. What is the area of a garden 240 ft. long and 120 ft. wide?　　　　Ans. 28800 sq. ft.

OPERATION.
240 ft. long.
120 ft. wide.
—————
28800 sq. ft., Ans.

Explanation.— In all square or rectangular figures, we multiply the length by the width, in the same units of measure, and in the product we have the required area.

2. How many square feet in a *right-angled triangle* whose base is 12 feet and perpendicular height is 8 feet?

OPERATION.
12 ft. long.
8 ft. high.
——
96÷2=48 sq. ft., Ans.

Explanation.—In all problems of this kind, we multiply the length by the height, which gives the sq. ft. of a rectangle of equal length and height. This result is then divided by 2, since a right angled triangle is equal to but *one-half* of a rectangle of equal length and height.

3. What is the number of square yards in a circular piece of ground which is 20 yards in diameter?　　　　Ans. 314.16 sq. yds.

OPERATION.
20 yds. diameter = length.
20 yds. diameter = width.
——
400 = the sq. yds. in a square which is 20 yds. long and 20 yards wide.
400 × .7854 = 314.1600 sq. yds.
　　　　　　　Ans.

Explanation. — In all problems of this kind, we multiply the diameter by itself as is shown in the operation, and then multiply this product by .7854, which has been demonstrated in geometry to be the ratio between the area of a square and the area of a circle, the diameter of which is equal to one side of the square.

GENERAL DIRECTIONS.

591. From the foregoing elucidations, we derive the following general directions for the measurement of surfaces:

1°. *To find the area of a square or rectangle, multiply the length by the width.*

2°. *To find the area of a right-angled triangle multiply the length by the height, and divide the product by 2.*

3°. *To find the area of a circle, multiply the diameter by itself, and the product thus obtained by .7854.*

4. A water pipe is 50 feet 9 inches long, and its diameter is 30 inches. What is its concave surface?
Ans. 57397.032 sq. inches.
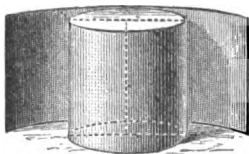
OPERATION.

$30 \times 3.1416 = 94.248$ circumference, or linear width of pipe.
94.248×609 in., length $= 57397.032$ sq. in., Ans.

5. How many square feet in a floor 22 feet 4 inches long and 16 feet wide?   Ans. $357\frac{1}{3}$ sq. ft.

6. How many square yards of plastering in one wall of a house 32 feet 4 inches long and 15 feet 3 inches high?   Ans. $54\frac{85}{108}$ sq. yds.

7. How many square feet in the bottom of a cistern whose diameter is 11 feet 8 inches?
Ans. $106.9016\frac{2}{3}$ sq. ft.

8. What is the area of the base of a cylinder whose circumference is 62.832 inches?
Ans. 314.16 sq. in.

9. A lumberman has 25 boards each 14 feet long and 15 inches wide. How many square feet in all?
Ans. $437\frac{1}{2}$ sq. ft.

10. Find the area in square feet of a plank 22 feet 3 inches long, $2\frac{1}{4}$ feet wide at one end and 14 inches wide at the other. Ans. $38\frac{1}{96}$ sq. ft.

11. Find the area of a piece of ground which is 406 feet long and is of the following width at different points, equally distant from each other : at the wider end, 210 feet; at the narrower end, 165 feet; near the wider end, 180 feet; near the narrower end, 142 feet; in the middle, 300 feet.
Ans. $82164\frac{1}{4}$ sq. ft.

NOTE.—Draw a diagram of the ground before working the problem.

OPERATION INDICATED
to find the average width.
$$\frac{210+165}{2}=187\frac{1}{2},\ \frac{+180+142+300}{4}=202\frac{2}{3}\ \text{ft. average}$$
width.

PAVING YARDS.

1. How many bricks will be required to pave a sidewalk 64 feet long and 11 feet 8 inches wide, each brick being 8 inches long and 4 inches wide?
Ans. 3360 bricks.

OPERATION INDICATED.

Length of brick=8 | 768=length of sidewalk in inches.
Width " " =4 | 140=width of sidewalk in inches.
— | 3360=number of bricks to pave the sidewalk.

Explanation —In all problems of this kind, we first make

the statement to ascertain the number of square inches in the sidewalk, by multiplyin the length by the width in the unit of inches ; and ther we divide this result by the product of the length and the width of a brick.

2. How many German flags, each 16 in. by 16 inches, will it take to pave a yard 45 feet square ?
Ans. 1139$\frac{1}{16}$ flags.

3. A circular court is 30 feet in diameter. How many tiles, each 6 inches square, will it require to cover the court, making no allowance for waste ?
Ans. 2827.44 tiles.

SLATING AND SHINGLING ROOFS.

1. The roof of a building is 72 feet 6 inches long and measures 46 feet 9 inches from eave to eave. How many slates and how many squares of slating are there in the roof, allowing a slate to cover a space 4$\frac{1}{2}$ by 8 inches and not allowing for the double course at the eaves ?   Ans. 13557$\frac{1}{2}$ slates.
33$\frac{143}{160}$ sq. of slating.

Operation to find the number of slates.

Width of slate = 8	870 in.=length of roof.
	561 in.=width " "
Length of slate 9	2
exposed on the —	———
roof.	13557$\frac{1}{2}$ slates, Ans.

Explanation.—In all problems of this kind, we first find the number of square inches in the roof by multiplying together the length and the width of the roof in the unit of inches ; and then divide the same by the number of square inches that each slate covers.

Operation to find the number of squares of slating.

12	870
12	561
100	

$33\frac{43}{60}$ squares, Ans.

Explanation.—Here we first make the statement to find the number of square feet and then divide by 100, which is the number of square feet in a *square*, as per Article 373, page 264.

2. How many shingles will it require to shingle a house that is 54 feet long and 35 feet 10 inches from eave to eave, estimating that 5 inches of each shingle will be laid to the weather, and allowing for the double course at the eaves on each side?

NOTE.—The unit width of a shingle is 4 inches.

Ans. 14256 shingles.

3. A flat roof is 208 feet 2 inches long and 28 feet 5 inches wide. How many square yards of tin will be required to cover it, and what will be the cost at $1.15 per square yard?

Ans. $657\frac{11}{12}\frac{3}{4}$ sq. yds.
$755.85+cost.

CARPETING FLOORS.

1. How many yards of carpeting that is 27 inches wide, will be required to cover the floor of a parlor that is 32 feet 4 inches long and 25 feet 6 inches wide, making no allowance for waste in matching or turning under? Ans. $122\frac{4}{27}$ yds.

FIRST OPERATION.

36	388
27	306

$122\frac{4}{27}$ yds., Ans.

Explanation.—In this solution, we first find the number of *square inches* in the floor, by multiplying together the length and width in the unit of inches. We then divide by the product of the length and width of a yard of the carpet, which is the number of square inches in one yard of it.

SECOND OPERATION.

12	388		3	97
12	306	or	2	51
9			9	
27	36		27	36

$122\frac{4}{27}$ yds., Ans. | $122\frac{4}{27}$ yds., Ans.

Explanation.—In this solution, we first find the number of *square yards* in the floor, which would be the number of yards required if the carpeting was 1 yard, or 36 inches, wide. But since the carpeting is not 1 yard wide, we multiply by 36, which gives the number of yards required if the carpeting was but 1 inch wide, and we then divide by 27, the width of the carpeting, and produce the correct result.

We reason as follows: Since it requires this expressed number of yards when it is 36 inches wide, if it was but 1 inch wide it would require 36 times as many yards, and if 27 inches wide, the 27th part.

2. A room is 28 ft. 6 inches long and 18 feet 9 inches wide. Making no allowance for waste, how many yards of matting will it take to cover the floor, the matting being $1\frac{1}{4}$ yards wide, and what will be the cost at 55¢ per linear yard for the matting? Ans. $47\frac{1}{2}$ yards.

$26.12\frac{1}{2}$¢ cost.

MENSURATION OF SOLIDS.

DEFINITIONS.
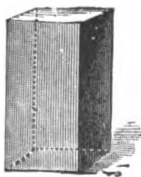
592. A Rectangular or Quadrilateral **Solid** is a solid which has length, width, and thickness, and is bounded by six *sides* or *faces*.

593. A Cube is a solid whose sides or faces are all equal squares.
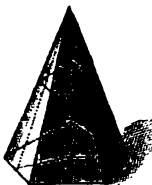
594. A Pyramid is a solid whose base is any kind of a polygon, and its other faces, triangles united at a common point called the *vertex*.

595. A Frustum of a Pyramid is the part which remains after the top is cut off by a plane parallel to the base.

596. A Cylinder is a solid *having two* faces or bases, which are equal parallel circles, and which have an equal diameter in any parallel plane between them.
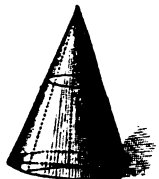
597. A Cone is a solid having one face or base which is a circle, and a convex or curved surface terminating in a point, called the *vertex*.

598. A Frustum of a cone is the part which remains after the top is cut off by a plane parallel to the base.

599. A Sphere is a solid bounded by a curved surface, all the points of which are equally distant from a certain point within called the *center*.

The **Radius** of a sphere is a line drawn from the center to any part of the circumference. The **axis, or diameter of** a sphere is a line passing through the center and terminated by the circumference.
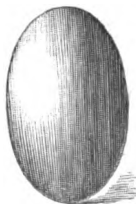
600. A Prolate Spheroid is a solid elongated in the direction of a line joining the poles; or, it is a solid generated by the revolution of an ellipse about its longer axis.

601.. An Oblate Spheroid is a solid flattened or compressed at the poles; or, it is a solid generated by the revolution of an ellipse about its shorter axis.

PROBLEMS.

602. 1. A Rectangular Box is 4 feet long, 3 feet 4 inches wide, and 2 feet high. How many solid, or cubic feet does it contain ? Ans. 26⅔ cu. ft.

OPERATION.

$$3 \begin{vmatrix} 4 \\ 10 \\ 2 \end{vmatrix} \quad \text{or} \quad 12 \begin{vmatrix} 4 \\ 40 \\ 2 \end{vmatrix}$$

26⅔ cu. ft., Ans.   26⅔ cu. ft., Ans.

Explanation. — In all Cubical or Rectangular solids, we multiply together the length, width, and height, in the same units of measure, and in the product we have the required solidity.

2. A Square or Rectangular Pyramid is 8 feet high and each side of the base is 4 feet 6 inches. How many cubic feet does it contain ?

Ans. **54 cu. ft.**

OPERATION.

	8			8
2	9		12	54
2	9	or	12	54
3				3
	——		——	——
	54 cu. ft., Ans.			**54 cu. ft.,** Ans.

Explanation.— In all problems of this kind, we multiply the height by the *area* of the base, and then divide by 3, because a *square* pyramid has been demonstrated in geometry to be ⅓ of a rectangular solid of equal height and area of base.

3. A Frustum of a Square or Four-sided Pyramid is 8 feet high, lower base 7 feet, and upper base 6 feet. How many solid feet does it contain ?

Ans. 338⅔ cu. ft.

OPERATION.

$7^2 = 49 =$ square of the greater base.
$6^2 = 36 =$ " " " lesser "
$7 \times 6 = 42 =$ geometrical mean proportional between the two bases.

——————

3) 127

42⅓ = average area of the frustum of the
8 = height.   pyramid.

——————

338⅔ cu. feet, Ans.

Explanation.—In all problems of this kind, we first find, as shown in the operation, the average area of the frustum of the pyramid, and then multiply the same by the height, or altitude.

NOTE.—For a full elucidation of the various measurements of all kinds of pyramids, see Soulé's Philosophic Work on Practical Mathematics.

4.   A cylinder is 8 feet high and 4 feet 6 inches in diameter.   How many cubic feet does it contain?

<div align="right">Ans. 127.2348 cu. ft.</div>

OPERATION.   *Explanation.*

	8			8
2	9		12	54
2	9	or	12	54
	.7854			.7854
—			—	
	127.2348 cu. ft.,			127.2348 cu. ft.,
	Ans.			Ans.

In all problems of this kind, we multiply the height by the square of the diameter, which gives the solidity of a rectangular solid whose height and width of sides are equal to the height and diameter of the cylinder. Then we multiply by .7854, the ratio between the area of a square and that of a circle whose diameter is equal to one side of the square.

5.   A cone is 8 feet high and its base is 4 feet 6 inches in diameter.   How many solid feet does it contain?

<div align="right">Ans. 42.4116 cu. ft.</div>

OPERATION.   *Explanation.—*

	8			8
2	9		12	54
2	9	or	12	54
	.7854			.7854
3			3	
—			—	
	42.4116 cu. ft.,			42.4116 cu. ft.
	Ans.			Ans.

In all problems of this kind, we make the same solution as in the preceding problem and then divide by 3, because the solidity of a cone is ⅓ of a cylinder whose height and diameter are equal to that of the cone.

6. A frustum of a cone is 8 feet high, lower base 7 feet in diameter, and upper base 6 feet in diameter. How many solid feet does it contain ?

<div align="right">Ans. 265.9888 cu. ft.</div>

<div align="center">OPERATION.</div>

$7^2 = 49 =$ square of greater diameter.

$6^2 = 36 =$ "   "   lesser   "

$7 \times 6 = 42 =$ geometrical mean between the two diameters.

$3 \,) \, 127$

$42\frac{1}{3} =$ area of a rectangular solid, the sides of whose bases are equal to the average diameter of the frustum of the cone.

$$3 \,\big|\, \begin{array}{l} 127 \\ .7854 \\ 8 \\ \hline \end{array}$$

$-$ 265.9888

<div align="center">cu. ft.,
Ans.</div>

Explanation.—In all problems of this kind, we first find, as shown in the operation, the area of a rectangular solid the sides of whose bases are equal to the average diameter of the frustum of the cone; then we multiply this by .7854 for reasons given in problem 4 above, and then by the height.

NOTE.—For a full discussion of this kind of problems, see **Soulé's** Philosophic Work on Practical Mathematics.

7. A sphere is 4 feet in diameter. How many cubic feet does it contain ?   Ans. 33.5104 cu. ft.

<div align="center">OPERATION.</div>

4^3, or $4 \times 4 \times 4 = 64 =$ cu. ft. in a cube which is 4 feet on each side.

$64 \times .5236 = 33.5104$ cu. ft., Ans.

Explanation.—In all problems of this kind, we cube the diameter by multiplying it by itself 3 times, as shown in the operation, and then multiply this result by .5236, which is the ratio between the solidity of a cube and that of a sphere whose diameter is equal to one side of the cube.

8. A prolate spheroid has a transverse, or longer, diameter of 8 feet, and a conjugate, or shorter, diameter of 5 feet. How many cubic feet does it contain ?   Ans. 104.72 cu. ft.

<div align="center">OPERATION.</div>

	8=height.	8
	5=diameter. or	5
	5=diameter.	5
	.7854=ratio of cir., etc.	.5236
3	2=ratio bet. C. & P. S. —	———
—	———	104.72 cu. ft.
	104.72 cu. ft., Ans.	Ans.

Explanation.—In the first statement, we indicate the solution for a cylinder of equal height and diameter as the prolate spheroid, and then multiply by ⅔, because a prolate spheroid is equal to ⅔ of a cylinder of equal height and diameter.

In the second statement, we indicate the solution for a rectangular solid, by multiplying together the three dimensions, and then multiply by .5236, which is the ratio between the solidity of a cube and that of a sphere, the diameter of which is equal to one side of the cube.

9. An oblate spheroid has a height or shorter diameter of 5 feet, and a width or longer diameter of 8 feet. How many solid feet does it contain ?
                    Ans. 167.552 cu. ft..

<div align="center">OPERATION.</div>

	5=height.	5
	8=diameter.	8
	8=diameter. or	8
	.7854=ratio of cir., etc.	.5236
3	2=ratio bet. C. & O. S.	———
—	———	167.552 cu. ft.
	167.552 cu. ft., Ans.	Ans.

Explanation.—In the first statement, we indicate the solution for a cylinder of equal height and diameter as the oblate spheroid, and then multiply by ⅔, since an oblate spheroid is equal to ⅔ of a cylinder of equal height and diameter.

For an explanation of the second statement, see the explanation in the preceding problem.

GENERAL DIRECTIONS.

603. From the foregoing elucidations, we derive the following general directions for the measurement of the above class of solids :

1°. *To find the solidity of a* Cube *or of* Rectangular Solids, *multiply together the length, width, and height, in the same units of measure.* (See problem 1).

2°. *To find the cubical contents of a* Square *or* Rectangular Pyramid, *multiply the height by the area of the base, and then divide the product by* 3 (See problem 2).

3°. *To find the solidity of a* Frustum of a Pyramid, *multiply the average area of the frustum by the height.* (See problem 3).

4°. *To find the solidity of a* Cylinder, *multiply the height by the square of the diameter and then multiply this product by* .7854. (See problem 4).

5°. *To find the solidity or volume of a* Cone, *multiply the height by the square of the base and this product by* .7854, *and then divide by* 3. (See problem 5).

6°. *To find the solidity or volume of a* Frustum of a Cone, *multiply the average area of the frustum by* .7854 *and this product by the height.* (See problem 6).

7°. *To find the volume or solidity of a* Sphere, *cube the diameter and multiply by* .5236. (See problem 7).

8°.   *To find the solidity of a* Prolate Spheroid, *multiply the* Height, *or* Transverse *diameter, by the square of the* shorter *diameter; then multiply by* .7854 *and then by* ⅔. *Or, multiply the longer diameter by the square of the shorter diameter, and this product by* .5236.   (See problem 8).

9°.   *To find the volume or solidity of an* Oblate Spheroid, *multiply the* height, *or* shorter diameter, *by the square of the* longer *diameter; then multiply by* .7854 *and then by* ⅔. *Or, multiply the shorter diameter by the square of the longer diameter, and this product by* .5236.   (See problem 9).

10°.   *To find the solidity of a hemisphere or the half of a prolate or of an oblate spheroid, first find the solidity for the whole solid and then divide by* 2.

604.   PRACTICAL PROBLEMS. '

1.   A rectangular box is 6 ft. 3 in. long, 3 feet wide, and 4 feet 6 inches high, or deep.   How many cubic yards, how many cubic feet, and how many cubic inches does it contain?   Also how many bushels, and how many gallons will it hold?

Ans. 3⅛ cu. yds.; 84⅞ cu. ft.; 145800 cu. in.;
67.8+bus.; 631.17—gals.

OPERATIONS INDICATED.

First to find cu.yds.		Second to find cu. ft.		Third to find cu. in.		Fourth to find bus.		Fifth to find gals.	
4	25						75		75
	3	4	25		75		36		36
2	9		3		36		54		54
27		2	9		54	2150.42		231	
—		—		—					
	3⅛		84⅞		145800		67.8+		631.17—
	cu.yds.		cu. ft.		cu. in.		bus.		gals.

Explanation.—To find cubic yards, we multiply the three dimensions together in the unit of feet, and then divide by 27, because 27 cubic feet make 1 cubic yard.

To find cubic feet, we multiply together the three dimensions in the unit of feet.

To find cubic inches, we multiply together the three dimensions in the unit of inches.

To find bushels, we find the number of cubic inches and then divide by 2150.42 because 2150.42 cubic inches make a bushel.

To find gallons, we find the number of cubic inches and then divide by 231, because 231 cubic inches make a gallon.

2. A box is 5 feet 6 in. long, 3 feet 4 in. wide, and 2 ft. 8 in. deep. How many cubic feet does it contain and how many gallons will it hold ?

Ans. 48⅜ cu. feet; 365⅚ gallons.

3. A rectangular pyramid is 6 feet high, and has a base 4 feet 3 inches by 3 feet 9 inches. How many of each, cubic yds., cubic feet, and cubic inches does it contain ? Also, how many bushels and how many gallons will it hold ?

Ans. 1 4/13 cu. yds.; 31⅞ cu. ft.; 55080 cu. in.; 25.61+bus.; 238 34/44 gals.

OPERATIONS INDICATED.

cu. yds.		cu. ft.		cu. in.		bus.		gals.	
	6						72		72
4	17		6		72		51		51
4	15	4	17		51		45		45
3		4	15		45	3		3	
27		3		3			2150.42		231
—		—		—		—		—	
	1 4/13		31⅞		55080		25.61+		238 34/44
cu. yds.		cu. ft.		cu. in.		bus.		gals.	

4. A cellar in the form of a frustum of a rectangular pyramid is 40 feet 4 inches long on the top and 30 feet long at the bottom ; it is 24 feet wide at the

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top and 18 feet 8 inches wide at the bottom; and it
is 7 feet 6 inches deep. How many of each, cubic
yards, cubic feet, and cubic inches does it contain?
Also, how many bushels, and how many gallons
will it hold?   Ans. $209\frac{1}{2}\frac{6}{4}\frac{2}{3}$ cu. yds.; $5661\frac{1}{5}$ cu. ft.;
9782400 cu. in.; $4549.06+$bus.; $42348\frac{1}{2}\frac{1}{3}\frac{2}{1}+$gals.

<div align="center">OPERATION.</div>

$40\frac{4}{12}\times24$ =   968 = area of the top of the cellar.
$30\ \times18\frac{8}{12}$ =   560 = "   "   bottom of the cellar.

2)$70\frac{4}{12}$ $42\frac{8}{12}$

$35\frac{2}{12}\times21\frac{4}{12}=750\frac{3}{8}\times4=3000\frac{5}{8}=4$ times the middle section be-
                                            tween the two areas.

6)$4528\frac{5}{8}$

$754\frac{2}{3}\frac{3}{4}=$average area of the cellar.

$754\frac{2}{3}\frac{3}{4}\times7\frac{1}{2}$ (ft. deep)$=5661\frac{1}{5}$ cubic feet.

$5661\frac{1}{5}$ cu. ft.$\div27=209\frac{1}{2}\frac{6}{4}\frac{2}{3}$ cu. yards.

$5661\frac{1}{5}$ cu. ft.$\times1728$ (cu. in.)$=9782400$ cubic inches.

9782400 cu. in.$\div2150.42=4549.06+$bus.

9782400 cu. in.$\div231$ $=42348\frac{1}{2}\frac{1}{3}\frac{2}{1}$ gals.

NOTE 1.—If it is desired, the different dimensions in the
above problem may all be reduced to inches and the work
performed in the same manner, thus avoiding much frac-
tional work.

NOTE 2.—The above solution is in accordance with the
Prismoidal Formula, by which the solidity of Cubes, Rec-
tangular Solids, Cones, Cylinders, Pyramids, Frustums of
Cones or Pyramids and several other forms of solids may be
determined.

The Prismoidal Formula is as follows: Add together the
areas of the two ends or bases and four times the middle
section parallel to them. Then divide this sum by 6, and
multiply the quotient by the height, or depth.

5. How many cubic yards were excavated from
a cellar 92 feet long and 50 feet wide at the top, 86
feet long and 44 feet wide at the bottom, and 8 feet
4 inches deep?   Ans. $1291\frac{7}{8}\frac{2}{4}$ cu. yds.

6. A cylinder is 7 feet 4 inches high and 3 feet 5 inches in diameter. How many of each, cubic yards, cubic feet, and cubic inches does it contain? Also, how many bushels and how many gallons will it hold?

> Ans. 2.4902+cu. yds.; 67.2353+cu. ft.;
> 116182.6512 cu. in.; 54.028—bus.;
> 502.9552 gals.

OPERATIONS INDICATED.

cu. yds.			cu. ft.	
3	22		3	22
12	41		12	41
12	41		12	41
	.7854			.7854
27				
	2.4902+cu. yds.			67.2353+cu. ft.

cu. in.		bush.		gals.	
			88		88
	88		41		41
	41		41		41
	41		.7854	231	.7854
	.7854	2150.42			
	116182.6512 cu. in.		54.028—bus.		502.9552 gals.

NOTE.—By inspection, we find that 231 always cancels .7854 and gives a quotient of .0034.

7. How many gallons in a cylinder that is 9 feet 3 inches high and 8 feet 4 inches diameter.

> Ans. 3774 gals.

8. A cone is 134 inches high and 45 inches in diameter at the base. How many of each, cubic yards, cubic feet, and cubic inches does it contain? Also, if it is a vessel and these dimensions are

inside measurements, how many bushels and how many gallons will it hold ?

Ans. 1.5226+cu. yds.; 41.1108—cubic ft.;
71039.43 cu. in.; 33.035+bus.; 307.53 gals.

OPERATIONS INDICATED.

cu. yds.	cu. ft.	cu. in.	bus.	gals.
12|134			|134	
12|45	12|134	|134	|45	|134
12|45	12|45	|45	|45	|45
|.7854	12|45	|45	|.7854	|45
3|	|.7854	|.7854	3|	3|.7854
27|	3|	3|	2150.42	231|
1.5226+	41.1108—	71039.43	33.035+	307.53
cu. yds.	cu. ft.	cu. in.	bus.	gals.

9. How many cubic inches in a cone that is 10 feet 2 inches high and 8 feet 3 inches in diameter at the base ?   Ans. 313040.0196 cu. in.

10. A cistern in the form of a frustum of a cone is 10 feet 2 inches high, 9 feet 6 inches diameter of lower base and 8 feet 6 inches diameter of upper base. How many bushels, and how many gallons will it hold ?   Ans. 520.26+bus ;
4843.2048 gals.

OPERATIONS INDICATED.

		Bushels.	Gallons.
114^2=12996		|11676	|11676
102^2=10404		|122	|122
114×102 =11628		|.7854	|.7854
———	2150.42	231|	231|
3)35028	———	———	———
11676		520.26+	4843.2048
		bus.	gals.

11. How many gallons in a cistern of the shape of a frustum of a cone, and which is 9 feet 9 inches high, 7 feet 6 inches diameter of lower base, and 6 feet 8 inches diameter of upper base ?

Ans. 2877.42 gallons.

12. A sugar kettle in the form of a half of a prolate spheroid is 42 inches deep and 50 inches in diameter. How many bushels, and how many gallons will it hold ?　　　Ans. 25.566+bushels.
238 gallons.

OPERATIONS INDICATED.

Bushels.		Bushels.		Gallons.		Gallons.
			42				42
			50				50
	42		50		42		50
	50	or	.7854		50	or	.7854
	50		3|2		50		3|2
2150.42	.5236	2150.42		231	.5236	231	
	25.566+		25.566+		238		238
	bus.		, bus.		gals.		gals.

13. How many cubic inches in a semi-oblate spheroid which is 70 inches in diameter, and 30 inches deep ?　　　Ans. 76969.2 cu. inches.

14. A yard is 144 feet long and 32 feet 4 inches and 7 lines wide, American measure. It is desired to fill the yard 27 inches deep with earth. Allowing the yard to be a perfect plain, how many cubic yards of earth will be required to fill it; and at 70¢ per cubic yard, what will be the cost ?
Ans. $388\frac{7}{12}$ cu. yds.; \$272.00$\frac{5}{6}$, cost.

OPERATION INDICATED
to find the cu. yds.

	144		
12	4663		$\dfrac{(144 \times 12) \times 388\frac{7}{12} \times 27}{1728 \times 27}$ =cu. yds.
12		or	
12	27		
27			

15. What is the freight on 8 boxes, which are each 3 feet 3 inches long, 2 feet 8 inches wide, and 21 inches deep, at 20¢ per cu. foot ?
Ans. \$24.26$\frac{2}{3}$.

16. A large orange is 4 inches in diameter, and a smaller one is 2 inches in diameter. Allowing each to be a perfect sphere, how many of the smaller oranges are equal to the larger ?   Ans. 8.

17. An orange peddler sells two oranges which are each 3 inches in diameter, or 3 oranges which are each 2 inches in diameter, for 5¢. Allowing the oranges to be perfect spheres, which is the better purchase, and how many cubic inches of orange would be gained ?   Ans. The better purchase would be the 2 oranges each 3 in. in diameter. The gain would be 15.708 cu. in.

18. How many gallons will a tub hold which is 34 inches upper diameter, 29 inches lower diameter, and 21 inches deep ?   Ans. 70.9954 gallons.

19. A rank of wood is 46 feet 4 inches long, 6 feet 6 inches high, and 3 feet 8 inches deep or length of stick. How many cords does it contain ?
          Ans. $9\frac{79}{92}$ cords.

NOTE.—In commerce the length of stick, when less than 4 feet, is estimated as if it were 4 feet long, and the price is graded accordingly.

20. How many cords of wood in two ranks, each 44 feet long and 6 feet 3 inches high ?
          Ans. $17\frac{3}{16}$ cords.

LUMBER AND BOARD MEASURE.

605. A **Standard Board** is one that is 12 feet long, 12 inches wide, and 1 inch thick. Hence 1 **Board Foot** is 12 inches long, 12 inches wide, and 1 inch thick, or 1 foot long, 1 foot wide, and 1 inch thick, and contains 144 square or board inches.

606. A Standard Saw Log is 12 feet long and 24 inches in diameter.

NOTE.—Since 1 board foot contains but 144 board inches, there are 12 times as many board feet as cubic feet in lumber, timber, and logs. Hence to change board feet to cubic, divide by 12; and to change cubic feet to board feet, multiply by 12.

607. *To find the Board or Square Feet in Planks, Girders, Scantling, Joists, and Square Timber.*

PROBLEMS.

1. How many square feet of lumber in a board 16 feet 4 inches long and 15 inches wide ?

Ans. $20\frac{5}{12}$ sq. ft.

OPERATION.

3	49		12	196
4	5	or	12	15
—			—	
	$20\frac{5}{12}$, Ans.			$20\frac{5}{12}$, Ans.

Explanation.—In all problems of this character, we multiply the length and width together in the unit of feet.

2. A board is 20 feet 6 inches long, 21 inches wide at one end and 15 inches at the other end. How many square feet does it contain ?

Ans. $30\frac{3}{4}$ sq. ft.

OPERATION.

21 in., wider end.
15 in., narrower end.

2	41
12	18

2)36
—
18 in., average or mean width.

$30\frac{3}{4}$ sq. ft.
Ans.

3.   How many square feet in a plank 24 feet long,
22½ inches wide, and 3 inches thick?

Ans. 135 sq. ft.

OPERATION.

$$
\begin{array}{c|c}
 & 24 \\
2 & 45 \\
12 & 3 \\
\hline
 & 135 \text{ sq. ft. Ans.}
\end{array}
$$

Explanation—As the board foot is 1 inch in thickness, it is clear that when the thickness exceeds 1 inch, the measurement must be increased accordingly; hence, whenever the thickness exceeds 1 inch, we multiply by the thickness. When the thickness is less than 1 inch, by custom, no deduction is made, the measurement being in that case the same as if the lumber was 1 inch thick.

4.   What is the number of board feet in 16 pieces
of scantling each 20 feet long, 4 inches wide, and 3
inches thick?   Ans. 320 board feet.

5.   How many board feet in 200 girders each 30
feet long, 15 inches wide, and 2 inches thick?   And
what will they cost at $18 per M.?

Ans. 15000 board feet; $270 cost.

6.   What will be the cost of 4 black walnut boards,
each 10 feet 9 inches long, 28½ inches wide, and 1¾
inches thick, at $55 per M.?   Ans. 9.829\frac{1}{3}\frac{7}{2}$.

7.   A piece of timber is 34 feet long, 16 inches
wide, and 15 inches thick.   How many solid feet
does it contain?   Ans. 56⅔ cu. ft.

8.   A rectangular telegraph pole is 60 feet long,
16 inches square at the larger end and 6 inches
square at the smaller end.   How many cubic feet
does it contain?   Ans. 53⅜ cu. ft.

NOTE.—See problem 4, pages 441 and 442.

9.   A circular telegraph pole is 60 feet long, 16
inches in diameter at the larger end, and 6 inches in
diameter at the smaller end.   How many cubic feet
does it contain, and what is the cost at 15¢ per
cubic foot?   Ans. 42.3243⅛ cu. ft.

$6.34865 cost.

SYNOPSIS FOR REVIEW:

Define the following words and phrases :
575. Mensuration. 576. A Point. 577. A Line. 578. A Surface. 579. A Polygon. 580. An Ellipse. Transverse Diameter. Conjugate Diameter. 581. A Square. 582. A Rectangle. 583. A Right-Angled Triangle. 584. A Circle. 585. The Ratio between Diameter and Circumference. 586. Area. 587. The Ratio between Area of Circle and of Square. 589. General Directions for Linear Measure. 591. General Directions for Mensuration of Surfaces. 592. A Rectangular or Quadrilateral Solid. 593. A Cube. 594. A Pyramid. 595. A Frustum of a Pyramid. 596. A Cylinder. 597. A Cone. 598. A Frustum of a Cone. 599. A Sphere. 600. A Prolate Spheroid. 601. An Oblate Spheroid. 603. General Directions for Mensuration of Solids. 605. A Standard Board. A Board Foot. 606. A Standard Saw Log.

BILLS AND INVOICES.

608. Bills, in a general sense, embrace all written statements of accounts and many legal instruments of writing · but in a more common and limited sense, they are statements of goods sold or delivered, services rendered, or work done, with the price or value, quality or grade, of each article or item. Bills or Invoices of Merchandise should state the place and date of each sale, the names of the buyer and the seller, the price, the extra charges, or the discount to be allowed, the marks and numbers on the goods, and the terms of the sale.

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When goods are bought to sell again, or when bills are rendered to a jobber or retailer, or consigned to an agent, the bill is then called an **invoice.**

It is the custom of accountants and merchants, when making bills, to commence the name of each article with a capital.

When a charge is made for the box, barrel, jar, etc., containing goods, it is customary to write ts price above and to the right of it, and add the same to the cost of the goods it contains.

In making extensions, fractions of cents are not used in the product; when they are ½ or more, they are counted cents; when they are less than ½, they are not counted.

In making the following bills, students should use pen and ink and give earnest attention to the proper form and spacing; to plain, neat, and rapid penmanship of both words and figures; and above all, to the accuracy of extensions and additions.

When notes or bills of exchange are given in payment, the student should draw the same and correctly mature them.

No. 1.

NEW ORLEANS, Jan'y 2, 1886.

H. A. & R. C. Spencer,

Bot. of *A. L. & E. E. Soulé.*

TERMS—Cash.

1886						
Dec.	16	2 bags Rio Coffee, 325 lbs. @ $	23½¢	$	76	38
		50c. 1 bbl. Sugar, 234½ lbs. "	9 ¢		21	61
		½ Chest Black Tea, 35 lbs. "	87½¢		30	63
		1 bbl. Rice, 243–16 =227 "	8 ¢		18	16
		40 gals. N. O. Molasses "	75 ¢		30	00
		6 doz. Brooms "	4.15		24	90
		3 bbls. XXX Family Flour "	8.12½		24	38
		25 lbs. Cream Crackers "	16 ¢		4	00
		50 lbs. Graham do. "	15 ¢		7	50
		20 lbs. W. Butter "	30 ¢		6	00

Rec'd pay't, $243 56

A. L. & E. E. SOULÉ,

Per F. Richardson.

No. 2.

NEW ORLEANS, Jan'y 31, 1886.

S. C. Hepler and E. G. Folsom,

Bot. of *W. H. and Frank Soulé.*

TERMS—Note at 30 days.

1886					
Jan.31	453½ lbs.	Mocha Coffee,	@ $	25	¢
	241 "	Rio Coffee,	"	18¾¢	
	316¼ "	C. Sugar,	"	12½¢	
	72 "	Duryea's Starch,	"	6¼¢	
	64 "	N. Y. C. Cheese,	"	17½¢	
✓	52 "	W. F. Cheese,	"	15 ¢	
	180 "	B. Sugar,	"	7½¢	
✓	80 doz.	C. Eggs,	"	37½¢	
	42 gals.	N. O. Molasses,	"	62½¢	
	320 lbs.	G. Butter,	"	35 ¢	
✓	23 "	Almonds,	"	27 ¢	
	76 "	Y. H. Tea,	"	74 ¢	
✓	68 boxes	Shrimp,	"	48 ¢	
	84 boxes	Lobsters,	"	34 ¢	
✓	92 gals.	N. O. G. Syrup,	"	96 ¢	
	114 "	B. Whiskey,	"	1.08	
✓	112 bags	Salt,	"	93 ¢	
	320 bbls.	Sweet Potatoes,	"	1.25	
	82 kits	No. 1 Mackerel,	"	2.50	
	63 lbs.	S. Crackers,	"	11 ¢	
	24¾ "	P. L. Soap,	"	8½¢	
	18½ "	Codfish,	"	9¼¢	
	Drayage $31.25, boxes $2.50.				

Rec'd pay't by note at 30 days, $1,492 09

W. H. & F. SOULÉ,

Per Jas. Tucker.

NOTE.—All the extensions of this bill should be made mentally. For rapid mental work in Computations, see Appendix.

No. 3.

NEW ORLEANS, Jan'y 31, 1886.

J. M. Butchee,

Bot. of *J. B. Cundiff.*

TERMS—60 days credit.

875 bbls. Nes. Potatoes, @$4.25	
440 " P. B. Potatoes, " 3.87½	
325 " Perfect'n Flour, " 8.50	
1324 " St. L. XX " " 6.62½	
112 " F'ily Clear Pork," 17.50	
650 " Prime Pork, " 13.75	
220 kegs Pig Feet, " 7.50	
124 half bbls. F. M. Beef," 11.	
1872 lbs. Choice Ham, " 14′ ¢	
289 " B. Bacon, " 9½ ¢	
106 Pig Tongues, " 8 ¢	

Rec'd pay't $31167 27

NOTE—All the extensions of this bill should be made mentally.

———

No. 4.

NEW YORK, Dec. 8, 1886.

H. C. Spencer & Co.,

Bot. of *B. D. Rowlee & Co.*

TERMS—Cash.

20 doz. Missionary Bibles, @ 15.25	
108 " sm. New Testam't, " 2.50	
65 " Prayer Books, " 2.25	
65 " Hymn Books, " 3.	
3 Bible Dictionaries, " 4.	
½ doz. Webster's Dict'ry, " 50.	

Rec'd pay't, $ 953 25

B. D. ROWLEE & CO.,

Per E. Conrad.

No. 5.

New Orleans, Jan'y 31, 1886.

Wm. Melchert & Co.,

TERMS—Cash.

Bot. of *L. L. Williams & Co.*

321 lbs. Tobacco Low Lugs, @	6 ¢	
1140 " " Med. Lugs. "	7½¢	
509 " " Low Leaf, "	9¼¢	
965 " " Med. Leaf, "	11¼¢	
398 " " Good Leaf, "	13¾¢	
2416 " " Fine Leaf, "	15 ¢	
713 " " Selections, "	16⅝¢	
Rec'd pay't,		$ 798 49

L. L. WILLIAMS & CO.

No. 6.

New Orleans, Dec. 17, 1886.

F. L. Richardson, Jr.,

TERMS—Dft. 30 days.

Bot. of *C. J. Sinnott.*

1420 lbs. Sugar, Common, @	5½¢	
1927 " " Good, "	7¼¢	
2810 " " Fair, "	7⅝¢	
902 " " Prime, "	8½¢	
813 " " Choice, "	9¼¢	
2741 " " Yel. Cent'al "	10½¢	

Rec'd pay't, by dft. @ 30 days' sight, $ 878 78

C. J. SINNOTT.

No. 7.

New Orleans, Dec. 23, 1886.

Montgomery & Trepagnier,

TERMS—3 mos.

Bot. of *Sadler & Smith.*

7 Gross Chew'g Tobacco @ $13.		
180 lbs. Smoking do., " 1.40		
6 M. Havana Cigars, " 70.		
2 M. N. O. Man'f'ture do. " 30.		
Rec'd pay't, $		

No. 8.

NEW ORLEANS, Jan'y 21, 1886.

Jones & Weiss,

TERMS—Note 60 days.

Bot. of *Stewart & Henderson.*

34 bbls. La. Oranges, lar., @ $5.75	
27 boxes Messina Lemons, " 6.00	
63 cases Malaga Grapes, " 1.75	
45 boxes California Pears, " 4.50	
5 mats Dates, 593 lbs., " 7½¢	

Rec'd payt, by note at 60 days, $714 73

STEWART & HENDERSON.

No. 9.

NEW ORLEANS, Nov. 17, 1886.

Geo. B. Brackett & Co.,

TERMS—Cash.

Bot. of *R. Spencer Soulé.*

1427 bu. No. 1 Winter Wheat @ $1.55	
856 " No. 2 Winter Wheat " 1.47	
420 " Ill. No. 1 White do. " 1.41	
3145 " W. corn, " .70	
1040 " B. Oats, " .55	

Rec'd pay't, $6835 87

R. SPENCER SOULÉ.

No. 10.

NEW ORLEANS, Feb. 4, 1886.

F. L. & W. P. Richardson,

TERMS—1 mo.

Bot. of *P. W. Sherwood & Co.*

30 box. Sp. Candles, 596 lbs. @ .35½	
24 do. Adam Ex. do. 483 " " .28	
15 do. Sil. Gloss St'rch 360 " " .10¾	

Rec'd pay't, $ 385 52

No. 11.

NEW ORLEANS, Jan'y 19, 1886.

Heald & Howe,

Bot. of *Cole & Montague.*

TERMS—Due Bill 1 mo.

342 lbs. La. Pecans, @ $ .13	
289 " Taragona Almonds, " .21	
175 " Naples Walnuts, " .17	
196 " Brazil Nuts, " .11	
268 " Western Chestnuts, " .18	
160 boxes Figs, " .20	
585 Cocoanuts @ $45 per M.	
61 bunches Bananas, " 1.75	
14 do. Plantains, " .85	
327 Pine Apples @ $80 per M.	

Rec'd pay't, by due bill @ 1 month, **$407 84**

COLE & MONTAGUE.

No. 12.

NEW ORLEANS, Feb. 1, 1886.

Hibbard & Gray,

Bot. of *Odell & Faddis.*

TERMS—Cash.

2714 lbs. Black Moss, @ 4 ¢	
1829 " Gray do. " 1¼¢	
913 " Wool, " 24 ¢	
74 " Live Geese Feathers " 65 ¢	
1528 packages Broom Corn, " 6 ¢	
752 lbs. Baling Twine, " 14 ¢	
800 yds. Indian Bagging, " 11 ¢	

Rec'd pay't, **$683 60**

ODELL & FADDIS,

Per C. P. Meads.

No. 13.

NEW ORLEANS, Jan'y 7, 1886.

Spaulding & Musselman,

TERMS—60 days.   Bot. of *Warr & Bogardus.*

78½ yds. Black Silk,	@ $2.90		
148 " Muslin,	" 16 ¢		
62 " Cassimeres,	" 1.75		
38¼ " Blk.F.Br'dcloth	" 5.25		
45 " " " Doeskin,	" 1.50		
324 " Ameri'n Satinets	" 95 ¢		
3 cases, each 40 ps. Amos-			
keag Sheetings,			

11231
1204 } 3423¾ yds.   @   17½¢
1096²

142 yds. 6-4 Alpaca,	" 32 ¢	
560 " Union Ginghams,	" 11¾¢	
491 " Am. Fancy Prints	" 12 ¢	
107 " Manch'ter Delains	" 21¼¢	
10 doz. Handkerchiefs,	" 2.15	
16 " $8¾, $⁴⅕₀, $³⁴₇₅		
Ladies' Hose,		
2 ps. 61½ yds. Can. Flannel	" 18 ¢	

Rec'd pay't, $1825 57

No. 14.

NEW ORLEANS, Nov. 4, 1886.

New Orleans, St. Louis, and Chicago R. R.,

To *W. L. & H. Hall,* **Dr.**

For 150 Cisterns holding 766782.45 gals. @ 2½¢ per gal. The inside measurement of each cistern is as follows: 11 ft. 3 in. perpendicular height, lower base 9 ft. 2 in. in diameter and upper base 8 ft. 5 in. in diameter.	

Rec'd pay't, $19169 56

No. 15.

NEW ORLEANS, Feb. 8, 1886.

Tasker & Felton,

TERMS—Cash.   Bot. of *Allen & Shields.*

50 lbs. Casing Nails,	@ $ 7¢
1¾ doz. Mortice Locks,	" 7.50
½ " Porcelain Knobs,	" 4.75
50 pr. Butts,	" 25¢
3 Gross Screws,	" 75¢
8 bars, 1½ × ½ Bar Iron,	
254 lbs.,	" 5¢
2 Rowland No. 2 Spades,	" 1.

Rec'd pay't, $ 48 46

ALLEN & SHIELDS.

No. 16.

NEW ORLEANS, May 9, 1886.

Tillie McGuigin,

TERMS—Due end of month.   Bot. of *Katie Weiss.*

3 reams Cap Paper,	@ $3.25
2 doz. Ebony Rulers,	" 3.50
4 6 qr. Med. Ledgers 24 qrs.	" 1.75
3 3 " Demy Journals, 9 " "	1.25
3 3 " " Cash Books, 9 " "	1.25
3 6 " " Sales Books 18 " "	1.15
4 gross Pen-holders,	" 2.10
¾ doz. bottles Black Ink,	" 4.50
½ ream Blotting Paper,	" 2.50
2 doz. bottles Mucilage,	" 2.75
3 " " Carmine,	" 2.15
3 5⁄12 doz. Bill-books,	" 4.25

Rec'd pay't, $ 141 45

No. 17.

NEW ORLEANS, April 1, 1886.

E. J. & R. Paul,

TERMS—Due bill 30 days.　　　　　Bot. of *Gresham & Harp.*

½ doz. Comb's Con. of Man, @$12.	
2 " Dana's Geological Story briefly told, " 10.	
6 " Soulé's Phi. Arithmetics " 42.	
6 " " Con. in Numbers, " 18.	
6 " " Prim. Arithmetics " 9.	
¾ " Webster's Acad. Dict., " 18.	
2 " Swinton's Lan. Lessons, " 3.25	
1½ " Steel's Nat. Philosophy, " 12.	
⅓ " Spencer's Science of Sociology, " 10.50	
2 copies Wood's Byron, " 3.00	
4 " Dick's Shakspeare, " 4.50	
Drayage 75¢, box 50¢.	

Rec'd pay't by due bill @ 30 days,　$ 506 75

GRESHAM & HARP.

————

No. 18.

NEW ORLEANS, Jan. 1, 1886.

T. Janney,

To *A. Laborde,* Dr.

For furnishing, making, and laying Brussels carpet 27 in. wide, in 2 rooms measuring as follows:		
No. 1, 24 ft. 9 in. × 20 ft. 3 in.		
No. 2, 18 ft. × 16 ft. 6 in.		
*118¼ yards, @ $1.95		
68 " 6-4 Che. Mat. " 90¢		

Rec'd pay't,　$ 291 79

*In this bill, no allowance is made for waste in matching or otherwise.

No. 19.

New Orleans, Jan'y 25, 1886.

McDaniel & Keller,

Bot. of *Redfield & Halsmith.*

TERMS—Note at 60 days.

2144 lbs.——— bush. Yellow Corn,	@ $	63¢
1242 lbs.——— " Texas Wheat,	"	1.70
852 lbs.——— " White Oats,	"	56¢
792 lbs.——— " Barley"		83¢
1427 lbs.———Cwt. Bran "		75¢
3745 lbs.———Tons Timo-thy Hay,	"	18.50
1701 lbs.———Tons Clo-ver Hay,	"	20.

Rec'd pay't by note at 60 days, $ 150   27

REDFIELD & HALSMITH.

No. 20.

New Orleans, Jan'y 3, 1886.

Geo. F. Bartley & Co.,

To *Steamship Knickerbocker and Owners,* Dr.

For Freight on 439⅝ cubic ft. @ 25¢ The same being contents of 8 boxes measuring as follows :	
Nos. 1, 2 & 3; 5 ft. 4 in. × 4 ft. 6 in. × 2 ft. 8 in. =	
Nos. 4, 5 & 6; 6 ft. 2 in. × 3 ft. 0 in. × 2 ft. 11 in. =	
Nos. 7 & 8; 12 ft. 3 in. × 2 ft. 4 in. × 1 ft. 6 in. =	

Rec'd pay't, $ 109   91

No. 21.

NEW ORLEANS, Jan'y 4, 1886.

J. T. O'Quinn,

To *W. Hermann,* **Dr.**

For rent of house No. 386, Dryades St., from Oct. 7, '85, to Jan. 1, '86, 1st date included, $2\frac{2}{3}\frac{1}{3}$ mos. at $35	
For services as collector from Sept. 19, '85, to Jan. 4, '86, both dates included, $3\frac{1}{3}\frac{2}{0}$ months, at $75	
Rec'd pay't,	$ 363 00

No. 22.

NEW ORLEANS, Jan'y 9, 1886.

The La. Levee Co.,

To *W. H. Mills,* **Dr.**

For constructing $16206\frac{1}{2}$ cubic yds. of Levee @ 45¢, as per the following measurements:	
1st Sect'n: $893\frac{1}{4}$ ft. long, 70 ft. wide at the base and 30 ft. at the top, with an average depth of $8\frac{2}{3}$ ft.	
2nd Sect'n: 165 ft. long, 60 and 25 ft. respectively for the lower and upper widths, and $7\frac{1}{2}$, $5\frac{1}{2}$, 6, $8\frac{1}{2}$, 9, and $6\frac{1}{2}$ ft. in depth at different points.	
For excavating $1291\frac{7}{3}\frac{9}{1}$ cubic yds. of earth @ 45¢, the same being the contents of a cellar measuring as follows:	
92 ft. long and 50 ft. wide at the top, and 86 ft. long and 44 ft. wide at the bottom, average depth 8 ft. 4 in.	
Rec'd pay't,	$ 7874 14

No. 23.

NEW ORLEANS, Jan'y 16, 1886.

Geo. Soulé,

To *A. D. Hofeline,* Dr.

For composition and electrotyping 560 pages Soulé's Intermediate, Philosophic Arithmetic, @ $1.55 per page.	
For press work on 192 tokens @ 50¢	
For 50 rms. paper, 22×23 @ 4.00	
For binding 1000 copies, " 25¢	

Rec'd pay't, $

A. D. HOFELINE.

No. 24.

NEW ORLEANS, Jan'y 18, 1886.

Western Union Telegraph Co.,

To *F. D. Ross & Co.,* Dr.

For 3261⅒ cubic feet Timber @ $24 per 100; the same being the contents of 50 Telegraph poles measuring as follows:	
40 Poles are 70 feet long, 16×16 inches at the larger end and so remain for a distance of 10 feet, at which point they begin and taper regularly to the smaller end, which is 6×6 inches.	
10 Poles are 60 feet long, 16×12 inches at the larger end, 6×4 inches at the smaller end, and taper regularly the whole length.	

Rec'd pay't, $ 782 67

No. 25.

NEW ORLEANS, Jan'y 29, 1886.

H. J. Calvert,

To *L. B. Keiffer*, Dr.

Jan.	1	To old balance as per bill ren'd,		91	10
	6	12 cords Ash Wood @ $7.		84	00
	6	4 cords Oak Wood " $6.50		26	00
	14	50 bbls. Pittsburg Coal " 60¢		30	00
		Cr.		$231	10
	8	By Cash $50			
	29	" 6 days' Labor, at $4, $24		74	00
		Balance due Jan'y 29, 1886, $		157	10

Settled by note at 60 days.

L. B. KEIFFER.

———

No. 26.

NEW ORLEANS, Jan'y 12, 1886.

A. & S. H. Soulé,

To *S. & F. Cusimano*, Dr.

	To 4378 feet Com. Boards @		
	$21 per M.		
	" 1760 ft. Dressed Flooring "		
	$28.50 per M.		
	" 5125 Bricks "		
	$14.25 per M.		
	" 9250 Cypress Shingles "		
	$6.50 per M.		
	" Cartage and Labor,	14	25
	Rec'd pay't, $	289	51

No. 27.

NEW ORLEANS, Dec. 31, 1886.

A. J. Boyce,

To *R. & C. Rice,* Dr.

For $15\frac{1}{2}\frac{8}{1}\frac{7}{6}$ sq. yards North River Flags @ $7.50, as per the following measurements:
Nos. 1. 2 & 3, are each 4 ft. 3 in. by 3 ft. 6 in. = ... sq. ft.
Nos. 4, 5 & 6, are each 4 ft. 8 in. by 3 ft. 4 in. = ... sq. ft.
Nos. 7 & 8 are each 4 ft. 0 in by 3 ft. 0 in. = ... sq. ft.
Nos. 9. 10 & 11. are each 3 ft. 4 in. by 2 ft. 8 in. = ... sq. ft.
For $41\frac{2}{8}\frac{1}{1}$ sq. yds. German Flags @ $2.25, comprising 152 Flags, each 22 × 16 inches.
For $162\frac{35}{108}$ sq. yds. Brick Pavement @ $1.15, contained in a sidewalk measuring 124 ft. 4 in. long by 11 ft. 9 in. wide.
For 124 ft. 4 in. Curbing @ $ 1.30
For $3\frac{1}{2}\frac{4}{3}\frac{9}{3}\frac{8}{2}\frac{7}{6}$ cu. ys. Granite " $16.00 contained in 23 blocks of stone measuring as follows:
Nos. 1 to 7 inclusive, are each 26 × 15 × 10 inches, =
Nos. 8 to 20 inclusive, are each 23 × 16 × 9¼ inches, =
Nos. 21 to 23 inclusive, are each 42 × 35 × 21 inches, =

Rec'd pay't, $ 616 48

No. 28.

NEW ORLEANS, Jan'y 28, 1886.

Invoice of Sundries purchased by J. Simmons & Co., and shipped per Steamer La Belle, for acc't and risk of James Byrnes, Shreveport, La.

87 bbls. Molas's, 3498 gals. @ 60¢		
20 hhds. Sugar, 23780 lbs. " 9¢		
10 bbls. Rice, 2150 lbs. " 5¢		
————————*Charges:*————————	4346	50
Drayage	17	50
Insurance on $4800.40 @ ⅝%	30	00
Commission on $4364.00 " 2½%	109	10
Rec'd pay't, $ 4503 10		

No. 29.

NEW ORLEANS, Jan'y 14, 1886.

C. H. Reynolds,

To *H. Marsden,* Dr.

For slating a roof measuring 72 ft. 4 in. by 49 ft. 10 in., and containing 36.04⅛ sqs. @ $14.50		
For 239 ft. Guttering " .90		
Rec'd pay't, $ 737 77		

No. 30.

NEW ORLEANS, Feb. 1. 1886.

Mississippi Valley Transportation Co.,

To *Buck & Richardson,* Dr.

For services rendered in cause No. 55472. "Steamer R. E. Lee and Owners vs. Miss. V. T. Co."		
Rec'd pay't,		

THE METRIC SYSTEM
OF WEIGHTS AND MEASURES.

REMARKS.

609. The **Metric System of Weights and Measures** is based upon the **Meter** as the primary Unit, and all the Increasing and Decreasing Units are based upon the decimal scale.

The term *meter* is from the Greek *metron*, a measure.

610. The **Standard Meter** is a bar of platinum and is kept among the national archives in Paris; but duplicates of it have been furnished to the United States and other nations.

611. The **Metric System** is one of the products of the French Revolution of 1789, which resulted in the establishment of the French Republic, on the 22d day of September, 1792. The progressive spirits of the Revolutionists claimed that every thing needed reforming. They demanded a change in the calendar, a new classification of the seasons, a new order of months, with new names and a change of days, a new arrangement for Sunday and festive days, and above all they demanded a decimal system of weights, measures, and values.

By the laws of 1793 and 1795, a temporary meter and kilogramme were adopted, and in 1795 a commission was appointed, under the direction of the Academy of Sciences, for the purpose of perfecting the system.

The first and most important duty was to deter-

mine an invariable standard unit for all measures
of length, area, solidity, capacity, and weight. For
the consummation of this object, a trigometrical
survey was made, by two eminent mathematicians,
Delambre and Mechain, of the arc of the meridian
through Paris, from Dunkirk, France, to Mont
Jouy, near Barcelona, Spain, a distance of 9⅔
degrees, more than one-tenth of the quadrant of
the meridian. From the survey of this meridian,
the length of the quadrant from the equator to the
pole, measured on the earth's surface, was com-
puted.

The length of the Paris meridian quadrant, thus
obtained, was divided into 10,000,000 equal parts,
and one of these parts was called a *meter* and taken
as the primary unit of the French system of meas-
ures. This meter was adopted as the unit of
length, and from it all the other units of measure
are derived by the application of the decimal scale.
It is equal to 39.37079, practically 39.37 inches in
length, and at the time of its adoption, it was
thought to be exactly the one ten-millionth part of
the distance from the equator to the pole, on the
meridian of Paris. But by more accurate measure-
ments, based upon the fact that the earth's equator
is not a perfect circle, but slightly elliptical, a fact
not considered by the French mathematicians and
astronomers, it is found to be $\frac{1}{163}$ part of an inch
too short. This very small error, almost imper-
ceptible in a single meter, amounts to 5124 feet in
the length of the quadrant measured. But this
slight discrepancy does no injury to the system.
The meter has a fixed length and is decimally
divided, and that is all that is required.

In 1799, the French nation formally adopted the
revised Metric System; and in 1837, a law was

enacted and promulgated making it compulsory throughout France.

Previous to the adoption of the Metric System by the French nation, their units of weights and measures were, in many respects, quite similar to those of the English, many of which came down the centuries through Egypt, Greece, and Rome, and were originally largely derived from different parts of the human body; such as the foot,—the length of the foot of Hercules or of a King; the ulna or yard,—the distance from the middle of the chest or lips, to the tip of the middle finger; the palm or hand,—the width of the hand; the span,—the distance from the tip of the thumb to tip of the middle finger when extended; the digit or the finger,—the length of the first finger; the cubit,—the distance from the elbow to the tip of the middle finger; the fathom,—the distance from the tips of the middle fingers of the two hands when extended in opposite directions, etc. Other units were taken from familiar objects in nature; such as the barley corn, the grain of wheat, shells, horns, etc.

These once variable ancient and English units of length have now certain fixed values based upon the Imperial *Standard Yard*, the length of which is 36 inches. This standard yard is such that a pendulum equal in length to 39.13929 of its inches, will vibrate seconds, in a vacuum, at the level of the sea, in the latitude of London, the thermometer being 62° Fahrenheit. The Standard Yard of the United States is a copy of the English Standard Yard, marked upon a brass scale, and deposited in the Treasury Department at Washington. Copies of this yard have been supplied to all the States.

By reason of the decimal divisions of the Metric System, it possesses many advantages over all other systems and is now used wholly, or in part,

by nearly all civilized countries. It was legalized and adopted, but not made compulsory, in the United States, by an act of Congress passed in 1866. And it is the only system ever authorized by the Government of the United States.

It is adopted by the U. S. Coast Survey, is used in the Mints and the Post Offices, and largely by all Scientists, Colleges, and Universities.

612. The **Meter** is the Unit of **Length,** and from it are derived the *Are,* the *Stere,* or *Cubic Meter,* the *Liter,* and the *Gram.* From these five units, all others are formed.

613. The **Are** (air), is the unit of **Surface,** or **Square Measure,** and consists of a square whose side is 10 meters; hence, it contains 100 square meters.

NOTE.—The Air is from the Latin, *area,* a surface.

614. The **Cubic Meter,** or **Stere** (stair), is the unit of **Solidity,** and consists of a cube whose edge is one meter.

615. The **Liter** (Leeter), is the unit of the **Capacity** of vessels, etc., and is a vessel whose volume is equal to a cube whose edge is one-tenth of a meter.

616. The **Gram** is the unit of **Weight,** and is the weight of a cube of distilled water (weighed in a vacuum, $39.2°$ F., or $4°$ C.) whose edge is one-hundredth of a meter.

Each of these five units has its multiples and sub-multiples, or its higher and lower metric denominations.

617. The Multiple Units, or **Higher Denomi-nations,** are formed by prefixing to the name of the *base units,* the Greek numerals, *Deka,* (10), *Hecto,* (100), *Kilo,* (1000), and *Myria,* (10000).

They are used as follows :

Dekameter,	10 meters.	Kilometer,	1000 meters.
Hectometer,	100 meters.	Myriameter,	10000 meters.

618. The Sub-Multiple Units, or lower denom-inations, are formed by prefixing to the name of the *base units,* the Latin numerals, *Deci* ($\frac{1}{10}$), *Centi* ($\frac{1}{100}$), and *Milli* ($\frac{1}{1000}$).

They are used as follows :

Decimeter, $\frac{1}{10}$ meter.	Centimeter, $\frac{1}{100}$ meter.
Millimeter, $\frac{1}{1000}$ meter.	

NOTE.—The student should memorize these Greek and Latin prefixes, and also the names of the primary or base units, before proceeding farther.

619.   METRIC TABLES.

Table of Linear Measure, of which the METER is the Base Unit.

METRIC UNITS.	EQUIVALENTS IN ENGLISH MEASURES.
1 Millimeter ($\frac{1}{1000}$ of a M.) =	.03937+ inch.
10 mm. = 1 Centimeter ($\frac{1}{100}$ of a M.) =	.3937 + inch.
10 cm. = 1 Decimeter ($\frac{1}{10}$ of a M.) =	3.93707+ inches.
10 dm. = 1 **Meter** (1 meter) =	39.37079 in., prac-tically 39.37 in., or 3.28089+ feet.
10 M. = 1 Dekameter (10 meters) =	32.80899+ ''
10 Dm. = 1 Hectometer (100 meters) =	19.88423+ rods.
10 Hm. = 1 Kilometer (1000 meters) =	.62138+ mile.
10 Km. = 1 Myriameter [Mm.] (10000 meters) =	6.21382+ miles.

The **Meter,** like our yard, is used in measuring short distances, cloths, etc.

The **Kilometer** is used in measuring long distances and is about ⅝ of a common mile.

The **Centimeter** and **Millimeter** are used by artisans and scientists in measuring very small lengths.

620. Table of SURFACE or SQUARE MEASURE, of which the ARE is the Base Unit.

100 sq. Millimeters, (sq. mm.)=
   1 sq. Centimeter, (sq. cm.) =   .155+ sq. in.

100 sq. Centimeters=1 sq. Deci-
   meter,   (sq. dm.)   =   15.5+ sq. in.
                           or .1076+ sq. ft.

100 sq. Decimeters = 1 sq. *Meter*,
   (sq. M.)   =   1.19603+ sq. yds.

100 sq. Meters = 1 sq. Dekame-
   ter, (sq. Dm.) or *Are*, (A.) = 119.6034+ sq. yds.
                           or 3.95383+ sq. rods

100 sq. Dekameters, or Ares,=1 sq.
   Hectometer, (sq. Hm. or
   Hectare,)   (Ha.)   =   2.47114+ acres.

100 sq. Hectometers or Hectares,
   = 1 sq. Kilometer   =   .3861+ sq. mile

This measure is used for measuring land, flooring, ceilings, etc. The lower denominations are seldom used.

621. Table of CUBIC, or SOLID MEASURE, of which the Cubic Meter or STERE is the Base Unit.

1000 cu. Millimeters, (cu. mm.)=
   1 cu. Centimeter, (cu. cm.) =   .061027+ cu. in.

1000 cu. Centimeters = 1 cu. De-
   cimeter, (cu. dm.)   =   61.02705+ cu. in.

1000 cu. Decimeters = 1 **Cubic**
   Meter, (cu. M.) or STERE,
   (St.)   =   35.31658+ cu. ft.,
   or 1.30802+ cu. yds., or .2759 + cord.

The Cubic Meter is the unit used for measuring ordinary solids, as boxes, excavations, etc. The Cubic Centimeter and Cubic Millimeter are used for measuring very minute bodies.

622. When the *Cubic Meter* is applied to the measurement of wood, it is called the *Stere,* and has the following units:

TABLE.

1 Decistere, (dst.)		= 3.53165+ cu. ft.
10 Decisteres, (dst.)= 1 STERE,		= 35.31658+ cu. ft.
10 Steres, (St.) = Dekastere, (Dst.)= 353.1658+ cu. ft.		
		or 13.0802+ cu. yds.

623. Table for Measuring the Capacity of Vessels, etc., of which the LITER is the Base Unit.

1 Milliliter, ($\frac{1}{1000}$ of a liter)=	.06102+ cu. in.
10 ml. = 1 Centiliter, ($\frac{1}{100}$ of a liter)=	.61027+ " "
10 cl. = 1 Deciliter, ($\frac{1}{10}$ of a liter)=	6.1027+ cu. in.
10 dl. = 1 **Liter,** (1 liter) =	61.02705+" "
10 L. = 1 Dekaliter, (10 liters) =	610.2705+ " "
10 Dl. = 1 Hectoliter, (100 liters) =	6102.70502+" "
10 Hl. = 1 Kiloliter, (1000 liters) =	61027.05024+" "
10 Kl. = 1 Myrialiter Ml. (10000 lit.) =610270.5024+ " "

The Liter, or the cube of a decimeter, is the *unit* of capacity for both Liquid and Dry Measures. The Hectoliter is the *unit* for measuring liquids, grain, or fruits.

The following table shows the equivalents of the Liter units, in United States measures.

624.   TABLE OF EQUIVALENTS.

Metric Denominations.		Dry Measure.		Liquid Measure.
1 Milliliter	=	.001816+pts.	=	.0338 fl. oz., or .00845+ gi.
1 Centiliter	=	.01816+ pts.	=	.338 fl. oz., or .084539+ gi.
1 Deciliter	=	.181625+pts.	=	.84539+ gi.
1 Liter	=	.908128+qts.	=	1.056745+ qts.
1 Dekaliter	=	9.08128+ qts.	=	2.64186+ gals.
1 Hectoliter	=	2.8379+ bus.	=	26.4186+ "
1 Kiloliter, or Stere	=	28.379+ bus.	=	264.186+ "
1 Myrialiter	=	283.79+ bus.	=	2641.86+ "

625. Table for Measuring Weight, of which the GRAM is the Base Unit.

	1 Milligram ($\frac{1}{1000}$ of a gram) = .015432+ gr. Troy.	
10 mg.	= 1 Centigram ($\frac{1}{100}$ of a gram) = .15432+ " "	
10 cg.	= 1 Decigram ($\frac{1}{10}$ of a gram) = 1.54324+ " "	
10 dg.	= 1 **Gram** (1 gram) = 15.43248+ " "	
10 G.	= 1 Dekagram (10 grams) = .35273+ oz. Avoir.	
10 Dg.	= 1 Hectogram (100 grams) = 3.52739+ " "	
10 Hg.	$\left\{\right.$ = 1 Kilogram or Kilo $\left.\right\}$ (1000 grams) = 2.20462+ lbs. "	
10 Kg.	= 1 Myriagram (10000 grams) = 22.04621+ " "	
10 Mg. or 100 Kg.	$\left.\right\}$ = 1 Quintal (100000 grams) = 220.46212+ " "	
10 Q. or 1000 Kg.	$\left.\right\}$ = 1 Tonneau or Ton (T.) $\left.\right\}$ (1,000,000 grams)= 2204.62124+ " " or 1.10231+ Tons.	

The **Gram** is the *unit* used in weighing gold, silver, jewels, and letters, and in compounding medicines. It is a little less than 15½ grains Troy.

The **Kilogram,** or **Kilo,** is the *unit* used in weighing common articles in trade; as grain, sugar, butter, etc. It is a very little less than 2½ lbs. Avoirdupois.

The **Tonneau, or Ton,** is used for weighing very heavy articles, and is a little more than $\frac{1}{10}$ of the United States ton.

NOTE.—The pound of Germany, Austria, and Denmark is ½ of a Kilogram.

626. TABLE OF EQUIVALENTS,

By which **Units of the American Measure** may be easily changed to **Metric Units,** and *vice versa.*

1 inch	=	2.54 cm.	1 cm.	=	.3937 in.
1 foot	=	3.0487 dm.	1 dm.	=	.328 ft.
1 yàrd	=	9144 M.	1 M.	=	1.0936 yds. =
1 rod	=	5.029 Dm.		3.2808 ft.	= 39.3707 in
1 mile	=	1.6095 Km.	1 Dm.	=	1.9884 rods.
1 sq. in.	=	6.4516 sq. cm.	1 Km.	=	.6213 mi.
1 sq. ft.	=	9.2936 sq. dm.	1 sq. cm.	=	.155 sq. in.
1 sq. yd.	=	.8361 sq. M., or centare.	1 sq. dm.	=	.1076 sq. ft.
			1 sq. M.	=	1.196 sq. yds.
1 sq. rd.	=	.2531 sq. Dm. or Are.	1 Are	=	3.9538 sq. rds
				=	119.6034 sq. yds.
1 acre	=	.4048 Hectare, or sq. Hm.	1 Hectare	=	2.4711 acres.
			1 sq. Km.	=	.3861 sq. mi.
1 sq. mi.	=	2.59 sq. Km.	1 cu. cm.	=	.061 cu. in.
1 cu. in.	=	16.3934 cu. cm.	1 cu. dm.	=	.0353 cu. ft.
1 cu. ft.	=	28.3286 cu. dm.	1 cu. M.	=	1.308 cu. yds.
1 cu. yd.	=	.7645 cu. M.	1 Stere	=	.2759 cord.
1 cord	=	3.6281 Steres.	1 cl.	=	.338 fl. oz.
1 fl. oz.	=	2.9585 cl.	1 L.	=	1.0567 l. qts.
1 l. qt.	=	.9463 L.	1 Dl.	=	2.6418 gals.
1 gal.	=	.3785 Dl.	1 L.	=	.9081 dry qts.
1 dry qt.	=	1.1012 L.	1 Dl.	=	1.1351 pecks.
1 peck	=	.8809 Dl.	1 Hl.	=	2.8379 bushels.
1 bushel	=	.3523 Hl.	1 mg.	=	.0154 gr. Troy
1 Troy gr.	=	64.935 mg.	1 Gram	=	15.4324 grs. Troy
1 oz. Troy	=	31.1526 G.		=	.0321 oz. Troy
1 Troy lb.	=	.3732 Kilo.	1 Kilo	=	2.6791 lbs. "
1 oz. Av.	=	28.409 G.	1 Gram	=	.0352 oz. Av.
1 lb. Av.	=	.4536 Kilo.	1 Kilo	=	2.2046 lbs. Av.
1 Ton	=	.9072 Tonneau	1 Tonneau	=	1.1023 ton =
1 lb. Av.	=	.9072 German lb.			2204.6212 lbs. Av.
			1 Ger. lb.	=	1.1023 lbs. Av

GENERAL PRINCIPLES AND DIRECTIONS.

627. Since the Metric System is based upon the decimal scale, wherein 10 units of a lower order or denomination make 1 of the next higher, therefore all operations in Addition, Subtraction, Multiplica-

60

tion, and Division are performed in the same manner and are governed by the same numerical law as the operations in Decimals or with Dollars, cents, and mills.

628. Abbreviations of the base unit and of its higher denominations begin with a *capital;* the lower denominations begin with a *small* letter. All abbreviations should be read in full. Thus 6 M., 7 dm., 5 cm., 3 mm., should be read 6 meters, 7 decimeters, 5 centimeters, 3 millimeters.

629. The units of **length, capacity,** and **weight** increase and decrease according to the decimal scale; hence, each order of units must occupy *one* place, when written in full.

Thus, 127.4215 meters would be written, 1 Hm., 2 Dm., 7 M., 4 dm., 2 cm., 1.5 mm.

630. The scale of **square** or **surface measure** is (10×10) 100; hence, each order of units must have *two* places of figures.

Thus, 43 Ha., 8 A., 6 ca. may be written 43.0806 Ha.; and read, 43 hectares and 806 centares. Or, they may be written 4308.06 A., and read, 4308 ares and 6 centares. Or thus, 430806 ca., and read, 430806 centares.

631. The units of **cubic or solid measure** increase by the scale of $(10 \times 10 \times 10)$ 1000; hence, each order of units must have *three* places of figures.

Thus, 36 cu. M., 8 cu. dm., 25 cu. cm., may be written, 36.008025 cu. M.; or thus, 36008.025 cu. dm.; or thus, 36008025 cu. cm.

632. TO WRITE AND READ METRIC NUMBERS.

1. Write and read in the unit of meters and the lower denominations, 127.4215 meters.

$\qquad$ Ans. 127 M. 4 dm. 2 cm. 1.5 mm.

2. Write and read in higher units 210524683 millimeters. Ans. 21 Mm. 0 Km. 5 Hm. 2 Dm. 4 M. 6 dm. 8 cm. 3 mm.

3. Write and read 46286 mm. as decimeters.
Ans. 462.86 dm.

4. Write and read 46286 mm. as dekameters.
Ans. 4.6286 Dm.

5. Write in lower units 5.46732 Kg.
Ans. 5 Kg. 4 Hg. 6 Dg. 7 G. 3 dg. 2 cg.

6. Write 5.46732 Kg. as grams.
Ans. 5467.32 G.

7. Write 604 L. in higher units.
Ans. 6 Hl. 0 Dl. 4 L.

8. Write 604 L. as centiliters. Ans. 60400 cl.

9. Write 604 L. as myrialiters. Ans. .0604 Ml.

10. Write 6 Dm. 3 dm. 4 mm. as meters and decimals. Ans. 60.304 M.

NOTE.—When there are omissions in any of the denominations of the number given, their places must be filled with naughts.

11. Write as Kilograms and decimals, 434278 dg.
Ans. 43.4278 Kg.

633. TO REDUCE METRIC NUMBERS FROM HIGHER UNITS OR DENOMINATIONS TO LOWER.

1. Reduce 8 meters to millimeters.

OPERATION.

| 8 M. |
| 10 |
| 10 |
| 10 |
| ———— |
| 8000 mm. Ans. |

Explanation. — Remembering the table for the Metric Linear Measure, we reason as follows. Since 1 meter = 10 dm., there are 10 times as many dm. as M.; then since 1 dm. = 10 cm., there are 10 times as many cm. as dm.; then since 1 cm. = 10 mm., there are 10 times as many mm.

as cm., which is, as shown by the operation, 8000 mm. Or we might reason thus: Since 1 M. = 1000 mm., there are 1000 times as many mm. as meters.

GENERAL DIRECTIONS.

634. From the foregoing elucidation, we derive the following general directions, to reduce Metric Numbers from Higher to Lower denominations:

For the Linear, Capacity, and Weight Measures, multiply by 10 *(annex* 1 *naught) for each lower denomination to which the given number is to be reduced.*

For Surface Measure, multiply by 100 *instead of* 10; *and for Cubic Measure, multiply by* 1000 *instead of* 10.

NOTE.—In reducing the denominations of the Stere, in wood measure, 10 is the multiplier.

PROBLEMS.

2. Reduce 42 Hm. to Meters. Ans. 4200 M.
3. Reduce 33 Dm. to decimeters.
　　　　　　　　　　　　　　　Ans. 3300 dm
4. Reduce 8 L. to milliliters. Ans. 8000 ml.
5. Reduce 1 Kl. to liters. Ans. 1000 L.
6. Reduce 4 G. to milligrams. Ans. 4000 mg.
7. Reduce 21 Kg. to centigrams.
　　　　　　　　　　　　　　　Ans. 2100000 cg.
8. Reduce 9.82 M. to millimeters.
　　　　　　　　　　　　　　　Ans. 9820 mm.
9. Reduce 16 A. to centares. Ans. 1600 ca.
10. Reduce 25 sq. M. to sq. millimeters.
　　　　　　　　　　　　　　Ans. 25000000 sq. mm.
11. Reduce 14 cu. M. to cu. millimeters.
　　　　　　　　　　　　　Ans. 14000000000 cu. mm.
12. Reduce 3 cu. dm. to cu. centimeters.
　　　　　　　　　　　　　　　Ans. 3000 cu. cm.

13. Reduce 22 Ha. to centares.   Ans. 22000 ca.

14. Reduce 54 Ds. to decisters.   Ans. 5400 ds.

15. Reduce 1 millier or tonneau to milligrams.

Ans. 1000000000 mg.

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635. TO REDUCE METRIC NUMBERS FROM LOWER UNITS OR DENOMINATIONS TO HIGHER.

1. Reduce 44505 mm. to meters.

OPERATION.

| 44505
10 |
10 |
10 |

44.505 M. Ans.

Explanation.—Here again remembering the Metric Linear Table, we reason as follows: Since 10 mm. = 1 cm. there are $\frac{1}{10}$ as many cm. as mm.; then since 10 cm. = 1 dm. there are $\frac{1}{10}$ as many dm. as cm.; then since 10 dm. = 1 M. there are $\frac{1}{10}$ as many M. as dm., which is, as shown by the operation, 44.505 M. Or we may reason thus: Since 1000 mm. = 1 M. there are $\frac{1}{1000}$ as many M, as mm.

GENERAL DIRECTION.

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636. From the foregoing elucidation, we derive the following general direction to Reduce Metric Numbers from Lower to Higher denominations:

For the Linear, Capacity, and Weight Measures, divide by 10 (point off 1 place) for each higher denomination to which the given number is to be reduced.

For Surface Measure point off 2 places instead of 1.

And for Cubic Measure point off 3 places instead of

1, *for each higher denomination to which the given number is to be reduced.*

NOTE.—In reducing the denominations of the Stere, in wood measure, 10 is the divisor.

PROBLEMS.

2. Reduce 48752 cm. to Meters.
<div align="right">Ans. 487.52 M.</div>

3. Reduce 12307 dm. to Km.   Ans. 1.2307 Km.

4. Reduce 120 M. to Hm.   Ans. 1.2 Hm.

5. Reduce 333444 cl. to liters.
<div align="right">Ans. 3334.44 L.</div>

6. Reduce 10234 dl. to Kl.   Ans. 1.0234 Kl.

7. Reduce 24 ml. to Ml.   Ans. .0000024 Ml.

8. Reduce 484500 mg. to G.   Ans. 484.5 G.

9. Reduce 2389 cg. to kilograms.
<div align="right">Ans. .02389 Kg.</div>

10. Reduce 10 G. to Mg.   Ans. .001 Mg.

11. Reduce 1 mg. to tonneaus.
<div align="right">Ans. .000000001 T.</div>

12. Reduce 3414864 sq. mm. to sq. M.
<div align="right">Ans. 3.414864 sq. M.</div>

13. Reduce 8.27 sq. cm. to sq. dm.
<div align="right">Ans. .0827 sq. dm.</div>

14. Reduce 55544433366 cu. mm. to cu. M.
<div align="right">Ans. 55.544433366 cu. M.</div>

15. Reduce 897568 cu. dm. to steres.
<div align="right">Ans. 897 568 S.</div>

16. Reduce 2245 cu. M., or Steres, to dekasteres.
<div align="right">Ans. 224.5 Ds.</div>

17. Reduce 444 ds. to dekasteres.
<div align="right">Ans. 4.47 Ds.</div>

637.   TO ADD METRIC NUMBERS.

1. What is the sum of 462 mm. 28 cm. 406 dm. and 16 Dm.   Ans. 201.342 M., or 201342 mm.

FIRST OPERATION.	SECOND OPERATION.
.462	462
.28	280
40.6	40600
160.	160000
201.342 M. Ans.	201342 mm.

Explanation.— In all problems of this kind, the numbers should be written in the *base unit* of the table and added as in decimals. Accordingly, as shown in the first operation, we wrote the numbers in meters and decimals of meters and then added.

In the second operation, we wrote and added the numbers as mm.

GENERAL DIRECTION.

638. From the foregoing elucidation, we derive the following general direction for Adding Metric Numbers :

Write the numbers to be added in the BASE UNIT, *and decimals of the same, of the table to which they belong and then add as in decimals.*   Art. 306, page 239.

PROBLEMS.

2. Add 68 M. 42 dm. 3204 mm. and 63 Hm.
Ans. 6375.404 M.

3. Add 7 Mm. 2 Km. 5 Hm. 7 Dm. 8 M. 5 dm. 4 cm. and 8.07 mm.   Ans. 72578.34807 M.

4. Find the sum of 21 L. 16 dl. 4 cl. and 27 Hl.
Ans. 2722.64 L.

5. A grocer has four boxes containing as follows: 1st, 8.5 L.; 2d, 7 Dl.; 3d, 21 cl.; 4th, 1 M. and 8 ml. How many L. in all ?   Ans. 79.718 L.

6.   What is the sum of 12 G. 3 dg. 9 cg. and 6 mg ?   Ans. 12.396 G.

7.   What is the weight in Kg. of 3 bales of cotton which weigh respectively : 204.6 Kg.; 205 Kg. 4 Hg.; and 208 Kg. 8 G.?   Ans. 618.008 Kg.

8.   Add in Kg., 14 T. 6 Q. 8 Mg. 7 Kg. 4 Hg. 5 Dg. 9 G.   Ans. 14687.459 Kg.

9.   Add the above problem in the unit of *Tonneaus.*   Ans. 14.687459 T.

10.   What is the sum of 124 sq. M., 6 sq. dm., and 37 sq. cm.?   Ans. 124.0637 sq. M.

OPERATION INDICATED.

$$124.$$
$$.06$$
$$.0037$$

11.   Add 42.8 sq. M., 21.65 sq. M., 28 sq. dm., and 4 sq. cm.   Ans. 64.7304 sq. M.

12.   Add in the unit of *Ares*, 39.5 A., 25 Ha., and 84 ca.   Ans. 2540.34 A.

13.   What is the sum of 14.5 cu. M. 23 cu. dm. 123 cu. cm. and 24 cu. mm.?

              Ans. 14.523123024 cu. M.

OPERATION INDICATED.

$$14.5$$
$$.023$$
$$.000123$$
$$.000000024$$

14.   Add in the *unit* of *Steres*, 22 S., 12 Ds., and 15 ds.   Ans. 143.5 S.

639.   TO SUBTRACT METRIC NUMBERS.

1. What is the difference between 75.6 M. and 85 mm.?   Ans. 75.515 M.

OPERATION.
75.6 = M.
.085 = M.
_____
75.515 M. Ans.

Explanation.—In subtraction as in addition, we write the numbers to be subtracted in the *base unit* of the measure table to which the numbers belong, and subtract as in decimals.

GENERAL DIRECTION.

640. From the foregoing elucidation, we derive the following general direction for Subtracting Metric Numbers:

Write the numbers to be subtracted in the base unit and decimals thereof, of the table to which the numbers belong, and then subtract as in decimals. Art. 308, page 241.

PROBLEMS.

2. From 4324.08 Km. take 123.5 M. in the unit of M. and also of Km.   Ans. 4323956.5 M.
                                4323.9565 Km.

3. Find the difference between 274.25 L. and 44.5 cl.   Ans. 273.805 L.

4. A barrel contained 151.44 L., and 6 L. and 7 dl. leaked out. How many liters still remain?
                                Ans. 144.74 L.

5. What is the difference in L. between 1 Ml. and 1 ml.?   Ans. 9999.999 L.

6. From 264.5 G. take 28.4 cg.
                                Ans. 264.216 G.

7. From 1428 Kg. take 16.5 Hg.
                                Ans. 1426.35 Kg.

8. What is the difference in T. and Kg. between 2 tonneaus and 2 Kg.?   Ans. 1.998 T.; 1998 Kg.

. 9. What is the difference between 88.21 sq. M. and 38 sq. dm.?   Ans. 87.83 sq. M.

10. A plantation contains 2471.14 Ha. If 2471.14 A. are sold. how many hectares will remain ?
                    Ans. 2446.4286 Ha.

11. From 1528 cu. M. take 1 cu. M. 1 cu. dm. 1 cu. cm. and 1 cu. mm.
                Ans. 1526.998998999 cu. M.

12. A wood dealer bought 150 steres of wood; he sold 70.25 cu. M. How much has he remaining?
                    Ans. 79.75 S., or cu. M.

641.   TO MULTIPLY METRIC NUMBERS.

1. How many meters are there in 11 pieces of cloth, each containing 36.25 meters ?
                        Ans. 398.75 M.

OPERATION.
36.25
11
———
398.75 M. Ans.

Explanation—In multiplication of Metric Numbers, we proceed in the same manner as in simple and decimal numbers. Hence no extended elucidation and general directions are deemed necessary.

2. What will 204.5 meters cost at $2 per meter?
                    Ans. $409.

3. If a man walks 20000 meters per day, how many kilometers will he walk in 60 days?
                    Ans. 1200 Km.

4. What will 484 dm. cost at $1.50 per meter?
                    Ans. $72.60.

5. What will 484 dm. cost at $1.50 per dm. ?
                    Ans. $726.

6. What cost 75.2 liters of milk at 5¢ per L.?
                    Ans. $3.76.

7. What cost 200.52 Hl. of corn at $1.60 per Hl.?
Ans. $320.83.

8. A fruit dealer bought 6 Hl. of pecans, at $9 per Hl., and sold them at 15¢ per liter. How much did he gain?   Ans. $36.

9. Sold 26.4 Kg. of grapes at 45¢ per kilo. How much was received for them?   Ans. $11.88.

10. What cost 27.25 tonneaus of hay at $20.50 per T.?   Ans. $558.625.

11. What cost 416.3 Kg. of sugar at 20¢?
Ans. $83.26.

12. Bought 362.3122 G. of silver at 3½¢ per gram. What did it cost?   Ans. $12.68+.

13. How many kilograms in 49000 pills, the weight of each being .5 dg.?   Ans. 2.45 Kg.

14. How many sq. meters in a yard 40.5 M. long and 15.24 M. wide?   Ans. 617.22 sq. M.

15. A plantation is 2.42 Km. long and 1500 M. wide. How many hectares does it contain?
Ans. 363 Ha.

16. How many cubic meters in a box 2.8 M. long, 2.1 M. wide, and 8.5 dm. deep?
Ans. 4.998 cu. M.

17. What will be the freight on 4 boxes, each measuring 3 M. by 2.6 M. by .9 M. at $5.25 per cu. M.?   Ans. $147.42.

18. How many cubic meters of earth in a levee 140 M. long, 2.3 M. deep, and 20 M. wide at the base and 15.2 M. wide at the top?
Ans. 5667.2 cu. M.

19. How many steres in a pile of wood 28.5 M. long, 3.2 M. high, and 4.3 M. wide?
Ans. 392.16 S., or cu. M.

20. How many cubic meters of earth will it take to fill a lot .5 meter deep, the lot being 60.2 meters long, and 25 meters wide?   Ans. 752.5 cu. M.

642. TO DIVIDE METRIC NUMBERS.

1. A man walked 1600 meters in 20 minutes. How many meters did he walk in 1 minute?

Ans. 80 M.

OPERATION.

$1600 \div 20 = 80$ M. Ans.

Explanation—In Division of Metric Numbers, we proceed with the operations in the same manner as in simple and decimal numbers. Hence no extended elucidation and general directions are deemed necessary.

2. The Steamer Katie ran 500 Km. in 22 hours. How many kilometers did she run per hour?

Ans. 22.727+Km.

3. If 6.5 meters make a suit, how many suits can be made from 195 meters?

Ans. 30 suits.

4. In 425 liters, how many hectoliters?

Ans. 4.25 Hl.

5. Bought 45 liters of strawberries for $3.375. What was the price per L.? Ans. 7½¢.

6. If you divide 180 hectoliters of potatoes equally among 24 persons, what will each receive?

Ans. 7.5 Hl.

7. 21 kilograms of sugar cost $4.62. What was the cost of 1 Kg.? Ans. 22¢.

8. How many days will 42.5 T. of coal last a family, if they burn 150 Kg. per day?

Ans. 283.33+days.

9. A druggist has 2.45 Kg. of medicine which he wishes to make into pills, each to contain .5 dg. How many pills will there be?

Ans. 49000 pills.

10. A garden contains 900 sq. M., and is 22.5 M. wide. How long is it? Ans. 40 M.

11. A side-walk is 80.4 M. long by 4.2 M. wide. How many tiles, each 2.4 dm. long and 1.2 dm. wide, will be required to pave the side-walk ?

Ans. 11725 tiles.

12. How many *ares* in a piece of land 60 meters long and 42.2 meters wide ?   Ans. 25.32 A.

13. A box is 2.5 dm. long, 2 dm. wide, and 1.5 dm. deep. How many of such boxes may be put in a larger box which is 2.5 M. long, 2 M. wide, and 1.5 M. deep ?   Ans. 1000 boxes.

14. If you buy 5.6 dekasteres of wood and use 1.1 cu. meters per day, how long will it last ?

Ans. $50\frac{9}{11}$ days.

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643. TO REDUCE METRIC TO AMERICAN WEIGHTS AND MEASURES.

1. Reduce 2.6 meters to feet.   Ans. 8.53+ feet.

FIRST OPERATION.

1 M. = 39.37+ inches, practically.
        2.6
------
12) 102.362 inches.

        8.530+ feet Ans.

or thus :

          | 39.37
     12 | 2.6

Explanation.—Referring to the table of equivalents, we find that 1 M. = (practically) 39.37 in. We then reason as follows: Since 1 M. = 39.37 in., 2.6 M. are equal to 2.6 times as many, which is 102.362 in.; then, since 12 in.= 1 ft., there are $\frac{1}{12}$ as many feet as inches, which is 8.53 + feet.

SECOND OPERATION.

3.2808+ = feet, the equivalent of a meter.
2.6

8.53008+ = feet, Ans.

GENERAL DIRECTION.

644. From the foregoing elucidations, we derive the following general direction for Reducing Metric to American Weights and Measures:

Multiply the equivalent value of the metric unit given, by the metric number to be reduced, and then, if necessary, reduce the product to the denomination required.

NOTE.—In working the following problems, four decimal places have been taken as a standard for all unit equivalents of more than that many decimals. The calculations are made directly upon the equivalency of the denomination to be reduced.

2. Reduce 408.2 kilometers to yards.
Ans. 446407.52 yds.

3. In 24.5 cm. how many inches?
Ans. 9.6456+in.

4. Reduce 70 M. to yards. Ans. 76.552+yds.

5. Reduce 25.5 liters to dry quarts.
Ans. 23.1565+d. qts.

6. In 40 kiloliters how many gallons?
Ans. 10567.44+gals.

7. In 200 hectoliters how many bushels?
Ans. 567.58+bus.

8. Reduce 25 grams to Troy grains.
Ans. 385.81+grs.

9. Reduce 75.2 kilograms to pounds Avoir.
Ans. 165.7859+lbs.

10. In 444 tonneaus how many tons?
Ans. 489.4212+tons.

11. Reduce 5.5 mg. to Troy grains.
Ans. .0847+grs.

12. Reduce 24 *ares* to square rods.
<div align="right">Ans. 94.8912+sq. rds.</div>

13. Reduce 85.5 hectares to acres.
<div align="right">Ans. 211.279+Acres.</div>

14. In 500 hectares, how many sq. miles?
<div align="right">Ans. 1.9305+sq. mi.</div>

15. Reduce 150 cu. meters to cu. feet.
<div align="right">Ans. 5297.475+cu. m.</div>

16. Reduce 7.9 cu. mm. to cu. inches.
<div align="right">Ans. 0004819+cu. in.</div>

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845. TO REDUCE AMERICAN TO METRIC WEIGHTS AND MEASURES.

1. Reduce 40 feet to meters.
<div align="right">Ans. 12.192+M.</div>

OPERATION.

40 ft.

12

39.37) 480 in. (12.192+M.

or thus:

$$39.37 \,\bigg|\begin{array}{l}40\\12\end{array}$$

Explanation—Remembering that a meter = 39.37 inches, practically, we first reduce the feet to inches, and then reason as follows: Since in 39.37 in. there is 1 meter, in 480 in. there are as many meters as 480 in. are times = to 39.37 in., which is 12.192+,

SECOND OPERATION.

3.048 decimeters = the equivalent of 1 foot.

40

121.920 decimeters, which divided by 10 =
12.192 meters, Ans.

2. Reduce 1 mile, 8 rods, 10 feet, and 6 inches to kilometers.
<div align="right">Ans. 1.652781+Km.</div>

OPERATION.

1 mi., 8 rds., 10 ft., 6 in.   39.37) 65070 inches.
$\cdot$ 320
——–
328   rods.   1652.781 + M.
16½   1.652781 + Km.
——   or,
5422   feet.   65070 in. × 2.54 cm. = 165278.1 +
12   cm. = 1.652781 + Km.
——

65070   inches.

Explanation.—In this problem we first reduce the given number to inches; then to meters, as above explained; and then to kilometers by dividing by 1000, the number of meters in a kilometer.

GENERAL DIRECTIONS.

646. From the foregoing elucidations, we derive the following general directions for reducing American to Metric Weights and Measures:

1°. *Reduce the given number to the lowest unit named, or to a convenient denomination, of which the equivalent value of the metric unit is known ; then divide the same by the equivalent value of the metric unit, and when necessary reduce the quotient to the required denomination.*

Or 2°. *Multiply the given number, reduced to its lowest denomination, by the equivalent value of such denomination in the metric unit.*

3. In 70 yards, how many meters ?
                          Ans. 64.008 + M.

4. Reduce 35 gallons, 3 quarts, and 1 pint of molasses to liters.   Ans. 135.8001 + liters.

5. Reduce 5 bushels, 2 pecks, 5 quarts, and 1 pint to liters.   Ans. 199.8678 + liters.

6. In 20 gallons of milk, how many liters?
Ans. 75.7073+L.

7. In 400 bushels, how many hectoliters?
Ans. 140.9493—Hl.

8. In 3 gills, how many centiliters?
Ans. 41.4364+cl.

9. Reduce 200 pounds Avoirdupois to kilograms.
Ans. 90.7194+Kg.

10. Reduce 15 ounces Avoirdupois to Hectograms.
Ans. 4.2525+Hg.

11. Reduce 10 ounces Troy to grams.
Ans. 311.0339+G.

12. Reduce 50 tons to tonneaus.
Ans. 45.3597+T.

13. Reduce 20 grains Troy to centigrams.
Ans. 129.6176+cg.

14. Reduce 240 sq. yards to sq. meters.
Ans. 200.6689 — sq. M.

15. In 500 sq. feet, how many sq. decimeters?
Ans. 4646.8401+sq. dm.

16. In 200 acres, how many hectares?
Ans. 80.9356 Hectares.

17. Reduce 1610 cu. feet to cu. meters.
Ans. 45.5877+cu. M.

18. Reduce 2500 cu. yards to cu. meters.
Ans. 1911.3149+cu. M.

19. In 190 cords, how many steres?
Ans. 688.6553+Steres.

SYNOPSIS FOR REVIEW.

Define the following words and phrases :

609. Metric System of Weights and Measures. 610. The Standard Meter. 611. Origin of Metric System. 612. The Meter. 613. The Are. 614. The Cubic Meter. 615. The Liter. 616. The Gram. 617. The Multiple Units. 618. The Sub-Multiple Units. 619. Table for Metric Linear Measure. 620. Table of Metric Square Measure. 621. Table for Metric Cubic Measure. 622. Table for Wood Measure. 623. Table for Metric Measures of Capacity. 624. Table of Equivalents of Metric Denominations in Dry and Liquid Measures. 625. Metric Weight Table. 626. Table of equivalents of American and Metric Units. 627. Numerical Law of the Metric System. 628. Manner of writing Abbreviations of Metric Denominations. 629. Ratio of Increase or Decrease of Metric Units of Length, Capacity, and Weight. 630. Of Metric Square Measure. 631. Of Metric Cubic, or Solid Measure. 632. To Write and Read Metric Numbers. 634. General Directions to Reduce Metric Numbers from Higher to Lower Denominations. 636. To Reduce Metric Numbers from Lower to Higher Denominations. 638. To Add Metric Numbers. 640. To Subtract Metric Numbers. 641. To Multiply Metric Numbers. 642. To Divide Metric Numbers. 644. To Reduce Metric to American Weights and Measures. 646. To Reduce American to Metric Weights and Measures.

Miscellaneous Practical Problems.

1. What will 26½ pounds coffee cost at 16¾¢ per pound ? Ans. $4.43⅜.

2. 10½ yards cost $28.87½. What was the cost of 1 yard ? Ans. $2.75.

3. Sugar is worth 6¼¢ per pound. How many pounds can be bought for $1.62½ ? Ans. 26 lbs.

4. A jar filled with butter weighs 22 lbs. 5 oz· The weight of the jar alone is 4 lbs. 14 ounces. Buy this butter at 32½¢ per pound, and pay for the same as follows: Cash $1, 13½ lbs. rice at 9¼¢, and the balance in flour at 4½¢ per pound. How many pounds of flour must be given ?
Ans. Practically, 76 lbs.

5. Make and receipt a bill for rent of house 457 Baronne street, from April 17th to June 15th, inclusive, at $42 per month. What is the amount of the bill ? Ans. $81.20.

6. Buy 4 bushels 3 pecks 5 quarts 1 pint of cherries at $2.25 per bushel and state the cost.
Ans. $11.07.

7. Sell 1 bushel 3 quarts 1 pint of cherries at 8 cents per quart, and state the amount received.
Ans. $2.84.

8. Cloth is $2.50 per meter. What is it worth per yard ? Ans. $2.286+.

9. Cloth cost in Mexico $1.80 per varra. What is the cost per yard ? Ans. $1.94+.

10. Sell 15¢ worth of coffee at 22½¢ per pound. How much will you sell ? Ans. 10⅔ ounces.

11. A lady wishes to buy 35¢ worth of silk which is $3.85 per yard. How much will you sell her? Ans. $3\frac{3}{11}$ inches.

12. Sell 3½ inches of silk at $2.90 per yard, and state the price. Ans. $28\frac{7}{36}$¢.

13. Wheat is $1.90 per bushel. What is the price per cental? Ans. $3.16⅔.

14. Corn is 85¢ per cental. What is it per bushel? Ans. $ .47¾.

15. What is the cost of 1465 pounds of corn at 84 cents per bushel, and how many bushels are there? Ans. $21.97¼ cost.
26 bush., 9 lbs.

16. Sold 5294 pounds of hay at $23.75 per ton. How many tons were there, and what was the value of it? Ans. 2 tons, 1294 lbs.
$62.86⅝ value.

17. Bought 320$\frac{11}{10}$ bushels of wheat at $1.95 per bushel. What was the cost? Ans. $625.36¼.

18. Bought 1136½ pounds of dried peaches at $5.80 per bushel. How many bushels were there, and what did they cost? Ans. 34 bush., 14½ lbs.
199.74\frac{33}{14}$ cost.

19. Bought 15 bushels and 31 pounds of corn at 78½ cents per bushel. What was the cost? Ans. 12.20\frac{107}{112}$.

20. Bought 3 coops of chickens containing 2 dozen and 7 chickens each, at $4.35 per dozen. What did they cost? Ans. $33.71¼.

21. What will 74 pounds and 11 ounces of butter cost at 42½ cents per pound? Ans. 31.74\frac{7}{13}$.

22. Bought 36 pounds and 7 ounces of tea at $1.12½ per pound. What did it cost? Ans. 40.99\frac{7}{13}$.

23. Butter is worth 45 cents per pound. How much can be bought for 20 cents?

Ans. 7⅕ ounces.

24. What is the cost of 31845 feet of lumber at $22.25 per M.? Ans. $708.55¼.

25. What will 183 feet of lumber cost at $25.75 per M.? Ans. $4.71₂₀.

26. Bought 3 bales of hay weighing as follows: (1) 421 pounds, (2) 394 pounds, (3) 487 pounds, at $22.50 per ton. What did it cost?

Ans. $14.64¾.

27. Sold 3¼ dozen boxes Spencerian pens at $108 per gross. What did they amount to?

Ans. $29.25.

28. How much coffee can I buy for 5 cents, when a pound costs 28 cents? Ans. 2⁴⁄₇ ounces.

29. What is the cost of 400 T. 2 cwt. 3 qrs. 20 lbs. of iron at $60 per ton of 2240 pounds?

Ans. $24008.78¼.

30. A planter shipped 6 dozen dozen boxes of peaches to market, but being delayed on the way, ½ a dozen dozen boxes spoiled; the remainder were sold at 70 cents per box. What did they amount to? Ans. $554.40.

31. Bought 12 dozen and 5 hats at $11 per doz. What did they cost? Ans. $136.58⅓.

32. What is the amount due for the freight of 40000 pounds of merchandise for 965 miles, at 5¢ for 100 pounds for 100 miles? Ans. $193.

33. What is the cost of 2381¾ pounds of cotton at 17¹³⁄₁₆ cents per pound? Ans. $424.24⅖⅞.

34. What is the cost of a 14 carat gold chain that weighs 4 oz. 7 pwt. 15 grs. at $1.20 per pwt. for pure gold, allowing ½¢ per grain on full weight for manufacturing and the alloy? Ans. $71.85¼.

35. Bought 4692 pounds of barley at $.88 per bushel. How many bushels were there and what was the cost? Ans. 97 bush. 36 lbs.
Cost $86.02.

36. Bought 2765 pounds of oats at 76¢ per bushel. What was the cost, and how many bushels were there? Ans. $65.66⅝ cost.
86 bush. 13 lbs.

37. What is the cost of 4878 pounds of wheat at $2.45 per cental? Ans. $119.511.

38. What is the cost of 200 sacks of guano each weighing 162 pounds, at $52¼ per ton? Ans. $846.45.

39. What is the value of 5790 hoop-poles at $18 per M.? Ans. $104.22.

40. What is the value of 8750 shingles at $8.75 per M.? Ans. $76.56¼.

41. What is the value of 11428 fence pickets at $9 per M.? Ans. $102.852.

42. What is the value of 1364 pine apples at $11½ per C.? Ans. $156.86.

43. What is the cost of 2417 cocoanuts at $8.25 per C.? Ans. $199.40¼.

44. What is the value of 78420 railroad ties at $75 per M.? Ans. $5881.50.

45. What is the freight on 540 bales cotton, weighing 243084 pounds, at ⅝d. per pound from New Orleans to Liverpool? Ans. £933 0s. 7½d.

46. What is the freight in United States currency on 25000 bushels corn from New Orleans to Liverpool, at 24s. per imperial quarter of 480 pounds, allowing £1 to be equal to $4.87? Ans. $17045.

47. What is 2½% for selling 38482 lbs. of sugar at 6¼¢? Ans. $60.13.

48. Goods cost $24. At what price must they be sold to gain 25%? Ans. $30.

49. Goods cost $24. At what price must they be sold to lose 25%? Ans. $18.

50. Sold goods for $30 and gained 25%. What was the cost? Ans. $24.

51. Sold goods for $18 and lost 25%. What was the cost? Ans. $24.

52. Goods cost $24 and were sold for $30. What was the per cent gain? Ans. 25%.

53. Goods cost $24 and were sold for $18. What was the per cent loss? Ans. 25%.

54. Bought in New York an invoice of goods amounting to $2400. Paid freight and other charges to deliver them in New Orleans, $102. In the invoice are 8 dozen hats which cost in New York $27 per dozen. 1. What was the % of the charges? 2. What was the cost of 1 hat in New Orleans? 3. What is the retail price per hat, from which 20% may be deducted and the hats sold at wholesale at a gain of 25% on New Orleans cost?
Ans. 4¼% charges.
$2.34+cost of 1 hat in New Orleans.
$3.66—retail price of hat.

55. Bought a cargo of coffee at 10¢ a pound. Paid 10% duty. What must I ask for the coffee per pound, so that I may fall 10% on my asking price, allow 10% for loss by shrinkage of coffee, lose 10% of sales in bad debts, and still gain 10% on my investment (full cost)? Ans. $16\frac{438}{729}¢.

NOTE.—See problem 11, page 383.

56. What is the interest on $1800 for 64 days at 5 per cent? Ans. $16.00.

57. What is the interest on $1200 for 72 days at 6 per cent? Ans. $14.40.

58. What is the interest on $3000 for 33 days at 8 per cent ? Ans. $22.00.

59. What is the interest on $1600 for 124 days at 9 per cent ? Ans. $49.60.

60. What is the interest on $1560 for 51 days at 12 per cent ? Ans. $26.52.

61. How many square feet in a pavement 120 feet 4 inches long and 10 feet wide ?

Ans. 1203⅓ sq. feet.

62. How many square yards in a plat of ground 140 feet 3 inches long and 64 feet 6 inches wide ?

Ans. 1005⅓ sq. yards.

63. How many *squares* in the roof of a building 78 feet 6 inches long, and 48 feet 4 inches wide ?

Ans. 37.94⅓ squares.

64. How many sq. feet in 8 boards, each measuring 16 feet long and 17 inches wide, and what will they cost at 2½¢ per sq. foot ? Ans. 181⅓ sq. feet.

$4.53⅓ cost.

65. How many board feet in 13 pieces of plank, each measuring 20 feet 6 inches long, 14 inches wide, and 3 inches thick, and what is the cost at $23 per M.? Ans. 932¾ bd. feet.

$21.453¼ cost.

66. How many sqare feet in a circle, the diameter of which is 12 yards ? Ans. 1017.8784 sq. ft.

67. How many shingles will it require to shingle a building, the roof of which measures 44 ft. 7 inches from eave to eave, without allowances, by 50 feet 4 inches long, allowing a shingle to cover a space 4 inches wide and 5 inches long ? Ans. 16157.

68. A yard is 24 feet 3 inches long by 11 feet 5 inches wide. How may bricks, 4 by 8 inches will it take to pave it, no allowance to be made for the opening between the bricks ? Ans. 1245⅜⅞.

69. How many square yards of paving in a side-walk 64 feet long and 11 feet 8 inches wide?
<div align="center">Ans. 82⅔⁴ square yards.</div>

70. How many flags, each 16 inches square, will it require to flag a walk 22 yards 1 foot 4 inches long and 6 feet 8 inches wide?   Ans. 252½ flags.

71. How many yards of carpeting that is 27 inches wide, will it take to cover the floor of a room that is 25 feet 6 inches long, and 22 feet 9 inches wide, making no allowance for waste in matching or turning under?   Ans. 85¹⁷⁄₁₈ yards.

72. How many cubic feet in a box 5 feet long, 3 feet wide, and 4 feet deep?   Ans. 60 cubic feet.

73. What is the freight on a box 6 feet 4 inches long, 4 feet wide, and 3 feet 9 inches deep, at 25 cents per cubic foot?   Ans. $23¾.

74. What will be the freight on a box 9 feet 3 inches long, 4 feet 6 inches wide, 2 feet 10 inches deep, at 30 cents per cubic foot?   Ans. $35.38⅛.

75. How many bushels will a bin hold, that is 10 feet long, 8 feet 6 inches wide, and 5 feet 2 inches deep?   Ans. 352.90+ bushels.

76. How many cords of wood in two ranks, each 60 feet long and 8 feet 3 inches high?
<div align="center">Ans. 30¹⁵⁄₁₆ cords.</div>

77. How many barrels will a quadrilateral cistern hold, whose height is 12 feet and width of side 5 feet 8 inches?   Ans. 91⁸²¹⁄₁₆₁₇ barrels.

78. How many cubic yards in a levee 80 rods long, 60 feet wide at the base, 12½ feet at the top, and 5 feet 4 inches average depth?
<div align="center">Ans. 9451²³⁄₂₇ cu. yds.</div>

79. How many gallons will a box hold, that is 5 feet long, 2 feet 4 inches wide, and 3 feet deep ?
Ans. 261.81+ gallons.

80. How many cubic feet in a cylinder 6 feet long, 3 feet 4 inches in diameter ?
Ans. 52.36 cubic feet.

81. How many gallons in a cylindrical cistern, 9 feet 6 inches high, and 7 feet 2 inches in diameter ?
Ans. 2866.6896 gals.

82. How many pints in a cylindrical vessel, whose height is 14 inches and diameter 12½ inches ?
Ans. 59.5 pints.

83. How many bushels in a cylinder shaped box, whose height is 10 feet, and diameter 10 feet ?
Ans. 631.125 bu.

84. How many cubic feet in a frustum of a cone, whose height is 6 feet, diameter of the greater end is 4 feet and of smaller end 3 feet ?
Ans. 58.1196 cubic feet.

85. How many gallons in a cistern which is in the form of a frustum of a cone, whose height is 9 feet 6 inches, lower base 7 feet 2 inches, and upper base 6 feet 8 inches ?   Ans. 2671.3392 gals.

86. A farmer has a heap of grain in a conical form, the diameter of which is 14 feet 4 inches, and the depth 5 feet 3 inches. How many bushels does it contain ?   Ans. 226.906 bus.

87. *Report of the condition of the Soulé College*
National Bank of New Orleans, July 1, 1886.

RESOURCES.

Loans and discounts......................	$1560541.25	
United States Bonds..................	556000.00	
Due from National Banks............	61720.00	
Stocks and other securities...........	105009.26	
Real Estate..........................	221380.00	
Bank Furniture and Fixtures........	12010.38	
Legal Tender and U. S. Notes, including fractional currency......	915721.45	
Gold Coin	61200.00	
Exchange for Clearing House	162040.27	
Redemption Fund with U. S. Treasurer	22500.00	
		$3678122.61
Taxes paid.........................	7843.06	
Current expenses....................	23630.71	
		31473.77
		$3709596.38

LIABILITIES.

Capital Stock	$800000.00	
Reserve Fund........................	5462.12	
Circulation outstanding..............	450000.00	
Dividends unpaid....................	8400.00	
Individual deposits..................	2180329.43	
Due to Banks and Bankers...........	110900.87	
		$3608092.42
Exchange	40208.09	
Interest	26871.11	
Discount	28324.62	
Profit and loss......................	6100.14	
		$ 101503.96
		$3709596.38

FRANK SOULE. Cashier.

The foregoing statement is for 6 months business; what
has been the *net gain;* the *gain per cent* on capital and Reserve
Fund, and the *gain per cent per annum?* The Bank, through
its directors, declares a semi-annual dividend of 8 per ct, on

the capital stock and passes the remainder of the gain to the credit of Reserve Fund. Now if the current rate of interest in the money market is 12 per ct., what is the *market value* of the Bank Stock, the shares being $100 par, and what is the *intrinsic value* of the stock, based upon the foregoing figures?

Ans. $70030.19 net gain.

8.1576+ per ct. gain.

16.3152+ per ct. gain per annum.

$135.96 market value of stock.

$108.0615+ intrinsic value of stock.

SOLUTION.

First operation to find the net gain.	Second operation to find the net gain.
Resources........$3678122.61	Total gain........$101503.96
Liabilities.........$3608092.42	Total loss......... 31473.77
Net gain.......... $70030.19	Net gain $70030.19

Operation to find the gain *per cent* and the gain *per cent per annum.*

The Capital Stock is $800000
"   Reserve fund is 58462.12

"   Aggregate Capital is...........................$858462.12
which, according to the question is the amount to measure the gain with.

$858462.12 | $70030.19
           | 100

           | 8.1576+ per ct. gain for 6 months.
           | 2

           | 16.3152+ per ct. gain per annum.

Operation to find the market value of the stock.

$100
6 | 8.1576   or thus   12 | 16.3152
  ---------   ---------
  $135.96 market value $135.96 market value.

Operation to find the intrinsic value of the stock.

The net gain as above is...........................$70030 19
From which we deduct 8 per ct. dividend on
$800000 Capital Stock.......................... 64000 00

And obtain for the reserve fund.................... $6030 19
To which we add the previous reserve fund........$58462 12

And obtain the present amount of reserve fund.....$64492 31
To this we add the Capital Stock..................800000 00

And obtain the Net Resources of the Bank........$864492 31
   This $864492.31 divided by 8000, the number of shares, gives **$108.0615+** as the *intrinsic value* of the stock.

88. A merchant borrows **$50000** for five years at **10%** and agrees to pay the principal and interest in 5 equal annual installments. What are the yearly payments?   Ans. **$13189.87.**

   NOTE.—See problem 10, page 409.

<table>
<tr><td>OPERATION.</td><td>Explanation—</td></tr>
</table>

$50,000÷$3.79078068=$13189.874+   *Explanation—* By the conditions of the problem, we observe that the $50000 is the *present worth* of an annuity, the time being 5 years and the rate per cent 10. Hence, as the present worth of $1, multiplied by the annuity, would give the full present worth, it is clear that if we divide the given *present worth* ($50000) by the present worth of $1, for the given time and rate per cent, the quotient will be the required annuity. The present worth of an annuity of $1 for five years at 10% is $3.79078068, and $50000 divided by 3.79078068 gives $13189.87 as the yearly payment.

   The present worth of an annuity of $1 may be obtained from Annuity Tables, or produced by the operation of Compound Interest.

   To produce it by compound interest, we first find the *amount* of an annuity of $1 for five years at 10%. This we do by adding $1.4641 which is the *first* payment of $1 and its compound interest for four years.

plus $1.3310 which is the *second* payment of $1 and its compound interest for three years.

plus $1.2100 which is the *third* payment of $1 and its compound interest for two years.

plus $1.1000 which is the *fourth* payment of $1 and its compound interest for one year.

plus $1.0000 which is the *fifth* payment of $1.

   $6.1051   This total sum is the amount of an Annuity of $1 for five years at 10% and to find the *present worth* of the same, we divide it by the compound interest on $1 for five years at 10%, which is $1.6105100; thus, $6.1051÷$1.61051=$3.790780676+.

89. Sold a watch for $17.16 and lost as much per cent as the watch cost. What was the cost?

<div align="center">1st, Ans. $78. 2d, Ans. $22.</div>

NOTE 1.—It is generally believed that an arithmetical problem can have but one correct answer; but this belief is clearly erroneous. For if we apply the conditions of the sale to both of the answers to this problem, we prove their correctness.

NOTE 2.—All problems of this kind, provided the selling price does not exceed $25 or 25 of any other monetary unit, may be solved and the two answers obtained by the following method, which is derived from an Algebraic solution:

Subtract the selling price from 25, and to the square root of the remainder add 5; then multiply this sum by 10 and the product will be the greater answer. Or, from 5 subtract the square root above obtained and multiply the difference by 10; the result will be the lesser answer.

<div align="center">OPERATION.</div>

$25 - 17.16 = 7.84$; then the square root of 7.84 is 2.8; then $2.8 + 5 = 7.8$; $7.8 \times 10 = \$78$ the greater cost. And $5 - 2.8 = 2.2$; then $2.2 \times 10 = \$22$ the lesser cost.

<div align="center">PROOF.</div>

$78 cost $-(78\% = \$60.84) = \$17.16$ selling price.
$22 cost $-(22\% = \$4.84) = \$17.16$ selling price.

90. Sold goods for $12.75 and lost as much per cent as they cost. What did they cost?

<div align="right">Ans. $85; or $15.</div>

91. Sold goods for $39 and gained as much % as they cost. How much did they cost? Ans. $30.

NOTE.—All problems of this kind may be solved by the following method:

To the selling price add $25; from the square root of this sum subtract 5; then multiply the remainder by 10, and the product will be the cost.

<div align="center">OPERATION.</div>

$39 + \$25 = \$64$; the square root of $64 is $8; $8 - \$5 = \$3$; $3 \times 10 = \$30$ cost, Ans.

92. Sold goods for $56.90¼, and gained as much per cent as they cost. What did they cost?

<div align="right">Ans. $40¼.</div>

93. Hats cost $2¼ each. How many can be bought for $23½ and how much money will remain on hand? Ans. 10 hats, $1 remainder.

94. A commission merchant received $2000 to be invested in supplies. He charges 2½% commission for investing, 1% is allowed for charges on the supplies, and 2% for insurance on cost plus 10% of cost. What sum was invested in supplies, and how much was his commission? Ans. $1891.30 invested in supplies. $47.76 — commission.

OPERATION.

$100 = 1st cost assumed.
1 = 1% charges.
———
$101 = cost of supplies.
2½ = % commission.
———
$2.525 = 2½% commission.

Aggregate cost of $100 supplies.
$100.000 = cost assumed.
1.000 = charges.
2.525 = commission.
2.222 = insurance.
———
$105.747 = cost to buy and ship $100 supplies.

RECAPITULATION.
$1891.30 = 1st cost of supplies.
18.91 = 1% charges.
47.76 = 2½% commission.
42.03 = 2% insurance.
———
$2000.00 = sum received.

$101 = cost of supplies.
10 = % of cost of supplies
———
$10.10 = 10% of cost.
101. = cost of supplies
added.
———
$111.10 = cost + 10%.
2 = % insurance.
———
$2.2220 = 2% insurance.

$
100 = cost assumed.
105.747 | 2000.000
——— [supplies.
$1891.30 = invested in
18.91 = 1% charges
———
$1910.21 = cost and
charges of
supplies.
2½% = comm'on
———
$47.755¼ commission.

Operation to Find Insurance.
$1910.21 = cost and charges of
supplies,
191.02 = 10%.
———
$2101.23 = sum insured.
2 = % insurance.
———
$12.0246 = insurance.

95.   If a man counts $1 every second and counts 12 hours every day without stopping, how many days will it take him to count $1000000, and how many to count a billion dollars ?

Ans. $23\frac{4}{27}$ days to count a million.
$23148\frac{4}{27}$ days$=63$ years, $137\frac{43}{108}$ days to count a billion.

NOTE.—$365\frac{1}{4}$ days were considered a year, in reducing the days to years.

96.   How many years, of $365\frac{1}{4}$ days each, will it require to count a trillion, by counting 12 hours every day and counting $2 every second?

Ans. 31688 years, $32\frac{2}{27}$ ds.

97. $\frac{1}{5}$ of A's money and $\frac{1}{6}$ of B's money is $14, and A's money is $4 more than B's money.   How much has each ?   Ans. A. $40; B. $36.

OPERATION.

$\frac{1}{5}$ of 4=80¢, part of A's $4 excess in the $14;
$14.00 — 80¢=$13.20; then $\frac{1}{5}+\frac{1}{6}=\frac{11}{30}=$13.20.

$			$	
11	13.20		11	13.20
5	30		6	30
	1 =A's part.			1 =B's part.

$7.20+.80 A's excess= $8=$\frac{1}{5}$ of A's money, which is, therefore, $40.

$6=$\frac{1}{6}$ of B's money, which is therefore, $36.

98.   What is the interest on £50 12s. 6d. for 93 days at 6 per cent, allowing 365 days to the year ?

Ans. 15s. 6d.

OPERATION.

12s. 6d. = £0.625
12
240) 150 (.625

	£
365	.50.625
	6
	93
	15s. 6d., Ans.

NOTE.—Shillings and pence may be reduced to the decimal of a pound by the following short process:

Multiply the shillings by .05 and the pence by .004⅙ and add the results together. Thus:

12s.	6d.		12s.	6d.
.05	.004⅙	or thus	5 (hundredths)	4⅙ (thous'dths)

.60 + .025=£.625.   60 (hundred's)+ 25 (thous'dths)
                                          =£.625.

The above multipliers are produced thus:

Since 20s.=£1., 1s.=£$\frac{1}{20}$, or .05 of a £.
Since 240d.=£1, 1d.=£$\frac{1}{240}$, or .004⅙ of a £.

In like manner the decimal of a farthing would be £$\frac{1}{960}$, or .001$\frac{1}{24}$ of a £.

99.   What is 4 per cent of £5860 16s. 7d ?
                    Ans.   £234 8s. 8d.

FIRST OPERATION.	SECOND OPERATION.
£58.60 .16s. .07d.	£58.60 .16s. .07d.
4%	4%
£234.40 .64 .28	£234.43 6 4
20	20
8s. .64	8s. 66
12	12
7d. .96	7d. 96.

100.   What is 3½ per cent of £78 4s. 10d. ?
                    Ans.   £2 14s. 9d.

101.   What is ¼ per cent of £200 19s. 3d. ?
                    Ans.   10s. 0d. 2f.

64

102. If an article had cost 20% less the gain would have been 30% more. What was the % gain? Ans. 20%.

Note.—This and the following problem are purely Algebraic and can only be worked, arithmetically, by processes deduced or abbreviated from Algebraic equations. Hence, in the operations given, no arithmetical reasoning is presented.

OPERATION.

\$100 cost assumed.
20% less.

————

\$20.00 = 20% less.
100.00 = cost as above.

————

\$80.00 = decreased cost.
30% more.

————

\$24.00 = 30% more.
20.00 = 20% less.

————

\$ 4 = gain on \$20 less.

$$20 \,\Big|\, \begin{array}{l} 4 \\ \overline{100} \end{array}$$

\$20 gain = 20% gain Ans.

103. If the cost had been 12% more the gain would have been 15% less. What was the % gain? Ans. 40%.

OPERATION.

100 cost assumed.
12% more.

————

\$12.00 = 12% more.
100.00 = cost as above.

————

\$112.00 = increased cost.
15% less.

————

\$16.80 = 15% less.
12.00 = 12% more.

————

\$ 4.80 = gain on \$12 more.

$$12.00 \,\Big|\, \begin{array}{l} 4.80 \\ \overline{100} \end{array}$$

40% gain, Ans.

104. What is the value of $30 - (2.4 - 8.37 +$
$21.625) + 3.46 \times .12$? Ans. 14.7602.

OPERATION.

$2.4 - 8.37 = -5.97$; $-5.97 + 21.625 = 15.655$,
the result of the parenthesis; $3.46 \times .12 = .4152$.
Then $30 - 15.655 = 14.345$; $14.345 + .4152 =$
14.7602, Ans.

OR,

$2.4 + 21.625 = 24.025$; $24.025 - 8.37 = 15.655$,
the result of the parenthesis. $30 - 15.655 = 14.345$;
$3.46 \times .12 = .4152$; $14.345 + .4152 = 14.7602$, Ans.

NOTE.—The above and the following problem are Algebraic
equations, and the Algebraic principles involved in their
solution are as follows:

In all Algebraic and arithmetical equations or operations
indicated by the $+$, $-$, $\times$, or $\div$ signs, and in simplifying
fractions, the application of the *signs of operation* should be
well understood, as was explained on page 220. But to
understand thoroughly, the elucidation of the preceding prob-
lem, as well as that of the following and all others of a simi-
lar character, we deem a repetition of the principles neces-
sary and state them as follows:

The signs $+$ and $-$ affect only the number or expression
which immediately follows either of them.

The signs $\times$ and $\div$ indicate an operation to be performed
with the numbers or expressions between which either may
be, and this operation must be performed before that of any
$+$ or $-$ which may immediately precede or follow.

The parenthesis, ( ), or the vinculum, ———, indicate
that the quantities which they include must be simplified
into one expression before they can be connected to any
other quantity.

The operation of combining and simplifying always begins
with the ( ) or ——— if any, and then with the operations
indicated by the $\times$ and $\div$ signs. The operations of $+$ and
$-$ are then performed and are generally considered from left
to right.

It will be observed that the result of the parenthesis in
the above equation was subtracted from the quantity before
it, as indicated by the minus sign between them; this was
done because the result of the parenthesis was a plus quan-
tity. Had it been a minus quantity, i. e. had the result been

a quantity preceded by a minus sign, it would then have been added to the quantity before the parenthesis, for the reason that the minus sign before the parenthesis would cancel the minus sign before the result of the parenthesis and thereby produce a plus quantity. Should a plus sign precede the parenthesis, the operation to be performed with the result of the parenthesis would be in accordance with the sign before the resulting quantity and independent of the + sign before the ( ).

105. $20 - 30 + 2 \times 6 =$ what? Ans. 2.

106. Show that $\frac{11}{13}$ is greater than $\frac{11-3}{13-3}$ and less than $\frac{11+3}{13+3}$

107. Express decimally the value of $\frac{(\frac{3}{5}+\frac{3}{4})\div 2}{\frac{1}{4}\text{of}(\frac{1}{4}-\frac{1}{16})}$ Ans. 2.4293+.

108. Simplify $\left(1 + \frac{1+\frac{1}{4}}{5}\right) \div \left(1 + \frac{5}{1+\frac{1}{5}}\right)$ Ans. $\frac{9}{25}$.

109. A man, his wife, and two sons desire to cross a river. They have a boat that will carry but 100 pounds. The man weighs 100 pounds, the woman weighs 100 pounds and each boy weighs 50 pounds. How can they all cross the river in the boat?

OPERATION.

1. Let the 2 boys go over, and 1 boy return with the boat.

2. Let the man go over, and the boy return with the boat.

3. Let the 2 boys go over, and 1 boy return with the boat.

4. Let the woman go over, and the boy return with the boat.

5. Let the 2 boys go over.

110. What can you prefix to 1X to make it 6. Ans. S.

111. Three jealous husbands with their wives are to pass over a river in a boat which can carry but two at a time. How can these six persons row themselves over two at a time, so that none of the wives may be found in the company of one or two men unless her husband be present ?

112. How can 5 be taken from 5 and leave a remainder of 5 ?

113. How can 45 be taken from 45 and leave a remainder of 45 ?

<div align="center">OPERATION.</div>

$$9+8+7+6+5+4+3+2+1=45$$
$$1+2+3+4+5+6+7+8+9=45$$

$$8+6+4+1+9+7+5+3+2=45 \text{ Remainder.}$$

114. How long would it take a steamboat with the speed of fifteen miles an hour to travel seventy-five miles, going *half the distance* with and *half the distance* against the tide which flows at the rate of five miles an hour ?   Ans. 5 hrs., 37 min., 30 sec.

115. How can you arrange the 9 significant figures so that when added the sum will be 100 ?

<div align="right">Ans. 75\tfrac{2}{4}.
4
8\tfrac{4}{3}.
9
———
100.</div>

116. Prove that a pound and a half a cheese weighs more than 2 pounds of butter.

117. How much heavier is a pound of iron than a pound of gold ?

<div align="right">Ans. 1240   grains Av.
1020\tfrac{11}{16} grains Troy.</div>

118. Henry had 10 eggs, James had 25, and John had 30. Each boy made three sales and sold his eggs at the same price per egg and each received the same amount of money. How did each sell his eggs ?

OPERATION.

Henry sold as follows :	James sold as follows :	John sold as follows :
8 eggs @ 10¢=.80	2 eggs @ 10¢=.20	1 egg @ 10¢=.10
1 " @ 3¢=.03	19 " @ 3¢=.57	17 " @ 3¢=.51
1 " @ 2¢=.02	4 " @ 2¢=.08	12 " @ 2¢=.24
10 eggs sold for 85¢	25 eggs sold for 85¢	30 eggs sold for 85¢

119. A lady's grocery bill was $3.24. She bought salt, sugar, and coffee, buying 4 times as much sugar as salt and 8 times as much coffee as salt, and paid the same number of cents per pound for each commodity as she bought number of pounds of each. How many pounds of each did she buy ?

OPERATION.

1 lb. salt @ 1¢= 1¢, proportionate cost of salt.
4 " sugar @ 4¢=16¢, " " sugar.
8 " coffee @ 8¢=64¢, " " coffee.

81¢, proportionate cost of all =$3.24.

$$81 \,\Big|\, \frac{\$3.24}{1}$$

4¢, cost of salt.

$$81 \,\Big|\, \frac{\$3.24}{16}$$

64¢, cost of sugar.

$\sqrt{4}=2$ lbs. salt @ 2¢. $\sqrt{64}=8$ lbs. sugar @ 8¢.

$$81 \,\Big|\, \frac{\$3.24}{64}$$

$2.56, cost of coffee.

$\sqrt{2.56}=16$ lbs. coffee @ 16¢.

120. A student of mathematical logic proves that a cat has 3 tails by the following process of reasoning :

    1. No cat has two tails ;

    2. One cat has 1 tail more than no cat ;

    3. Now since no cat has 2 tails and since 1 cat has 1 tail more than no cat, therefore 1 cat has 3 tails.

Where is the error in the logic ?

121. A young hoodlum, a modern evolvement of the human race, stole a basket of peaches and divided them among 3 brother hoodlums and himself as follows :   To the first, he gave $\frac{1}{4}$ of the whole number and $\frac{1}{4}$ of a peach more ; to the second, he gave $\frac{1}{3}$ of what remained and $\frac{1}{3}$ of a peach more ; to the third, he gave $\frac{1}{2}$ of what remained and $\frac{1}{2}$ of a peach more.   The stealer retained what was then left, for himself, which was $\frac{1}{2}$ the number he gave to the first hoodlum.   What was the number of peaches stolen, and how many did each hoodlum receive ?   Ans. 7 peaches were stolen.

          1st hoodlum received 2.

          2d   "   "   2.

          3d   "   "   2.

          The stealer had   1.

SOLUTION.

In all problems of this kind the following principle prevails : That when the fractional parts of the successive portions are consecutively increasing, each of the parts or shares of the several persons are equal, except the last one, which from the nature of things must be one less than the average.   Hence in the given problem, the fact stated that the last one's share is half of the first (and each of the others also) and from the above principle that it must be one less, it must necessarily be ONE, and since each of the others is one greater, they must be 2 each, and since there are three persons having equal portions, there will be 6 + the 1 for the last, making 7 in all.   Hence, to solve problems of this kind :

Add 1 to the last number, (ONE), and multiply by the number of persons, and then subtract 1 from the product.

Contractions in Numbers.

The following Contractions are printed from the plates of " Soulé's Contractions in Numbers."

THE VICENARY SYSTEM OF ADDITION.

55. All expert calculators admit that the most rapid system of addition is by grouping two, three, or more numbers and mentally naming the result of the group.

But such a system requires a constant strain on the mind to retain the large and varying results and to make the several additions of the different groups, that with many it is wholly impracticable. To avoid this mental strain, several different methods of operation have been suggested by different calculators, but all have had some demerit that has rendered them more or less objectionable. With a view therefore to obviate the objections and difficulties of the usual methods and render the work simple, comprehensible, and practical, we present a new system, which from the character of the work we name the VICENARY SYSTEM. The leading peculiarity of this system consists in recording on the different joints of the fingers and thumb of the left hand, the 20's as soon as they are produced, and thus freeing the mind from all effort to retain large results, or of making the rather difficult additions resulting from the several groupings.

THE VICENARY SYSTEM ELUCIDATED.

56. We have be-
fore stated that the
leading characteristic
or peculiarity of the
Vicenary System of
addition consists in
recording the 20's as
soon as produced, on the joints of the fingers and thumb of
the left hand; and in order to render this part of the ope-
ration clear, we present a cut of a hand with the different joints
of the fingers and thumb numbered so as to represent the
value that we record on them in adding. The manner of
making the record of the 20's on these joints is by simply
placing the end of the thumb on the finger, or the end of
the first finger on the thumb, at such joint as represents the
value to be recorded. The manner of making the record,
and the value of the several joints, should be well under-
stood before the operation of adding is commenced. The
addition tables, on preceding pages, should also be well
understood.

In making the addition we first mentally group enough
figures to produce a result not less than 10, and then to
this result add the result of enough other figures to pro-
duce a second result not less than 20; then having 20 or an
excess of 20, we record the 20 on the hand, by placing the
end of the thumb on the first joint of the first finger which
represents the value of 20, then we add the excess, if any,
to the next group, and continue to produce and record the
20's until the column is added. *If it is desired, enough
figures to produce a result in excess of* 20 *may be grouped
at once, and the record made and the excess carried on as
above explained.*

57. To better elucidate the vicenary system we present
the following problems and explanations:

1. Add the following numbers:

82
97
76
89
64
38
36
92
74
53
45
27
84
65
36
87

1045
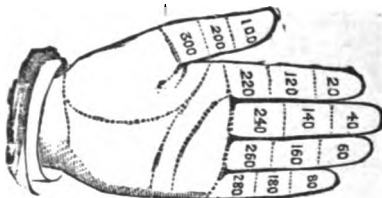
Explanation Commencing at the bottom of the units column we first mentally see 13, (7, 6), to which we add 9 (5 4), which make 22 ; we then record the 20 by placing the end of the thumb on the first joint of the first finger. Next, we see 14 (7, 5, and 2 in excess of 20) to which we add 9 (3, 4, 2), and obtain 23 ; then we record the 20 by passing the end of the thumb to the end of the second finger. We then see 17 (6, 8, and 3 in excess of 20), to which we add 4 and obtain 21, then we record the 20 by passing the end of the thumb to the end of the third finger. Next we see 16 (9, 6, and 1 in excess of 20), to which we add 9, (7, 2), and obtain 25, the 20 of which we record by passing the end of the thumb to the first joint of the fourth finger, and the 5 we write in the unit's place of the answer. We now have the first column added and by inspecting the position of the thumb and fingers of the left hand we find 80 recorded, which with the 5, the last excess of 20, make 85, the sum of the column. Then to add the second, or tens' column, we first mentally see 16 (8 and the 8 tens from the units' column) plus 9 (3, 6) =25 ; we then record the 20 on the first joint of the first finger, as above directed. Next we see 15 (8, 2, and the 5 in excess of 20), plus 9 (4, 5) = 24, then recording the 20 by moving the end of the thumb to the first joint of the second finger, we next see 20 (7, 9 and the 4 in excess), which we record by passing the end of the thumb to the first joint of the third finger. Then we see 12 (3, 3, 6), plus 8 = 20, this we record by placing the end of the thumb on the first joint of the fourth finger. Then we see 16 (7, 9), plus 8 = 24 ; the 20 we record by placing the end of the first finger on the first joint of the thumb, and as the addition of the column is now completed we inspect the position of the fingers and thumb of the recording hand and from the position last above named, find the record to be 100, which with the 4, last excess make 104, the result of the column. This 104 we write in the total result, and obtain 1045 as the sum of all the numbers.

2. Add the following numbers:

8 ⎫
5 ⎬ 16
2 ⎭
9 ⎫ — 21
8 ⎭
7   7 ⎫ 24
6 ⎰ 17 ⎭
4 ⎭
9 ⎫ 11 ⎫ 27
2 ⎬   ⎬
7 ⎬ 16 ⎭
8 ⎭
3 ⎫ 11 ⎫ 21
8 ⎬   ⎬
4 ⎬ 10 ⎭
5 ⎭
7 ⎫ — 21
6 ⎭
9 ⎫ 10 ⎫ 28
1 ⎬   ⎬
7 ⎬ 18 ⎭
8 ⎭
6 ⎫ — 23
9 ⎭
8 ⎫ 13 ⎫ 28
5 ⎬   ⎬
4 ⎬ 15 ⎭
9 ⎭
7   7 ⎫ 22
8 ⎰ 15 ⎭
2 ⎭
5 ⎫ 14 ⎫ 25
9 ⎬   ⎬
7 ⎬ 11 ⎭
4 ⎭

216

Explanation—Having made the explanation of the preceding example very full, we will therefore, in this, omit some of the details. To make the operation clearer. we have linked together with brackets the numbers used to produce the various intermediate results. Commencing at the bottom of the column we proceed thus, 11, 14, 25; then recording the 20 and adding the 5 excess to the next group, we have 15, 7, 22; then, recording the 20 and adding the 2 excess, we have 15, 13, 28; then, recording the 20 and adding the 8 excess to the next group, we have 23; then, recording the 20 and adding the 3 excess to the next group, we have 18, 10, 28; then, recording the 20 and adding the 8 excess to the next group, we have 21; then, recording the 20 and adding the 1 excess to the next group, we have 10, 11, 21; then, recording the 20 and adding the 1 excess to the next group, we have 16, 11, 27; then recording the 20 and adding the 7 excess to the next group, we have 17, 7, 24; then, recording the 20 and adding the 4 excess to the next group, we have 21; then recording the 20 and adding the 1 excess to the next group, we have 16. This completes the addition and by inspecting the recording hand, we find the end of the first finger on the second joint of the thumb, which shows that 200 have been recorded, which with the last obtained 16 make 216. the total sum of the column.

Add, separately, the following columns of numbers:

	(3)			(4)			(5)		
1 ⎫	13		1 ⎫	6 ⎫		4 ⎫		11	
8 ⎭			8 ⎪	⎬	13	8 ⎪			
3 ⎫	7 ⎫	24	3 ⎬	⎭		8 ⎬			
4 ⎭	⎬		4 ⎭			8 ⎭			
5 ⎫	17 ⎭		5 ⎫	7 ⎭		3 ⎫	9 ⎫	3	
9 ⎭			9 ⎭			6 ⎭	⎬		
7 ⎫	23		7 ⎫		3	2 ⎫	4 ⎭		
8 ⎭			8 ⎭			7 ⎭			
3 ⎫	11 ⎫	28	3 ⎫	1 ⎫		9 ⎫		5	
6 ⎬	⎬		6 ⎬	⎬	8	9 ⎪			
2 ⎭	⎬		2 ⎭	⎭		9 ⎬			
9 ⎫	17 ⎭		9 ⎫	7 ⎭		9 ⎪			
8 ⎭			8 ⎭			9 ⎭			
73			**73**			**91**			

Explanation.— In the 3d example we added in the same man-
ner as in the preceding example.

In the 4th and 5th examples, in stating the results of the
grouped figures, and also in the result in excess of 20, we have
set only the excess figure, or figures, which in practice are the
only figures that should be mentally noticed. In the 4th exam-
ple we grouped differently from the bracketing of the 3d example
in order to show that no particular form of grouping is necessary,
and that the more figures grouped, the more rapidly the addition is
made.

In the 5th example, we find *five* figures alike, and hence by
multiplying we instantly see 45; we then record the 40, and
continue in the usual manner, until we come to three figures
that are alike; we then multiply them and to the product add
the 3 excess, and 4, the last figure in the column, and produce
11 in excess of 20.

CONTRACTING METHODS.

58.   To be observed when adding by the Vicenary System

1. In *mentally* naming the results of the various groups, the
unit figure only should be named; thus, in the addition of the 4th
example instead of mentally naming or thinking 17, 11, &c.,
name or think only 7, 1, &c. Remember, in practice, these

results are never set on paper. It is here done to elucidate the work.

2. In adding the two results of the grouped figures to produce an excess of 20, add only the unit figures, and only mentally name the excess of 20.

These points were elucidated in adding the 4th and 5th examples.

3. Whenever the same figure occurs connectedly several times, the sum should be obtained by multiplying instead of adding. This principle was elucidated in example 5.

4. Where figures occur in regular order and the number is odd, thus: 5, 6, 7, or 9, 8, 7, 6, 5, then as many times the middle as there are figures in regular order, will be their sum.

5. When the number of figures in regular order is even, thus: 2, 3, 4, 5, or 7, 6, 5, 4, 3, 2, then one-half as many times the sum of the extremes as there are figures in regular order, will be their sum.

PROOF OF ADDITION.

59. The best proof of the correctness of addition is for the calculator to be proficient in his work, and then re-add the columns in the reverse direction. Casting out the 9's, as is sometimes done, is not positive proof of correctness, and hence many accountants in verifying their calculations, prefer to repeat, or go over in a reversed direction, the first operation.

(For full information in regard to this subject, see pages 68 to 71 of Soule's Analytic and Philosophic Commercial and Exchange Calculator.

The Accountant should always add in pencil, or on a separate piece of paper, and then verify his work.

ADDITION OF SEVERAL COLUMNS AT ONE OPERATION.

60. In many cases the addition of several columns at one operation will greatly expedite the accountant's work; but in long columns of solid figures, the addition by grouping, as above explained, is by far the most rapid and practical.

The following will illustrate the primary work of adding several columns at once:

1. Add:

26	*Explanation.* Commence at the bottom and
94	proceed by adding to the lower number 47, first
62	the tens of the next number above, then the units,
71	and in like manner continue till all are added,
18	thus. $47 + 10 = 57 + 8 = 65 + 70 = 135 + 1 = 136 + 60$
47	$= 196 + 2 = 198 + 90 = 288 + 4 = 292 + 20 = 312 + 6 =$
–––	318, the sum of the 2 columns.
318	

By naming only results, which in practice should only be named, we have 47, 57, 65, 135, 136, 196, 198, 288, 292, 312, 318.

2. Add:

Explanation. Commencing at the bottom we have.

4210	**1** 3691 **2** 5691 **3** 5791 **4** 5796
1587	
2105	**5** 6796 **6** 7296 **7** 7376 **8** 7383
3691	
––––	**9** 11383 **10** 11583 **11** 11593, the answer.
11593	

These operations show the basis of adding several columns simultaneously, and though the method is too laborious to be of material advantage in long columns, it should be practiced until the mind can easily and readily retain large and varying numerical results.

RAPID ADDITION.

In practice we very much shorten the work by combining and adding whole amounts at once. Thus, in the 2d problem, by combining, as we would naturally do in practice, we would have in the two lower numbers 5796, then adding the hundreds and thousands figures (15) of the third number we have 7296, then add the units and tens figures of the third number and we have 7383, to which add the fourth number at once, and we have 11593 the correct result.

To show clearly how to contract, and to practicalize the above system, we present the following combinations, which with many other similar numbers, should be carefully studied.

ADD THE FOLLOWING NUMBERS.

(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
23	52	84	68	98	88	97	96
16	36	95	72	59	46	89	99
39	88	179	140	157	134	186	195

Explanation. In the 3d 4th and 5th problems, the sum of the units figures being less than 10 the whole sum is instantly seen.

In the 6, 7, 8, 9 and 10th problems, we instantly see that the sum of the units figures is 10, or an excess of 10, and hence we know that the sum of the tens is to be increased by 1, and without thinking what the excess is, we write the result of the tens figures, which according to the combinations shown in table 2, we know without adding, and while writing this result we give an instantaneous thought to the exact sum of the units figures.

ADD THE FOLLOWING NUMBERS.

(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
74	86	88	91	126	342	485	1627
98	95	97	68	75	92	304	211
172	181	185	159	201	434	789	1838

Explanation. In these problems from 11 to 18 inclusive, we show how to facilitate and expedite the work when one number approximates 100. Thus in problem 11, we first mentally add 100 to the 74 which makes 174, and then, because the 100 added is 2 more than the 98, which should have been added, we mentally deduct 2 from 174, and write 172, the correct sum. In problems 12, 13, 14, 15 and 16, we mentally add to each upper number, 100, and then from the several sums thus produced we mentally deduct, respectively 5, 3, 9, 25 and 8. In problems 17 and 18 we first mentally add, respectively 300 and 200, and then to the respective sums thus obtained, we mentally add 4 and 11.

Combinations similar to the foregoing, from problem 1 to 18, must be practiced until the student can easily and rapidly perform the work; otherwise, proficiency in addition and multiplication cannot be attained.

ADDITION BY ONE OF THE PROPERTIES OF 9.

63.   1.   What is the sum of 5 lines of numbers the first being 467 ?   _   Ans. 2465

OPERATION.

467	1st. line
382	2d. "
815	3d. "
617	difference between 2d line and 9.
184	" " 3d line and 9.

2465 Ans.

Explanation. In all problems of this character with any number of odd lines, the answer may be produced as soon as the *first* line of numbers is given, by writing the *first* line minus the number of 9's to be produced. and then prefixing the number of 9's to the first line. Then, when the other lines are set, to insure this result, every two lines must make 9 or, in other words, one-half of the other lines must be such numbers as will, when added to the remaining half alternately, produce 9 in each column of the two lines added.

In this problem we set for the answer 467—2, the number of 9's = 465 to which we prefix the 2 and produce the answer 2465.   It should be borne in mind that 5 lines give *two* 9's, 7 lines *three* 9's, 9 lines *four* 9's, etc.

2.   What is the sum of 9 rows of numbers, the first being 10644 ?   Ans. 410640.

OPERATION.

10644	1st. line
23456	2d. "
13482	3d. "
96780	4th. "
25621	5th. "
76543	difference between 2d line and 9.
86517	" " 3d " and 9.
03219	" " 4th " and 9.
74378	" " 5th " and 9.

What is the sum of 7 rows of numbers the first being 86021 ?   Ans. 386018.

ADD THE FOLLOWING NUMBERS BY THE VICENARY SYSTEM

(25)	(26)	(27)	(28)
24	582	7563	1963
63	369	28325	3846
75	9473	461523	1215
82	208	6393	1872
29	960	21783	7312
76	2720	151672	910
84	843	3243	2311
57	1462	72	617
38	727	311	99
92	5106	3263	313
14	794	70015	4632
68	327	8063	516
74	8372	1000	3313
99	6481	56792	88
76	218	107331	200
57	1000	2441	3915
81	478	3457	617
92	66	67423	3129
25	4419	87635	28
63	1200	297521	319
48	6223	80000	4615
78	97	6751	313
97	328	324	3239
———	1462	233	272
	———	49	99

ADDING HORIZONTALLY.

In many lines of business, to economize time, Clerks and Accountants find it necessary to add numbers that are written horizontally; thus, add:

1. 568, 409, 328, 976, 4163, 87 and 615 =
2. 97, 4816, 518, 1273, 8964, 56 and 704 =
3. 124, 6070, 49, 1080, 5673, 483 and 680 =

SUBTRACTION BY THE COMPLEMENT OF 10.

In many lines of business, especially in the banking and in the operation of closing accounts, it is often convenient, and a saving of time, to be able to write the difference between the sums of several unadded debit and credit numbers, without first finding the amount of the respective debit and credit numbers of which the difference or balance is desired.

1. What is the difference or balance of the following account?

DR.	CR.
8472	3876
2168	4589
4607	764
596	1483
1659	——
——	6790 Balance.

Explanation.—Here we add the units figures of the smaller, or the credit side, and obtain 22, this we subtract from the next higher number of tens and obtain 8, which we add to the units figures of the debit side and produce 40. The 0 we write as the first figure of the difference. Here we observe that there were *three* tens in the credit side and *four* in the debit, and hence we have 1 ten to *add* to the tens of the debit or to *subtract* from the tens of the credit. We will subtract it from the credit. We now add the *second* column and obtain, minus the 1 ten, 28; this taken from 30, leaves 2, which added to the *second* column of the debit, gives 29; the 9 we write as the second figure of the balance. Here we observe that there is *one* more ten in the credit side than in the debit. This 1 ten we add to the *third* column of the credit and obtain 25, this taken from 30 leaves 5, which we add to the *third* column of the debit and produce 27; the 7 is written as the third figure of the balance. We now observe that there is an excess of *one* ten in the credit, which we add to the *fourth* column of credit and obtain 9, this taken from 10 leaves 1, which is added to the *fourth* column of debit and produces 16; the 6 is written as the fourth figure of the difference; and as the tens are now equal in the debit and credit numbers the difference or balance is complete.

2. What is the balance of the following account?

DR.	CR.
948	4351
793	
427	
——	2183 Balance.

OPERATION. *First.*—7 + 3 + 8 = 18; 20 — 18 = 2; 2+1 = 3, the first figure of the balance — 2 tens excess on the debit.

Second.—2+2+9+4=17; 20—17 =3. 3+5=8, the second figure of the balance—2 tens excess on the debit.

Third.—2+4+7+9=22. 30—22=8; 8+3=11. This gives 1 as the third figure of the balance, and as there were 3 tens in the debit and 1 ten in the credit there are 2 tens excess in the debit, which subtracted from the fourth figure of the credit gives 2 as the *fourth* and final figure of the balance.

3. A depositor has a credit balance of $7206. He draws the following checks: $527, $1318, $98, and $1642. What is the credit balance? Ans. $3621.

DIRECTIONS. *First.*—Add, horizontally, the *unit* figures of the checks=25; 30—25=5. Then 5+6=11. (2 tens excess on the checks).

Second.—Add, horizontally the *tens'* figures of the checks and the 2 tens excess,=18. 20—18=2. Then 2+0=2, the second figure of the balance. (2 tens excess in the check numbers). In the same manner proceed with the remaining orders.

4. What is the debit balance of the following account?

DR.	CR.
1375	356
8692	
——	9711 Balance.

DIRECTIONS. *First.*—10—6=4; 4+2+5=11.

Second.—10—5=5; 5+9+7=21.

Third.—(3—1=2) 10—2=8; 8+3+6=17.

Fourth.—8+1=9.

5. Find the difference between the following numbers by the same process: 38071 and 934068.

OPERATION.—9 & 8=17; 3 & 6=9; 9 & 0=9; 1 & 4=5; 6 & 3=9; 9 — 1=8.

6. What is the difference between the following accounts?

DR.	CR.	DR.	CR.	DR.	CR.
$ 482.45	$345.36	571	6416	4123.14	5421.45
1243.91	92.85	899			342.37
562.87		223			1691.52

SIMULTANEOUS, OR CROSS MULTIPLICATION.

71.   This system of multiplication is of inestimable value.   It is, so to speak, the accountant's and calculator's magic wand, by which he may produce results in multiplication operations with almost lightning rapidity.

No man can be proficient in the handling of numbers without a thorough knowledge of this system of work in addition to the several other contracted methods.

The operations of this system are based upon the following principles:

1.   Units × units produce units.
2.   Tens × units   "   tens. ⎱
3.   Units × tens   "   tens. ⎰
4.   Hundreds × units produce hundreds. ⎫
5.   Tens × tens   ··   hundreds. ⎬
6.   Units × hundreds   "   hundreds. ⎭
7.   Thousands × units   "   thousands. ⎫
8.   Hundreds × tens   "   thousands. ⎬
9.   Tens × hundreds   "   thousands. ⎭
10.   Units × thousands   "   thousands.
11.   Ten thousands × units "   ten thousands. ⎫
12.   Thousands × tens   "   ten thousands. ⎬
13.   Hundreds × hundreds "   ten thousands. ⎬
14.   Tens × thousands   "   ten thousands. ⎭
15.   Units × ten thousands "   ten thousands.
16.   Hundred thousands × units produce hundred thousands. ⎫
17.   Ten thousands × tens   "   hundred thousands. ⎬
18.   Thousands × hundreds   "   hundred thousands. ⎬
19.   Hundreds × thousands   "   hundred thousands. ⎬
20.   Tens × ten thousands   "   hundred thousands. ⎭
21.   Units × hundred thousands "   hundred thousands
                                 and so on for higher numbers.

PROBLEMS.

1.   Multiply 54 by 37.

OPERATION.
54
37
———
1998

Explanation.—We first multiply together in the ordinary manner, the units figures, thus, 7 times 4=28, and write the 8 in the first place of the product, and carry 2.   We next multiply the tens figure of the multiplicand by the units figure of the multiplier, thus, 7 times 5 (+2 to carry)=37, which we retain in the mind and add thereto

the product of the units figure of the multiplicand by the tens figure of the multiplier, thus, 3 times 4=12,+37=49; we write the 9 in the tens place of the product and retain the 4 hundreds in the mind to be added to the column of hundreds. The product of the multiplicand by the units figure of the multiplier, and also of the units figure of the multiplicand by the tens figure of the multiplier is now produced in the two figures (98) of the final product, and hence we have no further use for the units figure of either factor. We have therefore but to multiply the tens figures of the two factors, thus, 3 times 5 are 15, plus the 4 retained in the mind, makes 19, which we write to the left of the two product figures first obtained, and complete the full product.

To elucidate the operation by figures only, we present the following:

OPERATION.	*Explanation.*	Carrying figures.	Product figures.
54			
37	$7\times4=$	2	8 units.
——	$\overline{7\times5}+2+\overline{3\times4}=$	4	9 tens.
1998	$\overline{3\times5}+4=$	1	9 hund's.
4 2			

Multiply the following numbers:

(2)	(3)	(4)	(5)	(6)	(7)
35	62	87	76	93	89
46	23	42	58	64	97
—	—	—	—	—	—

8. Multiply 62549 by 53.

OPERATION.	*Explanation.*		
62549	$3\times9=2$	7
53	$\overline{3\times4}+2+\overline{5\times9}=5$	9
———	$\overline{3\times5}+5+\overline{5\times4}=4$	0
3315097	$\overline{3\times2}+4+\overline{5\times5}=3$	5
33452	$\overline{3\times6}+3+\overline{5\times2}=3$	1
	$\overline{5\times6}+3\quad\ =3$	3

Multiply the following numbers:

(9.)	(10.)	(11.)	(12.)	(13.)
3243	14907	897	52061	2981453
27	64	48	83	39
—	—	—	—	—

Multiply 7524 by 346

OPERATION.
7524
346
——————
2603304

Explanation.—In the elucidation of this problem, the principles used being the same as in the above examples, we will therefore condense the explanation of the work.

6 times 4=24, set the 4 and carry the 2.

6 times 2=12+2=14+4 times 4=30, write the 0 and carry the 3.

6 times 5=30+3=33+4 times 2=41+3 times 4=53, write the 3 and carry the 5.

6 times 7=42+5=47+4 times 5=67+3 times 2=73, write the 3 and carry the 7.

Having now as many figures in the partial and final product as we have figures in the multiplicand, we observe that the product of the multiplicand by the units figure of the multiplier is already produced in the final product, and hence we drop the units figure and commence with the tens figure of the multiplier; and as the product of the first three figures of the multiplicand has been taken by the tens figure of the multiplier, we commence with the fourth or last figure of the multiplicand. 4 times 7=28+7=35+3 times 5=50, set the 0 and carry the 5.

Now as the product of the multiplicand by the tens figure of the multiplier is produced in the final product, we drop the same and commence with the third or hundreds figure of the multiplier; and as the product of the first three figures of the multiplicand by the third or hundreds figure of the multiplier has already been produced, we commence with the fourth or thousands figure of the multiplicand.

3 times 7=21+5=26, which we set and complete the product.

14. Multiply 7524 by 346.

OPERATION.
7524
346
——————
2603304
57532

Explanation.

$$6 \times 4 = 2 \mid 4$$
$$\overline{6 \times 2} + 2 + \overline{4 \times 4} = 3 \mid 0$$
$$\overline{6 \times 5} + 3 + \overline{4 \times 2} \times \overline{3 + 4} = 5 \mid 3$$
$$\overline{6 \times 7} + 5 + \overline{4 \times 5} + \overline{3 \times 2} = 7 \mid 3$$
$$\overline{4 \times 7} + 7 + \overline{3 \times 5} = 5 \mid 0$$
$$\overline{3 \times 7} + 5 = 2 \mid 6$$

15. Multiply 74018 by 45602.

OPERATION.
<pre>
      78018
      45602
     ─────────
    3375368836
     56355401
</pre>

Explanation.

$$2 \times 8 = 1 \mid 6$$
$$\overline{2 \times 1} + 1 + \overline{0 \times 8} = \mid 3$$
$$\overline{2 \times 0} + \overline{0 \times 1} + \overline{6 \times 8} = 4 \mid 8$$
$$\overline{2 \times 4} + 4 + \overline{0 \times 0} + \overline{6 \times 1} + \overline{5 \times 8} = 5 \mid 8$$
$$\overline{2 \times 7} + 5 + \overline{0 \times 4} + \overline{6 \times 0} + \overline{5 \times 1} + \overline{4 \times 8} = 5 \mid 6$$
$$\overline{0 \times 7} + 5 + \overline{6 \times 4} + \overline{5 \times 0} + \overline{4 \times 1} = 3 \mid 3$$
$$\overline{6 \times 7} + 3 + \overline{5 \times 4} + \overline{4 \times 0} = 6 \mid 5$$
$$\overline{5 \times 7} + 6 + \overline{4 \times 4} = 5 \mid 7$$
$$\overline{4 \times 7} + 5 = 3 \mid 3$$

The multiplication sign ×, in the above elucidation, should be read "TIMES."

NOTE.—It will be observed that the general principles governing this system of multiplication have been strictly used in the operations and explanations. Thus, in the last problem, units are multiplied by units, (8 by 2); tens by units, (1 by 2); units by tens, (8 by 0); hundreds by units, (0 by 2); tens by tens, (1 by 0); units by hundreds, (8 by 6); thousands by units, (4 by 2); hundreds by tens, (0 by 0); tens by hundreds, (1 by 6); units by thousands, (8 by 5); ten thousands by units, (7 by 2); thousands by tens, (4 by 0); hundreds by hundreds, (0 by 6); tens by thousands, (1 by 5); units by ten thousands, (8 by 4); ten thousands by tens, (7 by 0); thousands by hundreds, (4 by 6); hundreds by thousands, (0 by 5); tens by ten thousands, (1 by 4); ten thousands by hundreds, (7 by 6); thousands by thousands, (4 by 5); hundreds by ten thousands, (0 by 4); ten thousands by thousands, (7 by 5); thousands by ten thousands, (4 by 4); ten thousands by ten thousands, (7 by 4).

NOTE.—By these lengthy elucidations, the work seems more tedious than it really is in practice. In performing the operation practically, we name results only and thereby very much lessen the labor.

Multiply the following numbers:

(16.)	(17.)	(18.)	(19.)	(20.)
2334	4354	270018	554422	9080706
136	809	30402	12345	1122033

(21.)	(22.)	(23.)	(24.)	(25.)
4120	5841	16190	2736	88997
54	325	2305	2107	11238

(26.)	(27.)	(28.)	(29.)	(30.)
894	2382	6709	4087	19205
143	751	284	2614	41003

(31.)	(32.)	(33.)	(34.)	(35.)
789	999	666	5566	5070608
567	888	777	4488	107024

(36.)   Multiply 222333444555 by 219543620324.

OPERATION.

$$222333444555$$
$$219543620324$$
_____

48811889336710025135820 **Ans.**

CONTRACTIONS.

73. Peculiar to problems when either factor is a convenient or aliquot part of 10, 100, or 1000. Thus, to multiply by

1⅑	multiply by	10	and	divide	the	product by	9
1⅐	"	10	"	"	"	8
1⅜	"	10	"	"	"	7
1⅔	"	10	"	"	"	6
2½	"	10	"	"	"	4
3⅓	"	10	"	"	"	3
6¼	"	100	"	"	"	16
8⅓	"	100	"	"	"	12
12½	"	100	"	"	"	8
14²	"	100	"	"	"	7
16⅔	"	100	"	"	"	6
18¾	"	300	"	"	"	16
25	"	100	"	"	"	4
31¼	"	500	"	"	"	16
33⅓	"	100	"	"	"	3
37½	"	300	"	"	"	8
62½	"	500	"	"	"	8
66⅔	"	100	and subtract ⅓ of product; or
	"	200	and divide by 3.
75	multiply by	100	and subtract ¼ of product.
83⅓	"	500	and divide by 6
87½	"	700	" 8, or
	"	100	and deduct ⅛ of product.
83⅓	"	1000	and divide by 12
112½	"	100	add ⅛ of product.
125	"	1000	and divide by 8
133⅓	"	100	add ⅓ of product.
166⅔	"	1000	and divide by 6
250	"	1000	" 4
333⅓	"	1000	" 3
375	"	3000	" 8
625	"	5000	" 8
833⅓	"	5000	" 6
875	"	700	" 8

(1.)   Multiply 289274 by 2½.

OPERATION.

4) 289274**0**

───────

723185   Ans.

Explanation. In answering the conditions of this simple problem, which is to repeat the multiplicand 2½ times, we first observe that 2½ is the ½ of 10, or that 4 times 2½ make 10, and hence to facilitate the work, we first multiply by 10, which is done by annexing one naught. This gives us a product of 2892740 which is four times too great; for the reason that 10 is 4 times 2½. To produce the correct result, therefore, we divide by 4, which gives us 723185.

(2)   Multiply 79862 by 12½.

OPERATION.

8) 79862**00**

───────

998275   Ans.

Explanation. The conditions of this problem require that the multiplicand be taken or repeated 12½ times; but in performing the operation, we first observe that 12½ is ⅛ of 100, and hence to save time and figures, we first take or repeat the multiplicand 100 times, as indicated by the small naughts, which gives us a product as many times too great as 100 is times, greater than 12½, which is 8 times. Therefore, we divide by 8 to produce the correct product.

(3.)   Multiply 937104 by 25.

OPERATION.

4) 937104**00**

───────

23427600   Ans.

Explanation We first multiply by 100, which is done by annexing two naughts to the multiplicand, and then divide by 4. The reason for this work is the same as that given in the preceding examples, which, to repeat, is that when you have multiplied by 100, your product is four times too large, for the reason that 100 is four times 25, and hence dividing by 4 produces the correct product.

In practice, the annexing of naughts may be made mentally only.

(4.)   Multiply 9426 by 66⅔.

OPERATION.

3) 9426**00**

314200

───────

628400   Ans.

Explanation. We first multiply by 100, and then deduct one-third of the product from itself; the remainder is the correct result or product. The reason for this is, that as 66⅔ is ⅔ of 100, when we have multiplied by 100, we have produced a product ½ too great, and hence by deducting ⅓ of the product, we have in the remainder the correct result.

Multiply the following numbers by the above contraction methods.

(5)	(6)	(7)	(8)	(9)	(10)
32	36	96	168	40	124
6¼	8⅓	16¾	62½	37½	125
200	300	1600	10500	1500	15500
(11)	(12)	(13)	(14)	(15)	(16)
816	640	264	640	800	696
375	625	875	75	18¾	833⅓

74. *To multiply by* 7½, *multiply by* 10, *and deduct* ¼ *of the product.*

75. *To multiply by* 17½, *multiply by* 20, *aud deduct* ⅛ *of the product.*

76. *To multiply by* 27½, *multiply by* 30, *and deduct* $\frac{1}{12}$ *of the product.*

The reason or philosophy of this is, 7½ being ¾ of 10, and 17½ being ⅞ of 20, and 27½ being 1½ of 30, multiplying by 10, 20 30, gives products respectively ¼ ⅛ and $\frac{1}{12}$ too large. Therefore, by deducting ¼ ⅛ and $\frac{1}{12}$ of the respective products we have the true results.

77. *To multiply by* 15, *multiply by* 10 *and add* ½ *of the product to itself, or multiply by* 30 *and divide by* 2.

78. *To multiply by* 35 *or* 45, *multiply respectively by* 70 *and* 90, *and divide the product by* 2, *or halve the multiplicand and double the multiplier.*

Multiply the following numbers.

(1)	(2)	(3)	(4)
247	214	586	637
15	35	45	55
7410	7490	26370	70070
3705			35035

79. *To multiply by any number between* 87 *and* 100, *first multiply by* 100 [*annex two naughts*], *then from this product deduct as many times the multiplicand as the multiplier is less than* 100.

(5)   Multiply 216 by 98.

OPERATION.
216⁰⁰
   432

21168 Ans.

Explanation. The conditions of this problem are that we repeat the multiplicand 98 times, but in performing the operation we first multiply by 100, which, for the reason that 100 is 2 more than 98, repeats the 216, 2 times more than was required; hence to produce the correct result, we deduct 2 times the multiplicand (432) from the product by 100, and have 21168, the correct product.

80.   *To multiply by any number between* 987 *and* 1000 *:*

(6)   Multiply 4268 by 997.

OPERATION.
4268⁰⁰⁰
   12804

4255196 Ans.

Explanation. In all problems of this kind, multiply by 1000 and then deduct as many times the multiplicand as the multiplier is less than 1000. In this problem, we first multiply the 4268 by 1000, which is done by annexing three naughts; this gives us a product 3 times 4268 too great, for the reason that 1000 is three more than 997. Hence, to produce the correct result, we deduct 3 times 4268, which is 12804, from the product by 1000, and in the remainder we have the correct product.

81.   *To multiply by any number between* 100 *and* 113 *:*

(7)   Multiply 5239 by 107.

OPERATION.
5239⁰⁰
   36673

560573 Ans.

Explanation. In all problems of this kind, first multiply by 100, then add to the product thus obtained, as many times the multiplicand as the multiplier is greater than 100.

82.   *To multiply by any number between* 1000 *and* 1013 *:*

(8)   Multiply 2871 by 1005 :

OPERATION.
2871⁰⁰⁰
   14355

2885355 Ans.

Explanation. In all problems of this kind, first multiply by 1000, then add to the product thus obtained, as many times the multiplicand as the multiplier is greater than 1000.

In the foregoing problems we have used small naughts, wherever we have annexed any, in order to show more clearly the operations of the work.

83. *To multiply together any two numbers of two or three figures each, when the hundreds and tens are alike and the sum of the units is ten:*

(9)   Multiply 86 by 84.

OPERATION.
86
84
———
7224

Explanation. In all problems of this kind, first multiply the unit figures and set the whole result, then add one to the multiplier of the tens, and multiply the other tens, or tens and hundreds, by it, and prefix the result to the product of the unit figure. In this problem we first multiply 4 times 6 are 24, which we set. Then add 1 to the 8 of the tens column, making it 9 ; we multiply 9 times 8 are 72, which is set to the left of the 24 and produces the correct product.

(10)   Multiply 132 by 138 :

OPERATION.
132
138
———
18216

Explanation. We first multiply, 8 times 2 are 16, which constitute the units and tens figures of the product, and are respectively written in the units and tens places.   We then add one to 13, making it 14, and multiply 14 times 13, are 182, which completes the product.   Or, after adding the 1 to the 13, we may multiply 14 times 3 are 42, set the 2, and multiply 14 times 1 plus 4 to carry are 18.

When the multiplication of the unit figures does not give a product of two figures, the tens place must be filled with a 0.

Multiply the following numbers.

(11)	(12)	(13)	(14)
38	61	223	149
32	69	227	141
1216	4209	50621	21009

(15)	(16)
326	417
324	413
105624	172221

84.   *To multiply two numbers of two or three figures
each, when the hundreds and tens are alike, and the sum of
the units approximate ten, more or less.*

Multiply the following numbers:

OPERATION.

(1)	(2)	(3)	(4)
47	74	123	228
42	78	124	223
2021	5624	15621	50616
47	148	369	228
1974	5772	15252	50844

Explanation. In problems of this kind, we first mentally add
to, or subtract from, the unit figures of the multiplier, such a
number as will make ten when added to the units figure of the
multiplicand; we then proceed as in the problems where the
unit figures add ten and the tens figures are alike, and then we
add to, or subtract from, the product thus obtained, as many
times the multiplicand as we added units to, or subtracted
them from, the units figure.

In he 1st problem, we mentally add 1 to the units figure of
the multiplier, and say 3 times 7=21, then 5 times 4=20; then
subtract one time the multiplicand from this product. In pro-
blem 2, we mentally subtract 2 from 8 and say 6 times 4=24;
then 8 times 7=56; then we add to this product 2 times the
multiplicand.   Problems 3 and 4, were similarly worked.

In practice, the adding to, or subtracting from, the first
obtained product, should be mentally performed.

85.   *To multiply two numbers of two figures each when
the tens figures add ten and the unit figures are alike.*

Thus, multiply the following numbers:

(1)	(2)	(3)
47	84	73
67	24	33
3149	2016	2409

Explanation. In all problems of this kind, first multiply the units figures and set the whole result in the product line, then multiply the tens figures and add to their product one of the units figures, and prefix the result to the product of the units figures. In problem 1. we first multiply 7 times 7=49; then 6 times 4=24 to which we add 7 and obtain 31. Problems 2 and 3, were similarly worked. When the product of the units figures is but one figure, the tens place in the product must be filled with a 0.

86. *To multiply two numbers of two or three figures each when the hundreds and tens, or the units or tens figures only are alike.*

Thus, multiply the following numbers:

(1)	(2)	(3)
54	87	124
34	82	123
───	───	─────
1836	7134	15252

Explanation. In all problems of this kind, first multiply the unit figures and write the unit result in the product line, then add the column of units or tens, that are not alike, or both where three figures are used, and with the sum multiply one of the numbers in the column where they are alike, and add to it the carrying figure; then multiply the figures in the tens or tens and hundreds, and add the carrying figure; the result will be the correct product. In problem 1, we first multiply 4 times 4=16; the 6 we set and carry 1; then 3 plus 5=8, and 8 times 4=32 plus 1 to carry make 33, the 3 we set and carry 3; then 3 times 5=15 plus 3 to carry make 18. In problem 2, we first multiply 2 times 7=14; then 7 plus 2=9 and 9 times 8=72 plus 1 to carry=73; then 8 times 8=64 plus 7 to carry=71. Problem 3 was similarly worked.

87. *To multiply by any number consisting of two figures the unit or ten of which is 1:*

(1.) Multiply 2657 by 31.

OPERATION.

```
 7971
 2657   or   2657 × 31
   31        7971
─────        ─────
82367        82367
```

Explanation. In all problems of this kind, first multiply by the tens figure and set the product under or over the multiplicand, one place to the left and add the two numbers together.

(2.) Multiply 4718 by 19.

OPERATION.

Explanation. In problems where the tens figure is 1, multiply by the unit figure and set the product under or over the multiplicand, one place to right and add.

42462
4718   or   4718 × 19
  19   42462
———   ———
89642   89642

This method of arranging the figures may be used to advantage when the multiplier is any number similar to the following: 601 or 105, 4001 or ₁009.

88.   *To multiply by any number one part of which is a factor or multiple of the other part:*

(1.) Multiply 3246 by 328.

OPERATION.
3246
 328
———
25968
1038720
———
1064688

Explanation. The conditions of this problem are that we take or repeat the 3246, 328 times, and in performing the operation we first multiply by 8, which repeats it 8 times; we yet have it to repeat 320 times, and ₂s 320 is equal to 8, 40 times, therefore, 40 times the product by 8 (25968) is the same as 320 times the multiplicand; hence, to shorten the operation we add to the product by 8, 40 times itself, instead of 320 times the multiplicand. The sum of these two products is the correct product.

(2.) Multiply 7251 by 618.

OPERATION.
7251
 618
———
4350600
130518
———
4481118

Explanation. In this problem we are required to repeat 7251, 618 times, and in performing the operation we first multiply by the 6 hundreds which repeats the 7251, 600 times, and leaves it unrepeated 18 times. To repeat it 18 times more we observe that 18 is equal to 6, 3 times, and having already repeated the 7251, 600 times, we therefore repeat the one hundreth part of the product by the six hundreds 3 times, which is equal to 18 times 7251, and add the same to the product by 6 hundreds. The result is the correct product.

(3.) Multiply 536183 by 27945.

OPERATION.

536183
27945

482564700
14476941000
24128235

14983633935

Explanation. In this problem we observe that the two first figures 45 are equal to the hundreds figure 5 times, and that the thousands and ten thousands figures 27 are equal to the hundred figure 3 times. We therefore, multiply first by the hundreds figure 9, which repeats the multiplicand 900 times. We have yet to repeat it 27045 times, and (using the 27000 first), since 27000 is equal to 9, 3000 times, we therefore repeat the product by 9, 3000 times, instead of the multiplicand 27000 times, which gives us 14476941000; and, since 45 is equal to 9 five times, we therefore repeat the one hundreth part of the product by the nine hundreds, 5 times, instead of repeating the multiplicand 45 times, which gives us 24128235. We then add the several products together, the sum of which is the true product.

Taking the last example, and omitting the naughts, which we always omit in practice, the figures would stand thus :

536183
27945

4825647
14476941
24128235

14983633935

89. *To multiply by the factors of the multiplier when it can be resolved into easy factors, is a saving of time and figures, and should be practiced whenever an opportunity presents itself, thus :*

(1.) Multiply 8421 by 64.

OPERATION.

8421
64

67368

538944 Ans.

Explanation. Instead of multipling by 64 we use the factors 8 and 8 ; first multiplying by 8 we produce a product of 67368, which we multiply by 8, and thus produce the correct result. By this system of work we save one line of figures and the addition of two lines.

90.   *To multiply any number by* 11 *or* 111.

(1) Multiply 62 by 11   *Explanation.* In multiplying any
number by 11 it is evident that we
have but to add the number to its
OPERATION.   decuple, i. e. to ten times the num-
682 Ans.   ber, hence in this and all operations
of two figures we have but to add the two figures together and
place the sum between them.   When the sum of the two figures
is in excess of 9, carry one to the left hand figure, thus, multi-
ply 87 by 11; 8 and 7 added make 15, place the 5 between the
figures and add the 1 to the 8 which gives a product of 957

(2)   Multiply 3456 by 11.

OPERATION.   *Explanation.* In this problem accor-
3456   ding to the principle stated in the first
11   problem, we add the number to its decu-
ple, thus, 0+6=6; 6+5=11: 5+4+1 to
carry=10; 4+3+1 to carry=8, then
38016   bring down the 3, and we have the cor-
rect product.

(3)   Multiply 3456 by 111.

OPERATION.   *Explanation.* In this problem we are
3456   required to repeat 3456, 100 times plus
111   10 times, plus 1 time.   Hence if we add
3456 to 10 times 3456 and 100 times 3456
we will have the correct result.   This
383616   we do by bringing down the first figure
6; and adding as follows : 6 plus 5=11;
6 plus 1 to carry, plus 5 plus 4=16; 5 plus 1 to carry, plus 4
plus 3=13 ; 4 plus 1 to carry, plus 3=8; and then bring down
the 3.   By performing these operations in the usual manner, the
work will appear plain.

PYRAMIDAL MULTIPLICATION.

91. Multiply 13245 by 22434.

```
13245
22434
─────
    0
  990
44980
6629735
225352980
──────────
297138330
```

Explanation. We commence with the left hand figure of the multiplier and the right hand figure of the multiplicand. Thus, 2 times 5 are ten, and place the 0 on the top line and make it the "keystone" of the pyramid, though we set it first. Then 2 times 4 are 8 and 1 to carry make 9, which we set on the second line, one place to the left of the first, then 2 times 2 are 4, which set in the third line one place to the left of the second, and thus we proceed until we get through the multiplicand. We take the second figure of the multiplier and multiply 2 times 5 are 10, and place the 0 on the second line, one place to the right of the keystone column; then 2 time 4 are 8 and 1 to carry make 9, which we place in the keystone column; then 2 times 2 are 4, which we place in the third line, one place to the left. Thus we proceed until we get through with all the multipliers, taking care to begin a new column to the right for each figure of the multiplier.

92. *To multiply together two numbers, both of which are convenient numbers under or above any number of hundreds.*

(1) Multiply 497 by 294.

```
        OPERATION.
              5
(30)     497    3 Complement,
 (9)     294    6 Complement,
         ───
              3
(39)    ───────
       146118
```

Explanation. We first multiply the complements of the two numbers and thus produce 18 which we set in the product line, then we mentally add the complement figures to their respective numbers, and thus produce 500 and 300; when we multiply the significant figure 5 and 3 of each number by the complement of the other, and add the products which gives 39, as shown in the figures in the brackets on the left; this 39 we then subtract from 100 and obtain 61 which we write in the product; then we multiply the 5 and 3 together, subtract 1 and write the remainder 14 in the product which completes the operation. The figures shown within the brackets should be only mentally made. All similar numbers of hundreds and thousands may be multiplied in a similar manner .

(2)   Multiply 704 by 305.

OPERATION.

            704
          · 305
        —————
        214720

Explanation. In this problem we multiply the excess figures 4 and 5, and set the result 20 in the ·product line; then we multiply the hundreds figures of each number by the excess figure of the other, and add the two products, thus 5 times 7=35, 4 times 3=12, 35 and 12=47; this 47 we set in the product line, then we multiply the hundreds figures and obtain 21 which we write in the product and complete the operation. All similar numbers of hundreds and thousands may be similarly worked.

Multiply the following numbers:

(3)	(4)	(5)	(6)
291 (9)	892 (8)	991 (9)	813
295 (5)	596 (4)	892 (8)	206
———	———	———	———
85845	531632	883972	167478

Note.—When the products of the increased hundreds figures by the complements add more than 100, then substract their sum from 200, and then from the product of the increased numbers deduct 2.

93.   *To multiply together two numbers, both of which are convenient numbers under* 100, 1000, *&c.*

(1.)   Multiply 97 by 94.

OPERATION.

    97 (3 Complement of 97)
    94 (6 Complement of 94)
    ——————
    9118

Explanation. In all problems of this kind, first add the two numbers, rejecting the left hand figure; then multiply the sum thus produced by 100 or 1000, according as the numbers approximate 100 or 1000 and add to the product of the complements of the two numbers.

Thus, in this problem we add the 4 and 7, and then the 9 and 9, rejecting the left hand figure; then we mentally multiply by 100 which gives 9100, but without setting the two 0s we add the products of the complements, 3 and 6, and thus produce 9118 the correct result.

The mental multiplication may be omitted if we remember that the product of the complements must fill as many places to the right of the sum produced by adding, as the multiplication by 100, 1000, &c., produced.

(2.) Multiply 991 by 995.

OPERATION.

991 (9 Complement of 991)
995 (5 Complement of 995)
⎯⎯⎯⎯
986045

Explanation. Here we first add 5 and 1; then 9 and 9, of the tens column, then 9 and 9, of the hundreds column, rejecting the left hand figure; then mentally multiplying by 1000 we add the product of the complements (9 and 5) and produce 986045 the correct product.

Multiply the following numbers:

(3)	(4)	(5)	(6)	(7)	(8)
99	89	88	996	988	991
91	92	89	994	997	985
9009	8188	7832	990024	985036	976135

(94.) The squaring of any number of 9s or 3s can be performed in the same manner.

What is the square of the following numbers, 99, 999, 9999, 33, 333, 3333?

OPERATION.

(1)	(2)	(3)	(4)	(5)	(6)
99	999	9999	33	333	3333
99	999	9999	33	333	3333
9801	998001	99980001	9)9801	9)998001	9)99980001
			1089	110889	11108889

Explanation. The square of 3 is one ninth of the square of 9. Hence in squaring 3's we square them as 9's and then divide by 9. From these results we see that in squaring 9's, the product is composed of as many 9's less 1, as there are 9's to square, an 8, as many 0's as there are 9's in the product and a 1. The product of the square of 3's is as many 1's less 1 as there are 3's, a 0 as many 8's as 1's and a 9.

(95.) *To multiply together two numbers both of which are a convenient number over* 100, 1000, *&c.*

(1.) Multiply 108 by 104.

OPERATION.

108 (8 excess)
104 (4 excess)
⎯⎯⎯⎯
11232

Explanation. In all problems of this kind, first cancel the left hand figure in one of the numbers, then add the remaining figures and multiply the sum by 100, 1000, &c., according as the numbers exceed 100, 1000, &c., and to

the product thus produced, add the product of the excesses of the two numbers.

In this problem we first canceled the hundreds figure of the 108, and then added the remaining figures and thus produced 112; then we mentally multiplied the 112 by 100 and thus produced 11200, to which we added the product of the two excess figures, 8 and 4, and obtained 11232 the correct product.

(2)   Multiply 1014 by 1009.

OPERATION.

    1014
    1009
    ———
    1023126

Explanation. Here we first cancel one of the left hand figures and then add and obtain 1023, which we mentally multiply by 1000, and without setting the three 0's, we add the product of the excess numbers 14 and 9, and produce 1023126 the correct product. The mental multiplication may be omitted, if we remember that the product of the excess numbers must occupy as many places to the right of the sum produced by adding, as the multiplication by 100, 1000, &c., would produce.

Multiply the following numbers:

(3)	(4)	(5)	(6)	(7)	(8)
103	114	109	1008	1024	1135
102	112	112	1007	1016	1010
———	———	———	———	———	———
10506	12768	12208	1015056	1040384	1146350

96.   *To multiply together two numbers, one of which is a convenient number over, and the other a convenient number under 100, 1000, &c.*

(1)   Multiply 105 by 93.

OPERATION.

    105   (5 excess.)
    93   (7 complement)
    ———
    9800
    35
    ———
    9765

Explanation. In all problems of this kind, first cancel the left hand figure of the number over 100, 1000, &c., and add the remaining numbers; then multiply their sum by 100, or 1000, &c., according as the numbers approximate 100, 1000, &c., and from the product thus produced, subtract the product of the excess and complement figures.

Multiply the following numbers:

(2)	(3)	(4)
112	123	1013
97	92	996
10900	184	052
36		
10864	11316	1008948

Explanation. In the 3d and 4th problems to shorten the operation we first multiply the excess and complement figures together, set their product in its proper place to the right, and make the subtraction before adding.

97. A PECULIAR PROPERTY OF THE NUMBER 9.

If we multiply the 9 figures in their order, 1 2 3 4 5 6 7 8 9 by 9, or any multiple of 9, not exceeding 9 times 9, the product will be in like figures, except the tens place, which will be a 0. The significant figure of the product will be the number, that the multiplier is equal to 9; thus to multiply by 9, will give a product of 1s; 18, which is 2 times 9, will give a product of 2s, and so on with the other multiples of nine, up to 81.

OPERATIONS.

(1)	(2)	(3)
123456789	123456789	123456789
27	45	72
3333333303	5555555505	8888888808

If we omit the 8 in the multiplicand the product figures will all be the same.

There are many peculiar properties belonging to the figure nine by reason of its being 1 less than the radix of our system of notation, but being mostly of no practical value, we cannot give them space. There is however one property of the nine, that may often be used with advantage by Accountants in the detection of errors in posting or transferring accounts, and we will state it.

98.   *The difference between any number, and the figures composing the number reversed in any manner, is a multiple of* 9.   *Thus the difference between* 6871, *and any transposition or reversing possible to be made with the same figures, will be a multiple of* 9.

EXAMPLES.

6871	6871	6871	6871
1786	6718	6781	8671

9)5085	9) 153	9) 90	9)1800
565	17	10	200

99.   TABLE OF SQUARES AND CUBES.

Numb	Squ're	Cube.	Numb	Squ're	Cub.
1	1	1	51	2601	132651
2	4	8	52	2704	140608
3	9	27	53	2809	148877
4	16	64	54	2916	157464
5	25	125	55	3025	166375
6	36	216	56	3136	175616
7	49	343	57	3249	185193
8	64	512	58	3364	195112
9	81	729	59	3481	205379
10	100	1000	60	3600	216000
11	121	1331	61	3721	226981
12	144	1728	62	3844	238328
13	169	2197	63	3969	250047
14	196	2744	64	4096	262144
15	225	3375	65	4225	274625
16	256	4096	66	4356	287496
17	289	4913	67	4489	300763
18	324	5832	68	4624	314432
19	361	6859	69	4761	328509
20	400	8000	70	4900	343000
21	441	9261	71	5041	357911
22	484	10648	72	5184	373248
23	529	12167	73	5329	389017
24	576	13824	74	5476	405224
25	625	15625	75	5625	421875
26	676	17576	76	5776	438976
27	729	19683	77	5929	456533
28	784	21952	78	6084	474552
29	841	24389	79	6241	493039
30	900	27000	80	6400	512000
31	961	29791	81	6561	531441
32	1024	32768	82	6724	551368
33	1089	35937	83	6889	571787
34	1156	39304	84	7056	592704
35	1225	42875	85	7225	614125
36	1296	46656	86	7396	636056
37	1369	50653	87	7569	658503
38	1444	54872	88	7744	681472
39	1521	59319	89	7921	704969
40	1600	64000	90	8100	729000
41	1681	68921	91	8281	753571
42	1764	74088	92	8464	778688
43	1849	79507	93	8649	804357
44	1936	85184	94	8836	830584
45	2025	91125	95	9025	857375
46	2116	97336	96	9216	884736
47	2209	103823	97	9409	912673
48	2304	110592	98	9604	941192
49	2401	117649	99	9801	970299
50	2500	125000	100	10000	1000000

SQUARING NUMBERS.

The usual methods of squaring numbers on the basis of complements, and supplements, are attended with so many variations that they are of very little practical value. In fact, they are of no value to those who understand the system of simultaneous multiplication, which we present a few pages further on, and hence we shall present but little work of this kind.

100. *To square numbers between 25 and 50.*

(1.) What is the square of 26?

OPERATION.

$26 - 25 = 1;$
$25 - 1 = 24;$
$24^2 = 576;$
$576 + 1^{00} = 676$ Ans.

Explanation. In all problems of this kind mentally subtract 25 from the number to be squared, and the remainder from 25; then square this *second* remainder and add to its hundreds figure the first remainder.

To be proficient in this work, the squares of numbers as high as 25 must be thoroughly learned from the table.

101. *To square numbers between 50 and 75.*

What is the square of 63?

OPERATION.

$63 - 25 = 38$
$63 - 50 = 13$
$13^2 = 169$
$169 + 38^{00} = 3969$ Ans.

Explanation. In all problems of this kind mentally subtract, first 25, and then 50 from the given number; then square the *second* remainder and add to its hundreds figure the first remainder.

102. *To square numbers between 75 and 100.*

What is the square of 89?

OPERATION.

$89 - 75 = 14$
$25 - 14 = 11$
$11^2 = 121$
$121 + (89^{00} - 11^{00}) = 7921$

Explanation. In all problems of this kind subtract 75 from the number and the remainder from 25; then square the *second* remainder and add to its hundreds figure the difference between the number and the second remainder.

103. *To multiply together two numbers on the principle that the square of the mean of two numbers minus the square of one half of the difference, is equal to the product of the two numbers.*

(1.) $68 \times 72 = (70-2) \times (70+2) = 70^2 - 2^2 = 4900 - 4 = 4896$ Ans.

(2.) $81 \times 79 = 80^2 - 1^2 = 6400 - 1 = 6399$ Ans.

(3.) $92 \times 108 = 100^2 - 8^2 = 10000 - 64 = 9936$ Ans.

SIMULTANEOUS INVOLUTION.

104. The following is an entirely new system of squaring numbers, which possesses many advantages over all others; is easily comprehended, and applicable to all numbers.

1. What is the square of 61345? Ans. 3763209025.

OPERATION.

61345
———
3763209025

Explanation. To square any number, by this method, we have only to observe that the successive figures of the product, are found by involving those orders that produce the same numerical result; remembering to mentally double the factor of one order, before multiplying by the other, but not to double the square of any order. Thus: the units squared, give units; the units by twice the tens, give tens; the units by twice the hundreds, plus the square of tens, give hundreds; the units by twice the thousands, plus the tens by twice the hundreds, give thousands, &c. After the highest order has been involved with the units, one figure from the right is canceled for each successive figure of the product.

In this problem we have

1st—$5^2 =$	$= 25$
2nd—$(2 \times 4) \times 5 + 2$ to carry	$= 42$
3rd—$(2 \times 3) \times 5 + 4^2 + 4$ to carry	$= 50$
4th—$(2 \times 1) \times 5 + 4 \times [2 \times 3] + 5$ to carry	$= 39$
5th—$(2 \times 6) \times 5 + 4 \times [2 \times 1] + 3^2 + 3$ to carry	$= 80$
6th—$(2 \times 6) \times 4 + 3 \times [2 \times 1] + 8$ to carry	$= 62$
7th—$(2 \times 6) \times 3 + 1^2 + 6$ to carry	$= 43$
8th—$(2 \times 6) \times 1 + 4$ to carry	$= 16$
9th—$6^2 + 1$ to carry	$= 37$

107. CONTRACTIONS IN DIVISION.

(1.)   Divide 9746521 by 634.

OPERATION.

634)9746521(15373$\frac{29}{634}$ Ans.

3406

2365

4632

1941

39

Explanation. The opera-
tion of this example is
abridged by performing
mentally, the subtraction
of the product of the divi-
sor by the quotient figure,
instead of placing it under
the dividend and then sub-
tracting, as is ordinarily
done.

Or, to arrange the figures differently, which we much
prefer, as follows:

3241
4369
0634
634)9746521

15373$\frac{29}{634}$ Ans.

In this and the following
example the subtraction is per-
formed mentally, as in the exam-
ple above, but the remainders are
placed in a vertical position over
the dividend, so as to avoid
repeating the figures of the dividend.

CONTRACTIONS.

108. Peculiar problems when the divisor is an aliquot part of 10, 100, or 1000, or some convenient number of 10, 100, or 1000. Thus, to divide by

	multiply the dividend by		and divide the product by
1⅑	"	9	10
1¼	"	8	10
1³⁄₇	"	7	10
1⅔	"	6	10
2½	"	4	10
3⅓	"	3	10
6¼	"	16	100
8⅓	"	12	100
12½	"	8	100
14²⁄₇	"	7	100
16⅔	"	6	100
18¾	"	16	300
22½	"	4	90
25	"	4	100
31¼	"	16	500
33⅓	"	3	100
37½	"	8	300
62½	"	8	500
66⅔	"	3	200
75	"	4	300
83⅓	"	6	500
87½	"	8	700
88⁸⁄₉	"	9	800
125	"	8	1000
133⅓	"	3	400
166⅔	"	6	1000
225	"	4	900
250	"	4	1000
333⅓	"	3	1000
375	"	8	3000
625	"	8	5000
833⅓	"	6	5000
875	"	8	7000
15	"	2	30
35	"	2	70
45	"	2	90

The reasons for these contractions are based upon the fact that in all division operations the result or quotient is not changed by multiplying the divisor and the dividend by the same number.

To elucidate the work we present the following examples:

(3.)   Divide 7242 by 2½

OPERATION.

$2\tfrac{1}{2}$ | 7242
4 | 4
———
10 | 2896.8 Ans.

Explanation. By inspection we observe that 4 times 2½ are 10; we therefore, to save time and labor, first multiply it by 4 to produce a new and more convenient divisor, and in order not to effect a change or error in the quotient, we also multiply the dividend by 4 for a new dividend, and then divide in the regular manner which in this case is done by simply pointing off one figure.

(4.)   Divide 179801 by 33⅓.

OPERATION.

$33\tfrac{1}{3}$)179801
————
5394.03 Ans.

Explanation. In this example we also see by inspection that 3 times the divisor make 100, hence for the reasons given above, we multiply the dividend by 3, and divide the product by 100. In practice we do not set the figure (3 in this example) by which we multiply the dividend.

(5.)   Divide 12358 by 37½

OPERATIONS.

$37\tfrac{1}{2}$)12358
————
300)98864
————
329$\tfrac{44}{300}$ Ans.

Explanation. We here observe by inspection that the divisor 37½ is ⅛ of a hundred, and hence if we multiply it by 8 we will have for a new divisor 300; therefore, for reasons given above, we multiply the divisor and the dividend by 8, and then divide in the usual manner.

(6.)   Divide 97450 by 75.

OPERATION.

75)97450
————
300)389800
————
1299⅓ Ans.

Explanation. By inspection we here observe that 4 times the divisor make 300; we therefore multiply the divisor and dividend by 4 and with their products proceed to divide.

(7.)   Divide 4307491 by 125.

OPERATION.

125) 4307491
_____

1000) 3445992̸8̸=1̸1̸4̸ Ans.

Explanation In this ex-
amples we observe that 8
times the divisor make
1000; we therefore pro-
ceed in the operation as in
the above examples, and for reasons therein given the student
should remember that in practice the multiplication of the
divisor should always be mentally performed.

PECULIAR CONTRACTIONS.

In the preceding contractions we based our work upon
the principle that multiplying the divisor and dividend by
the same number does not effect a change in the quotient.

109. In the following contractions we base our work
upon the principle that, to add to, or subtract from both
divisor and quotient, the same aliquot parts, will effect no
change in the result.

By the application of these principles we can often
increase or diminish either divisor or dividend, or both, by
aliquot parts, and thus obtain a more convenient divisor.

(8.)   Divide 426 by 7½.

OPERATION.

7½) 42.6
    14.2
_____

56.8=4/5 Ans.

Explanation. In this example we
first divide by 10, and then add ⅓
of the quotient to itself. This we
do because by inspection we ob-
serve that 7½ is ¾ of ten, or, that it
is ⅓ of itself less than 10; hence,
according to the above principles, and by the exercise of our
perceptive and reasoning faculties, we see that if we divide by
10, our quotient will be ⅓ of itself too small, and to obtain the
correct quotient we must therefore add ⅓ of the quotient by 10
to itself.

To add to, or deduct from, the dividend before dividing,
will produce the same result as adding to, or deducting
from, the quotient.

(9)   Divide 2548230 by 24.

OPERATION.

$\frac{1}{4}$ of 24 | 2548230
is   6
—
30 | 84941
212135¼

106176⅛ Ans.

Explanation. In this example we first divide by 30 and then add ¼ of the quotient to itself. The reason for the work is the same as that given in the above example.

(10.)   Divide 489723 by 54.

OPERATION.

54 | 489723
—
60 | 8162 $\frac{7}{10}$
906 $\frac{81}{180}$

9068 $\frac{179}{180}$ Ans.

Explanation. We here first divide by 60, which is $\frac{1}{9}$ of the true divisor too large; hence our quotient is $\frac{1}{9}$ of itself too small. We therefore add $\frac{1}{9}$ of the quotient by 60 to itself, and obtain the correct result.

In this example, by reason of the fractions, the operation is no shorter than our first system of contraction, or than dividing by the factors, would make it. The principle, however, is good and can often be applied with advantage.

(11.)   Divide 58042 by 35.

OPERATION.

35) 58042

11608 $\frac{2}{5}$

16581 $\frac{11}{15}$

Explanation. In this example our question is, how many times is 58042 equal to 35, and in the solution of the same, in order to lessen the work, we first divide by 5, which gives us a quotient 7 times too large, for the reason that 5 is but $\frac{1}{7}$ of 35, the true divisor. To produce the correct result we have therefore to divide the quotient by 5, by 7, which is performed as follows: 11 is equal to 7, 1 time and 4 remainder; 46 is equal to 7, 6 times and 4 remainder; 40 is equal to 7, 5 times and 5 remainder; 58 is equal to 7, 8 times and 2 remainder, which is reduced to fifths, equalling $\frac{10}{15}$, which added to the $\frac{2}{5}$ in the first quotient make $\frac{11}{15}$ which divided by 7 equals $\frac{11}{15}$.

MENTAL CONTRACTIONS IN MULTIPLICATION OF FRACTIONS.

118. In questions of multiplication of fractions, where the factors are small, the operation may be performed mentally, or with but few memorandum figures, without reducing or making the statement as above, and the result produced almost instantly.

(1.) Multiply 12¼ by 8¼.

OPERATION.

12¼
8¼
———
103¼ Ans.

Explanation. In this problem, by inspection and reason we see that 12¼ is to be repeated 8¼ times, and in performing the operation we first repeat 12, 8 times, which gives us 96; this we retain in the mind, and repeat the 12, ¼ of a time, which gives us 3; this we mentally add to the 96; and obtain 99; we then repeat the ¼, 8 times, (which is done by dividing the 8 by the 2): this gives us 4, which we mentally add to the 99 and obtain 103, which we set in the product line. This work multiplies the 12 only by 8¼, and the ¼ by 8; the ¼ remains to be multiplied by ¼, which we do and produce ¼, which affix'd to the 103 gives as the result of the problem 103¼.

(2.) What will 12¾ pounds cost at 9¾¢ per pound?

OPERATION.

9¾
12¾
———
$1.23¼ Ans.

Explanation. Here we have 9¾ to repeat 12¾ times. We first repeat the 9, 12 times, which gives us 108; this we retain in the mind and repeat the 9, ¾ of a time, which gives us 6; this we add mentally to the 108, and obtain 114; we then repeat the ¾, 12 times, which gives us 9; this we add to the 114 and obtain 123, which we set in the product line. We then multiply the ¾ by ¾, which gives us ₁₆⁶; this reduced gives us ⅜, which we annex to the 123 and obtain the correct result of the problem.

(3.) Multiply 13¼ by 8½.

OPERTION.

13¼ ¼
8½

116⅞ Ans.

Explanation. Here we first multiply the 13 by 8, which gives us 104, which we retain in the mind; then by ½, which gives us 6½; the 6 we add to the 104, and obtain 110, the ½ we set to the right of the ¼; then we multiply the ¼ by 8, which gives us 6, which we add to the 110 and obtain 116, which we set in the product line. We then multiply the ¼ by ½, which gives us ⅛ to which we add the ½ obtained by multiplying 13 by ½, which gives us ⅞; this we annex to the 116 and obtain the correct result.

(4.) Multiply 11¼ by 8½.

OPERATION.

11¼ ⅓
8½ ¼

96¾ Ans.

Explanation. Here we first multiply 11 by 8, and produce 88; we then multiply 11 by ½, and obtain 5½; the 5 we add to the 88, making 93, the ½ we set to the right of the ¼; we then multiply ⅓ by 8, and obtain 2⅔; the 2 we add to the 93, and obtain 95, the ⅔ we set to the right of the ½; we then multiply ¼ by ½ and produce ⅛, to which we add ⅓, making ⅔, to which we add ⅔ and obtain 1¼, which we add to the 95 and obtain the correct result

EXAMPLES.

(5.) Multiply 12½ by 4¼. Ans. 53¼.

(6.) Multiply 9¼ by 6¾. Ans. 61¾.

(7.) Multiply 10⅔ by 15¼. Ans. 160¼⅜.

PECULIAR CONTRACTIONS OF MULTIPLICATION OF FRACTIONS.

The preceding problems, and the explanations given, fully illustrate the work, or the general principles of contraction of fractions, without regard to any peculiar combination of numbers.

The principles upon which the work is based are so few and simple, the operations so short and easily performed, and the practical advantages of the work so great, that we specially commend it to the earnest attention of all classes of commercial men and accountants.

In the following methods of contraction, the process depends somewhat on the peculiar combination of numbers, and hence, although shorter than the first contractions, less valuable.

119.   *To multiply numbers of two or more figures each, when one or more of the right hand figures considered as tens, hundreds, &c., may be reduced to fractions of halves, fourths, or eighths.*

(1.)   Multiply 425 by 875.

OPERATION.

4¼ hundreds.
8¾ hundreds.
———
371875   Ans.

Explanation. In this problem we reduce the units and tens figures of both the multiplicand and multiplier to fractions of respectively ¼ and ¾ and thus obtain 4¼ and 8¾ to be multiplied together, which operation we perform according to the work elucidated in problem 1 page 87 and obtain a product of $37\frac{3}{16}$. Then we reduce the fractional part, $\frac{3}{16}$, of this product, to a whole number, and annex it to the integral part of the product; $\frac{3}{16} = 1875$, (see table on page 23), which annexed to 37 gives 371875, the correct product.

This $\frac{3}{16}$ is reduced to a whole number for the reason that it represents $\frac{3}{16}$ of 10000; and it represents this for the reason that we first reduced four figures, two in each factor, to fractions.

(2.)   Multiply 850 by 450.

OPERATION.	
8½	*Explanation.* By reducing the units and tens figures of both numbers as explained in the preceding problem, we have 8½ and 4½ to be multiplied together. The product of 8½×4½ is 38¼.
4½	

382500 Ans.

This ¼ represents, for reasons given in the preceding problem, ¼ of 10000, and hence we reduce it to a whole number and annex the same to the 38. ¼ of 10000=2500, which annexed gives 382500, the correct product.

(3.)   What will 2812½ gallons cost at $4.50 per gallon.

OPERTAION.	
28½	*Explanation.* The reason for the work of this problem is the same as elucidated in the two preceding, and hence omitted.
4½	

$12656.25   Ans.

(4)   What will 1650 bbls. Pork cost at $13.75 per bbl.

OPERATION.	
1⅜	*Explanation.* In this problem we use the units, tens and hundreds figures of the price as ⅜ and the units and tens figures of the barrels as ½, and thus produce a product of 22 11/16 ; the 11/16 we reduce and annex as above directed and obtain $22687.50.
16½	

$22687.50

Multiply the following numbers together.

5.   12250 by 4750.
6.   24250 by 8125.
7.   3456 by 2125.
8.   379 by 425.

OPERATIONS.

(5)	(6)	(7)	(8)
12¼	24¼	3456	379
4¾	8⅛	2⅛	4¼
58187500 A.	197031250 A.	6912	1516
		432	94½
		7344000 A.	161075 A.

51. TABLE V.

TABLE OF SIXTEENTHS AND THEIR EQUIVALENTS IN DECIMALS.

$\frac{1}{16}$ = .0625	$\frac{9}{16}$ = .5625
$\frac{2}{16}=\frac{1}{8}$ = .125	$\frac{10}{16}=\frac{5}{8}$ = .625
$\frac{3}{16}$ = .1875	$\frac{11}{16}$ = .6875
$\frac{4}{16}=\frac{1}{4}$ = .25	$\frac{12}{16}=\frac{3}{4}$ = .75
$\frac{5}{16}$ = .3125	$\frac{13}{16}$ = .8125
$\frac{6}{16}=\frac{3}{8}$ = .375	$\frac{14}{16}=\frac{7}{8}$ = .875
$\frac{7}{16}$ = .4375	$\frac{15}{16}$ = .9375
$\frac{8}{16}=\frac{1}{2}$ = .5	$\frac{16}{16}$ = 1.

52. TABLE VI.

TABLE SHOWING THE PRICE OF A SINGLE ARTICLE WHEN THE PRICE PER DOZEN IS GIVEN.

PER DOZ.	PER PIECE.	PER DOZ.	PER PIECE.
25¢ =	$2\frac{1}{12}$¢	$7.00 =	$58\frac{1}{3}$¢
50¢ =	$4\frac{1}{6}$¢	8.00 =	$66\frac{2}{3}$¢
75¢ =	$6\frac{1}{4}$¢	9.00 =	75¢
$1.00 =	$8\frac{1}{3}$¢	10.00 =	$83\frac{1}{3}$¢
2.00 =	$16\frac{2}{3}$¢	11.00 =	$91\frac{2}{3}$¢
3.00 =	25¢	12.00 =	$1.
4.00 =	$33\frac{1}{3}$¢		
5.00 =	$41\frac{2}{3}$¢		
6.00 =	50¢		

120. *To square numbers the sum of whose fractions add one.*

(1.) Multiply 9½ by 9½.

OPERATION.

$$9\tfrac{1}{2}$$
$$9\tfrac{1}{2}$$
___
90¼ Ans.

Explanation. In this example we add 1 to 9, making it 10.; then we multiply 10 times 9 are 90, and ½ of ½ is ¼, and thus produce the correct result. We do this because one-half of 9 taken twice and added to its square, is the same as to multiply the 9 by 1 more than itself. This principle and process of work are applicable to the multiplying of any two like numbers whose fractions add *unity* or 1.

EXAMPLES.

2. What will 12½ pounds cost at 12½¢ per pound?
Ans. $1.56¼.

3. What will 6¼ pounds cost at 6¾¢ per pound?
Ans. 42³⁄₁₆¢.

4. What will 8¾ pounds cost at 8½¢ per pound?
Ans. 72¾¢.

5. What will 19⅝ pounds cost at 19⅝¢ per pound?
Ans. $3.80¹¹⁄₆₄.

121. *To multiply any two numbers, the difference of which is* 1, *and the sum of whose fractions is* 1.

(1.) Multiply 5¼ by 4¾.

OPERATION.

$$5\tfrac{1}{4}$$
$$4\tfrac{3}{4}$$
___
24¹⁵⁄₁₆ Ans.

Explanation. In all problems of this kind, we add 1 to the *larger* number, and then multiply the sum by the *lesser* number, and set the result in the product line; then we square the fraction of the *larger* number and subtract the result from 1, and annex the remainder to the whole numbers in the product. In this example we added 1 to 5, which made 6; this we multiplied by 4, and produced 24; we then squared ¼, which gave us ¹⁄₁₆; we then subtracted the ¹⁄₁₆ from 1, and obtained ¹⁵⁄₁₆, which we annexed to the 24 and completed the correct result.

EXAMPLES.

2. What will 8¼ pounds cost at 7¾⊄ per pound?
Ans. 63¼⊄.

3. What will 10⅝ pounds cost at 9¼⊄ per pound?
Ans. 99$\frac{7}{16}$⊄.

4. What will 19⅜ pounds cost at 18⅝⊄ per pound?
Ans. 3.60\frac{55}{64}$⊄.

122. *To multiply any two like numbers, whose frac-tions have like denominators, or whose denominators may be easily reduced to the same denomination.*

(1.) Multiply 8¼ by 8¾.

OPERATION.

8¼
8¾
———
72$\frac{3}{16}$ Ans.

Explanation. In all problems of this kind, we add to the *multiplicand* the *fraction* of the *multiplier*, and then multiply by the *whole* number of the multi-plier and set the result in the product line; then we multiply the fractions together and annex the result to the whole numbers, and obtain the answer. In this problem, we added ¾ to 8¼ which gave us 9, which we multiplied by 8 and obtained 72; we then multiplied the frac-ions and obtained $\frac{3}{16}$. The reasons given in the first problem, under the head of contractions, cover all these special cases, and hence we here only give the directions for performing the operation.

EXAMPLES.

2. What will 4¾ yards cost at 4¼⊄ per yard?
Ans. 20$\frac{3}{16}$⊄.

3. What will 9½ yards cost at 9¾⊄ per yard?
Ans. 92⅝⊄.

4. What will 12⅞ yards cost at 12½⊄ per yard?
Ans. 1.60\frac{11}{16}$.

123. *To multiply any two numbers whose fractions are* ⅓, ⅔, ⅓, *etc.*

(1.)   Multiply 7½ by 5½.

OPERATION.

7½
5½
——
41¼ Ans.

Explanation. Here we multiply 7 by 5 and produce 35, to which we mentally add ½ of the sum of 7 and 5, and obtain 41, which we set in the product line; then we multiply the fractions together and obtain ¼, which we annex to the 41 and obtain the correct result. The reason for adding ½ the sum of 7 and 5 is, because ½ of either number added to ½ of the other is the same as ½ of their sum. Were the fractions ⅓, ¼, ½, etc., we would add ⅓, ¼, ½, etc., of the sum.

EXAMPLES.

2.   What will 9½ pounds cost at 5½¢ per pound?
Ans. 49⅞¢.

3.   What will 14¼ pounds cost at 12¼¢ per pound?
Ans. $1.74₂/₁₆.

4.   What will 31½ pounds cost at 11½¢ per pound?
Ans. $3.46¹¹/₁₄.

124.   *To multiply any two numbers together, whose fractions are of the same value.*

(1.)   Multiply 8⅔ by 4⅔,

OPERATION.

8⅔
4⅔
——
40⅘ Ans.

Explanation. In all problems of this kind, we add to the product of the whole numbers the product of their sum by either fraction, then annex the product of the fractions to the whole numbers. Should there be a fraction in multiplying the whole numbers by the fraction, it must be reserved and added to the product of the fractions.

EXAMPLES.

2.   What will 9¾ pounds cost at 11¾¢ per pound?
Ans. $1.14₉/₁₆.

3.   What will 15½ pounds cost at 10½¢ per pound?
Ans. $1.62¾.

4.   What will 40⅝ pounds cost at 22⅝¢ per pound?
Ans. $9.19₉/₆₄.

PRACTICAL APPROXIMATIVE CONTRACTIONS.

125. *To multiply any fractional numbers to the nearest unit.*

(1.) Multiply 9½ by 8¼.

OPERATION.

    9½
    8¼
    ——
    77 Ans.

Explanation. In this practical system of contraction, we first multiply the whole numbers, and retain in our mind the product, to which we mentally add, first the product, to the nearest unit, of the multiplicand by the fraction of the multiplier, and then the product, to the nearest unit, of the multiplier by the fraction of the multiplicand, and set the result in the product line, the same being the practical answer.

In this example, we see that the product of 9 and 8 is 72, that the product of 9 by the ¼ to the nearest unit is 2, that the product of the 8 by the ½ to the nearest unit is 3, and that the sum of the three products is 77.

(2.) Multiply 10½ by 7¼.

OPERATION.

    10½
     7¼
    ——
    73 Ans.

Explanation. Here we see that the product of the whole numbers is 70, that ½ of 10 to the nearest unit is 1, which added to the 70 makes 71, then that the ¼ of 7 to the nearest unit is 2, which added to the 71 makes 73, the practical answer; 73¹⁄₁₂ is the exact result.

(3.) What will 15¾ pounds cost at 13⅓₵ per pound?

OPERATION.

    15¾
    13⅓
    ——
    $2.09 Ans.

Explanation. Here the product of 15 and 13 is 195, and ⅓ of 15 to the nearest unit is 4, which added gives 199; and ¾ of 13 to the nearest unit is 10, which added to the 199 gives 209 as the practical result. The exact result is 208¹¹⁄₁₂.

(4.)   What will 25¾ yards cost at 17¾¢ per yard?

OPERATION.

25¾
17¾
———
$4.55   Ans.

Explanation. Here the product of 25 and 17 is 425; the product of 25 by ¾, to the nearest unit is 19, which added to 425 gives 444; the product of 17 by ¾, to the nearest unit is 11, which added to the 444, gives the practical answer, 455. The exact result is 455$\frac{7}{16}$.

In multiplying the 25 by ¾, we gained ¼¢, and in multiplying the 17 by ¾, we lost ⅓¢; this $\frac{1}{12}$¢ excess of loss, and the loss of ¼¢ by reason of not multiplying the fractions, account for the $\frac{7}{16}$¢ deficit in the answer.   In practice, it is easy to see whether the loss or gain by reason of the contractions exceeds a unit, and if so to increase or decrease the result accordingly.

With large numbers, the regular system as first presented under the head of Multiplication of Fractions, is preferable to this approximative method,

EXAMPLES.

7.   Multiply 9½ by 6¼.   Ans. 59⅜.
8.   What will 11⅜ yards cost at 12½¢ per yard?
                                                                     Ans. 1.42\frac{2}{16}$.
9.   What will 21¾ yards cost at 16½¢ per yard?
                                                                     Ans. $3.58¾.

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